A Generalized Computational Method for Multi-Ordered Geometric Series

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Abstract: This paper presents a generalized method for the summation of multi-ordered geometric series that is used for the application of computational science. Computational science is a rapidly growing interdisciplinary area where science, computation, mathematics, and its collaboration use advanced computing capabilities for understanding and solving the most complex real-life problems. Also, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. For this purpose, a new generalized method for the computation of multi-ordered geometric series is introduced in this article.

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1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new geometric series [1-10] is constructed for application [11] of computational science and engineering.

2. Generalized Computational Method

In this section, a new theorem is established on the computation of multi-ordered geometric series, which is used for applications of mathematical and computational sciences.

Theorem 2.1: Let $W = \{0,1,2,3,\cdots\}, k\&p_i \in W, 1 \le k \le n,$ and $p_0 = 1$.

$$\left(\sum_{j_k=0}^{p_k-1} \mathrm{x}^{j_k\left(\prod_{i=0}^{k-1} p_i\right)}\right)\left(\sum_{j_{k+1}=0}^{p_{k+1}-1} x^{j_{k+1}\left(\prod_{i=1}^{k} p_i\right)}\right)\left(\sum_{j_{k+2}=0}^{p_{k+2}-1} x^{j_{k+2}\left(\prod_{i=1}^{k+1} p_i\right)}\right)\cdots\left(\sum_{j_{n-1}=0}^{p_{n-1}-1} x^{j_{n-1}\left(\prod_{i=1}^{n-2} p_i\right)}\right)$$
$$\left(\sum_{j_n=0}^{p_n-1} x^{j_n\left(\prod_{i=1}^{n-1} p_i\right)}\right) = \sum_{j=0}^{\prod_{i=k}^{n} p_i-1} x^{j\left(\prod_{i=0}^{k-1} p_i\right)} = \frac{x^{\prod_{i=1}^{n} p_i}-1}{x^{\prod_{i=0}^{k-1} p_i}-1}, x \ne 1 \text{ } or \text{ } p_i \ne 0 \text{ for } x^{p_i}.$$

Proof.

$$\left(\sum_{j_k=0}^{p_k-1} \mathrm{x}^{j_k\left(\prod_{i=0}^{k-1} p_i\right)}\right)\left(\sum_{j_{k+1}=0}^{p_{k+1}-1} x^{j_{k+1}\left(\prod_{i=1}^{k} p_i\right)}\right)\left(\sum_{j_{k+2}=0}^{p_{k+2}-1} x^{j_{k+2}\left(\prod_{i=1}^{k+1} p_i\right)}\right)\cdots\left(\sum_{j_{n-1}=0}^{p_{n-1}-1} x^{j_{n-1}\left(\prod_{i=1}^{n-2} p_i\right)}\right)$$
$$\left(\sum_{j_n=0}^{p_n-1} x^{j_n\left(\prod_{i=1}^{n-1} p_i\right)}\right) = \left(\frac{x^{\prod_{i=1}^{k} p_i}-1}{x^{\prod_{i=0}^{k-1} p_i}-1}\right)\left(\frac{x^{\prod_{i=1}^{k+1} p_i}-1}{x^{\prod_{i=1}^{k} p_i}-1}\right)\left(\frac{x^{\prod_{i=1}^{k+2} p_i}-1}{x^{\prod_{i=1}^{k+1} p_i}-1}\right)\cdots\left(\frac{x^{\prod_{i=1}^{n-1} p_i}-1}{x^{\prod_{i=1}^{n-2} p_i}-1}\right)$$
$$\left(\frac{x^{\prod_{i=1}^{n} p_i}-1}{x^{\prod_{i=1}^{n-1} p_i}-1}\right) = \frac{x^{\prod_{i=1}^{n} p_i}-1}{x^{\prod_{i=0}^{k-1} p_i}-1}, x \ne 1 \text{ } or \text{ } p_i \ne 0 \text{ for } x^{p_i}. \quad (1)$$

