Binomial Series without Binomial Coefficients

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Abstract: Nowadays, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. Also, the computational science is a rapidly growing interdisciplinary area where science, computation, mathematics, and its collaboration use advanced computing capabilities for understanding and solving the most complex real-life problems. For this purpose, a novel binomial series is introduced for mathematical and computational applications.

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1. Introduction

Geometric series [1,9] played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, management and its applications. In this article, a new binomial theorem without binomial coefficients is introduced in this article for computational application [10].

2. Novel Binomial Series

The author of this article introduces a new binomial series given below.

Theorem 2.1: $$\sum_{k=0}^{n} x^k y^{n-k} = \sum_{k=0}^{n} x^{n-k} y^k = \frac{x^{n+1} - y^{n+1}}{x - y} = \frac{y^{n+1} - x^{n+1}}{y - x}, x \neq y.$$

Proof. Let's prove this theorem using the geometric series with rational numbers.

$$\sum_{k=0}^{n} \left(\frac{x}{y}\right)^k = \frac{\left(\frac{x}{y}\right)^{n+1} - 1}{\frac{x}{y} - 1} = \left(\frac{x^{n+1} - y^{n+1}}{y^{n+1}}\right)\left(\frac{y}{x - y}\right), x \neq y.$$

By simplifying the algebraic expression, we get

$$y^n \left(1 + \frac{x}{y} + \frac{x^2}{y^2} + \frac{x^3}{y^3} + \cdots + \frac{x^{n-1}}{y^{n-1}} + \frac{x^n}{y^n}\right) = \frac{x^{n+1} - y^{n+1}}{x - y}, x \neq y.$$

$$i.e., \quad y^n + xy^{n-1} + x^2 y^{n-2} + x^3 y^{n-3} + \cdots + x^{n-1} y + x^n = \frac{x^{n+1} - y^{n+1}}{x - y}, x \neq y. \quad (1)$$

$$i.e., \quad \sum_{k=0}^{n} x^k y^{n-k} = \frac{x^{n+1} - y^{n+1}}{x - y}, x \neq y. \quad (2)$$

By rearranging the binomial series (2), we obtain

$$\sum_{k=0}^{n} x^{n-1}y^{k} = \frac{y^{n+1}-x^{n+1}}{y-x}, x \neq y. \tag{3}$$

From the binomial series (2) and (3), we conclude that

$$\sum_{k=0}^{n} x^{k}y^{n-k} = \sum_{k=0}^{n} x^{n-k}y^{k} = \frac{x^{n+1}-y^{n+1}}{x-y} = \frac{y^{n+1}-x^{n+1}}{y-x}, x \neq y.$$

Hence, theorem is proved.

Corollary 2.1: $\sum_{k=0}^{n} x^{k}y^{n-k} = \sum_{k=0}^{n} x^{n-k}y^{k} = (n+1)x^{n}, \text{for } x = y.$

Corollary 2.2: $\sum_{i=k}^{n} x^{i}y^{n-i} = \sum_{i=k}^{n} x^{n-i}y^{i} = \frac{x^{n+1}-x^{k}y^{n+1-k}}{x-y} = \frac{y^{n+1}-y^{k}x^{n+1-k}}{y-x}, x \neq y.$

3. Conclusion

In this article, a novel binomial series has been introduced for mathematical and computational application. Also, this idea can enable the researchers for further involvement in the scientific research.

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