Computation of Algebraic Expressions and Geometric Series with Radicals

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Abstract: Computational science is a rapidly growing interdisciplinary area where science, computation, mathematics, and its collaboration use advanced computing capabilities for understanding and solving the most complex real-life problems. Also, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. For this purpose, algebraic expressions and geometric series of radicals are introduced in this article.

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1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management, and its applications. In this article, a new kind of geometric series [1-10] is formulated for the application [11] of mathematical and computational sciences.

2. Geometric Series and Radicals

$\sqrt[q]{x^p}$ is a radical, where x is any number, q is a natural number, and p is an integer.

$\sqrt[q]{x^p} = x^{\frac{p}{q}}$, where $\frac{p}{q}$ is the exponent of x.

Theorem 2.1: $\sum_{i=k}^{p-1} \sqrt[q]{x^i} = \frac{\sqrt[q]{x^p} - \sqrt[q]{x^k}}{\sqrt[q]{x} - 1}, x \neq 1.$

Proof. Let's prove this theorem using the following algebraic expression.

(i). $(\delta - 1)(\delta^k) = \delta^{k+1} - \delta^k$, where k is a positive integer.

(ii). $(\delta - 1)(\delta^k + \delta^{k+1}) = \delta^{k+2} - \delta^k$

(iii). $(\delta - 1)(\delta^k + \delta^{k+1} + \delta^{k+2}) = \delta^{k+3} - \delta^k$

(iv). $(\delta - 1)(\delta^k + \delta^{k+1} + \delta^{k+2} + \delta^{k+3}) = \delta^{k+4} - \delta^k$

(v). $(\delta - 1)(\delta^k + \delta^{k+1} + \delta^{k+2} + \delta^{k+3} + \delta^{k+4}) = \delta^{k+5} - \delta^k$

If continuing to write the above expression up to n, we obtain the n^{th} expression as follows:

(n). $(\delta - 1)(\delta^k + \delta^{k+1} + \delta^{k+2} + \cdots + \delta^{n-1} + \delta^n) = \delta^{n+1} - \delta^k$

If substituting $\delta = \sqrt[q]{x}$ and $\delta^i = \sqrt[q]{x^i}$, $(k \leq i \leq p)$ in the n^{th} expression, we get

$$\left(\sqrt[q]{x} - 1\right)\left(\sqrt[q]{x^k} + \sqrt[q]{x^{k+1}} + \sqrt[q]{x^{k+2}} + \cdots + \sqrt[q]{x^{p-2}} + \sqrt[q]{x^{p-1}}\right) = \sqrt[q]{x^p} - \sqrt[q]{x^k}$$

By simplifying this expression, we conclude that

$$\sum_{i=k}^{p-1} \sqrt[q]{x^i} = \frac{\sqrt[q]{x^p} - \sqrt[q]{x^k}}{\sqrt[q]{x} - 1}, x \neq 1.$$

Hence, theorem is proved.

Corollary 2.1: $$\sum_{i=-k}^{p-1} \sqrt[q]{x^i} = \frac{\sqrt[q]{x^p} - \sqrt[q]{x^{-k}}}{\sqrt[q]{x} - 1}, x \neq 1.$$

3. Conclusion

In this article, the computation of geometric series with radicals are introduced for mathematical and computational applications. Also, this idea can enable the scientific researchers for further involvement in research and development.

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