Binomial Geometric Series for Computational Application

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Abstract: Computational science is a rapidly growing interdisciplinary area where science, computation, mathematics, and its collaboration use advanced computing capabilities for understanding and solving the most complex real-life problems. Also, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. For this purpose, binomial geometric series and its theorem are introduced for mathematical and computational applications.

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1. Introduction

Geometric series played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new geometric series [1-5] is constructed for application [6] of computational science and engineering.

2. Binomial Geometric Series

The binomial geometric series is given below:

$$\sum_{k=0}^{n-1}(x+y)^k = \frac{(x+y)^n - 1}{(x+y)-1} = \frac{\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k - 1}{(x+y)-1}, (x+y) \neq 1, \quad (1)$$

where the binomial series states $(x+y)^n = \sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k = \sum_{k=0}^{n}\binom{n}{k}x^k y^{n-k}$. (2)

Theorem 2.1: $\sum_{k=0}^{n} x^k y^{n-k} = \frac{x^{n+1} - y^{n+1}}{x-y}, x \neq y.$

Proof. Let's prove this theorem using the geometric series with rational numbers.

$$\sum_{k=0}^{n}\left(\frac{x}{y}\right)^k = \frac{\left(\frac{x}{y}\right)^{n+1} - 1}{\frac{x}{y} - 1} = \left(\frac{x^{n+1} - y^{n+1}}{y^{n+1}}\right)\left(\frac{y}{x-y}\right), x \neq y. \quad (3)$$

By simplifying the expression (3), we get

$$y^n\left(1 + \frac{x}{y} + \frac{x^2}{y^2} + \frac{x^3}{y^3} + \cdots + \frac{x^{n-1}}{y^{n-1}} + \frac{x^n}{y^n}\right) = \frac{x^{n+1} - y^{n+1}}{x-y}, x \neq y. \quad (4)$$

By simplifying the above algebraic expression, we obtain

$$y^n + xy^{n-1} + x^2y^{n-2} + x^3y^{n-3} + \cdots + x^{n-1}y + x^n = \frac{x^{n+1} - y^{n+1}}{x - y}, x \neq y. \quad (5)$$

From the above algebraic expression, we conclude that

$$\sum_{k=0}^{n} x^k y^{n-k} = \frac{x^{n+1} - y^{n+1}}{x - y}, x \neq y. \quad (6)$$

Hence, theorem is proved.

3. Conclusion

In this article, a binomial geometric series has been introduced for mathematical and computational application. Also, this idea can enable the researchers for further involvement in the scientific research.

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