Computation of Mass-Energy Equation from Lorentz Factor and Kinetic Energy

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

Abstract: The relativistic momentum is concerned with the motion of a body whose velocity approaches the speed of light. Also, the velocity of a moving object with mass is less than the speed of light because light has no mass and the speed of light is the upper limit for the speeds of all objects with mass in the universe. In this article, the Annamalai's mass-energy equation is derived using the Lorentz factor and the kinetic energy of classical mechanics.

Keywords: speed of light, electromagnetics, light wave

1. Introduction

Quanta of light are the smallest discrete packets of electromagnetic energy. For instance, sunlight is a kind of electromagnetic energy. A quantum of light that has no mass is known as a photon. We know that relativistic mechanics [1-5] is concerned with the motion of bodies whose speed approaches the speed of light. Let us understand the relationship between the kinetic energy of classical mechanics and the Lorentz factor [5] in the following sections.

2. Kinetic Energy

Kinetic energy is a fundamental concept in classical mechanics that quantifies the mechanical energy performed by an object due to its motion.

The equation of kinetic energy is shown below:

$$E = \frac{1}{2}mv^2, \tag{1}$$

where *m* is the mass and *v* is the velocity.

By differentiating the equation of kinetic energy with respect to time *t*, we obtain

$$\frac{dE}{dt} = \frac{1}{2}\left(m2v\frac{dv}{dt}\right)$$

By simplifying the above differential equation, we receive

$$dE = vd(mv) \tag{2}$$

3. Derivation of the Lorentz Factor

The Lorentz factor is the factor by which time, length, and mass change for an object moving at a speed close to the speed of light. By the following experiment, the author of this article derives the equation of the Lorentz factor [5].

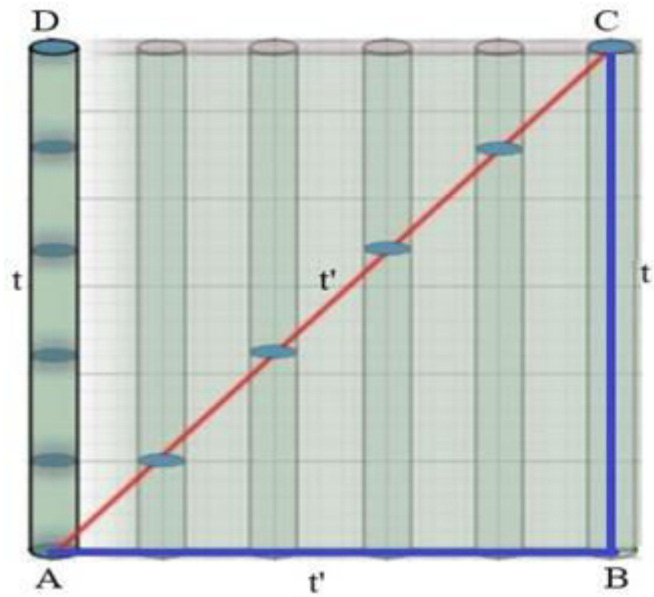

Fig 1. Relativistic speed of the metal tube

Let's consider that a metal tube of mass m_0 and length h is at a stationary position and the light takes time t to travel from point A to point D.

Here, $AD = BC = h = ct,$ where c is the speed of light.

Suppose the metal tube moves with velocity v horizontally and the light travels from point A along with the moving metal tube. When the light reaches point C at time t', the metal tube moves to point B at time t'.

$$AB = vt' \text{ and } AC = ct'$$

By the Pythagorean theorem, we can derive the time dilation and Lorentz factor as follows:

$$AC^2 = AB^2 + BC^2 \quad (3)$$

By substituting $AB = vt'$ and $AC = ct'$ in the equation (3), we obtain

$$(ct')^2 = (vt')^2 + (ct)^2$$

$$t'^2\left(1 - \frac{V^2}{C^2}\right) = t^2$$

By simplifying the above equation, we get

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{t'}{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (4)$$

Hence, γ is the Lorentz factor.

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4. Mass-Energy Equation

The equation of relativistic mass is m=$m_0\gamma$.

Here, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ is the Lorentz factor.

$$\text{Now, m} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{5}$$

Squaring both sides of the equation (5):

$$m^2 = \frac{{m_0}^2}{1-\frac{v^2}{c^2}} \Longrightarrow m^2(c^2 - v^2) = {m_0}^2 c^2$$

By simplifying the above equation, we obtain

$$m^2c^2 - m^2v^2 = {m_0}^2c^2 \tag{6}$$

where rest mass (m_0) and speed of light (c) are constant and relativistic mass (m) and relativistic velocity (v) are variables over time.

By differentiating the equation (6) with respect to time, we get

$$c^2 2m\frac{dm}{dt} - \left[2mv^2\frac{dm}{dt} + 2m^2v\frac{dv}{dt}\right] = 0$$

By simplifying the above equation, we receive

$$c^2dm = vd(mv) \tag{7}$$

Substituting the equation (7) in the differential equation (2) of kinetic energy:

$$dE = \ c^2dm = vd(mv) \tag{8}$$

Now, it is understood that $dE = vd(mv) = c^2dm \ \Rightarrow E = \frac{1}{2}mv^2 = \frac{1}{2}mc^2$.

In other words, c is a numeric constant and also one of the values of v.
Therefore, $v = c$. (9)

By substituting the equation (9) in the equation (1), we conclude that

$E = \frac{1}{2}mc^2$, where E is the energy of motion

Hence, the Annamalai's mass-energy equation is derived as follows.

$$E = \frac{1}{2}mc^2. \tag{10}$$

5. Conclusion

In this article, Annamalai's mass-energy equation has been derived using the Lorentz factor and the equation of kinetic energy in classical mechanics. This idea can enable the researchers for further involvements in the scientific research and development.

References

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