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$$\sum_{j=0}^{\prod_{i=k}^{n} p_i - 1} x^{j\left(\prod_{i=0}^{k-1} p_i\right)} = \frac{x^{\prod_{i=1}^{n} p_i} - 1}{x^{\prod_{i=0}^{k-1} p_i} - 1}, x \neq 1 \; or \; p_i \neq 0 \text{ for } x^{p_i}. \qquad (2)$$

From the series (1) and (2), we conclude that

$$\left(\sum_{j_k=0}^{p_k-1} x^{j_k\left(\prod_{i=0}^{k-1} p_i\right)}\right)\left(\sum_{j_{k+1}=0}^{p_{k+1}-1} x^{j_{k+1}\left(\prod_{i=1}^{k} p_i\right)}\right)\left(\sum_{j_{k+2}=0}^{p_{k+2}-1} x^{j_{k+2}\left(\prod_{i=1}^{k+1} p_i\right)}\right)\cdots\left(\sum_{j_{n-1}=0}^{p_{n-1}-1} x^{j_{n-1}\left(\prod_{i=1}^{n-2} p_i\right)}\right)$$

$$\left(\sum_{j_n=0}^{p_n-1} x^{j_n\left(\prod_{i=1}^{n-1} p_i\right)}\right) = \sum_{j=0}^{\prod_{i=k}^{n} p_i - 1} x^{j\left(\prod_{i=0}^{k-1} p_i\right)} = \frac{x^{\prod_{i=1}^{n} p_i} - 1}{x^{\prod_{i=0}^{k-1} p_i} - 1}, x \neq 1 \; or \; p_i \neq 0 \text{ for } x^{p_i}.$$

Hence, theorem is proved.

Now, let us find the summation of geometric series by substituting $k = 1, 2, 3, \cdots, n$ in the series given in the theorem 2.1.

Let $k = 1$. Then we obtain

$$\left(\sum_{j_1=0}^{p_1-1} x^{j_1}\right)\left(\sum_{j_2=0}^{p_2-1} x^{j_2 p_1}\right)\cdots\left(\sum_{j_n=0}^{p_n-1} x^{j_n p_1 p_2 \cdots p_{n-1}}\right) = \sum_{j=0}^{p_1 p_2 \cdots p_n - 1} x^j = \frac{x^{p_1 p_2 \cdots p_n} - 1}{x - 1}, x \neq 1.$$

Let $k = 2$. Then we get

$$\left(\sum_{j_2=0}^{p_2-1} x^{j_2 p_1}\right)\left(\sum_{j_3=0}^{p_3-1} x^{j_3 p_1 p_2}\right)\cdots\left(\sum_{j_n=0}^{p_n-1} x^{j_n p_1 p_2 \cdots p_{n-1}}\right) = \sum_{j=0}^{p_2 \cdots p_n - 1} x^{j p_1} = \frac{x^{p_1 p_2 \cdots p_n} - 1}{x^{p_1} - 1},$$

$p_1 \neq 0$.

Let $k = 1 \; and \; n = 1$. Then we obtain the following sum of geometric series.

$$\sum_{j_1=0}^{p_1-1} x^{j_1} = \sum_{j=0}^{p_1-1} x^j = \frac{x^{p_1} - 1}{x - 1}, x \neq 1.$$

Let $k = 1 \; and \; n = 1$. Then we obtain the following sum of geometric series.

$$\left(\sum_{j_1=0}^{p_1-1} x^{j_1}\right)\left(\sum_{j_2=0}^{p_2-1} x^{j_2 p_1}\right) = \sum_{j=0}^{p_1 p_2 - 1} x^j = \frac{x^{p_1 p_2} - 1}{x - 1}, x \neq 1.$$

Similarly, we can find the sum of geometric series by substituting the different values of $k$and n, $(1 \leq k \leq n)$, in the series given in the theorem 2.1.

3. Conclusion

This article provides the novel geometric series and its summations for mathematical and computational applications. Also, this idea can enable the scientific researchers for further involvement in research and development.

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