Geometric Progression-Based Binomial Series for Computing Application

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Abstract: This paper presents a computational technique for forming the binomial series using geometric series. Computational science is a rapidly growing interdisciplinary area where science, computation, mathematics, and its collaboration use advanced computing capabilities for understanding and solving the most complex real-life problems. Also, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. For this purpose, the geometric progression-based binomial series is introduced in this article.

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1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new geometric series [1-9] is constructed for application [10] of computational science and engineering.

2. Theorem on Geometric Series

The author of this article has already introduced the following geometric series in his previous papers [1,2].

$$\sum_{i=0}^{n} x^{pi} = \frac{x^{p(n+1)} - 1}{x^p - 1}, p \neq 0. \quad (1)$$

Now, let us construct the novel summations of multiple geometric series using (1).

Theorem 2.1: $\left(\sum_{\mathrm{i}=0}^{\mathrm{p}-1} \mathrm{x}^{\mathrm{i}}\right)\left(\sum_{j=0}^{n} x^{pj}\right) = \sum_{k=0}^{p(n+1)-1} x^k = \frac{x^{p(n+1)} - 1}{x - 1}, x \neq 1.$

Proof.

$$\left(\sum_{\mathrm{i}=0}^{\mathrm{p}-1} \mathrm{x}^{\mathrm{i}}\right)\left(\sum_{j=0}^{n} x^{pj}\right) = \left(\frac{x^p - 1}{x - 1}\right)\left(\frac{x^{p(n+1)} - 1}{x^p - 1}\right) = \frac{x^{p(n+1)} - 1}{x - 1}, x \neq 1. \quad (2)$$

$$\sum_{k=0}^{p(n+1)-1} x^k = \frac{x^{p(n+1)} - 1}{x - 1}, x \neq 1. \quad (3)$$

From (2) and (3), we conclude that

$$\therefore \left(\sum_{\mathrm{i}=0}^{\mathrm{p}-1} \mathrm{x}^{\mathrm{i}}\right)\left(\sum_{j=0}^{n} x^{pj}\right) = \sum_{k=0}^{p(n+1)-1} x^k = \frac{x^{p(n+1)}-1}{x-1}, x \neq 1. \quad (4)$$

Hence theorem is proved.

3. Binomial Series

In this section, the geometric progression-based binomial series is introduced for mathematical and computational applications.

Theorem 3.1: $(x+y-1)\sum_{i=0}^{n-1}(x+y)^i + 1 = (x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k$

Proof.

We know that the sum of the geometric series is: $\sum_{j=0}^{n-1} z^i = \frac{z^n-1}{z-1}$ (5)

Substituting $z = x + y$ in the geometric series (5), we obtain

$$\sum_{i=0}^{n-1}(x+y)^i = \frac{(x+y)^n-1}{x+y-1} \Rightarrow (x+y-1)\sum_{i=0}^{n-1}(x+y)^i + 1 = (x+y)^n \quad (6)$$

The binary series is given below:

$$(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k} \quad (7)$$

From (6) and (7), we conclude that

$$\therefore (x+y-1)\sum_{i=0}^{n-1}(x+y)^i + 1 = (x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k$$

Hence, theorem is proved.

4. Conclusion

This article provides the geometric progression-based binomial series for mathematical and computational applications. Also, this idea can enable the scientific researchers for further involvement in research and development.

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