Marine positioning independent of satellite systems using two consecutive solar elevations

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Abstract:

Oceanic navigation is highly dependent on the positioning of floating units. In the past, when satellite navigation did not exist, all floating units that navigated in the ocean calculated their position using celestial bodies and the sun. With the advent of satellite navigation, positioning using celestial bodies has faded and is only limited to emergency cases where the floating unit is unable to use satellite equipment.

In this study, the position of a floating unit in the sea is calculated using two consecutive solar elevations using a recursive algorithm. The results of this study show that by determining the appropriate timing between two consecutive elevations of about 10 to 20 minutes, the best positioning result can be obtained and the accuracy of this method is about 10^{-1} minutes.

Keyword : Positioning، Oceanic navigation، Sun، Elevation.

Introduction:

The use of global navigation satellite systems (GNSS), especially GPS, has reduced the use of traditional methods and substitute tables. However, in cases where GPS is not available or accessible due to solar flares, magnetic storms, or other reasons (GPS technical issues), the use of celestial navigation is inevitable. In celestial navigation, the position of a floating unit is calculated using the elevation of celestial bodies (the angle between the celestial body and the horizon) measured using a sextant.

The best current method in celestial navigation is the intersection method, in which a rough position is considered using estimated navigation. Then, the elevation of the celestial body at that approximate position is first measured and then calculated. The difference between these two measured and calculated elevations is called the intersection, and the correct position of the floating unit is obtained from the approximate point in the direction of the celestial body by the amount of the intersection. This method is still taught and used in floating units today.

To increase the accuracy of the position of the floating unit in a short period of time, the elevation of three bodies is measured and calculated for the approximate position. The intersection of the three position lines will create a more accurate position. One of the disadvantages of this method is that its usual accuracy in the best conditions is about one mile, but in general, sailors will experience an error of several miles (Malkin, 2014).

In addition to measuring the elevation of at least two celestial bodies with a sextant at the exact time using a timer, this method also requires the use of a marine nautical almanac to find the Greenwich hour angle (the west angle between the meridian passing through the celestial body and the Greenwich meridian, hereinafter referred to as GHA) and declination (the angle between the body and the celestial equator, hereinafter referred to as DEC) of the said bodies, as well as knowing the hypothetical or approximate position of the observer. The approximate position is obtained using estimated navigation. In estimated navigation, the expected future positions of the ship are obtained by recording and updating the position based on the ship's speed and course.

With the above, the observer usually finds the exact position by applying the drawing methods (position line) on the map. Methods that have historically been introduced mainly by Sumner in 1837 and Saint Hilaire in 1875 (Umland, 2015). It should be noted that estimated navigation and drawing methods are prone to errors and are sometimes challenging for a novice user.

There is a significant body of research on the calculation of a precise point using fully mathematical (non-drawing) and analytical (Rossano, 1977; Bennett, 1979; Gray, 1997; Kjer, 1981) and numerical methods (Daub, 1979; Kotlarick, 1981; Metcalf and Metcalf, 1991; Ogilvie, 1977). One of the most important analytical methods is the Allen (1981) that solved the intersection point of two astronomical position lines on the Earth in Cartesian coordinates. A comprehensive vector method has also been proposed by Gonzalez (2008) that calculates the intersection of astronomical position lines on the Earth based on vector methods. Some of these methods do not require an approximate position, while others require an approximate position or at least an approximate latitude that is used to transfer the position line.

Huxtable(2006) used the spherical trigonometry relationship to calculate the intersection point of two astronomical location lines on the Earth. His innovative method was more tangible for sailors due to the use of spherical trigonometry and received more attention. Pierros (2018) succeeded in solving the problem of astronomical location line transfer by using the idea of Huxtable (2006) and Daub (1979) and using a repetitive method. In the past, astronomical location line transfer was only used by using drawing methods, and mathematical calculations were exclusively devoted to calculating the location line.

The astronomical navigation method of Pierros (2018) includes the following two methods: (a) For a stationary observer, it directly and analytically calculates two possible accurate points (these two points are far apart from each other, and the navigator officer can eliminate one and choose the correct point by reasoning); and (b) It solves a position for a moving observer from a numerical iterative method without the need for an approximate position, and only needs two initial guesses for the latitude to start the iterative algorithm. These initial guesses do not need to be very accurate, and they can be obtained with previous information about what latitudes the navigation has taken place between. However, Pierros (2018) used the stationary observer method to calculate the relatively accurate latitude of the observer and used it as the initial guess for the moving observer, and practically eliminated the assumption of the initial guess to start the algorithm.

The advantages of this method, in comparison with similar ones, are as follows: (a) Relative mathematical simplicity and direct solution without any assumptions, which is even possible with a non-programmable calculator (at least for the stationary observer mode). (b) Generality, no need to choose specific celestial bodies, for example, any object that is located in the two eastern and western hemispheres relative to the observer's meridian, can be used in this method. (c) Use of equations and tables to calculate the location of celestial bodies, and as a result, the elimination of the need to use the marine astronomical calendar in calculating the location of bodies. (d) Accuracy, practically depends on the accuracy of observations, and the method used does not impose an error on the problem. (e) In the case of the location line transfer problem, the use of the secant iterative method and integration with the stationary observer method eliminates the need for an approximate latitude and significantly reduces the number of iterations required, and thus makes it possible to obtain a convenient solution with a non-programmable calculator.

In this research, using the Pierros (2018) method, astronomical navigation is performed using two lines of altitude from the Sun with a specified time difference. The distinctive part of this research with Pierros (2018) is focusing on the aspect of refining the calculated position and reducing the resulting error. Pyros had obtained the position of the Sun based on simple methods that are generally found in marine astronomical calendars, while the use of more accurate methods can lead to better results. On the other hand, the time difference between observations of the Sun is another factor that can affect the accuracy of solving the problem. Therefore, in this research, two accurate methods (Michalsky, 1988 and Jean Meeus, 1998) were used to calculate the position of the Sun in the sky. In addition, these two accurate methods were compared with the results of the Astro calculator application from DBG calculator company. In the next section, the theoretical foundations of the iterative method for transferring the location line of the Sun and the error resulting from the calculation of the location are discussed.

Theoretical foundations

In the following, it is necessary to talk about the principles and foundations of astronomical navigation, and then discuss the transfer of the location line on the Earth. In astronomical navigation, the Earth is considered as a perfect sphere. For any altitude of a celestial body, with geographical position GP1, which is determined by GHA1 and Dec1, there are an infinite number of astronomical positions that are equidistant from GP1 (Figure 1). The center of this circle is on GP1 and its radius (in spherical trigonometry terms) is equal to the zenith distance of the body $z1=90°-H1$, where H1 is the observed altitude (the angle of the celestial body with the observer's horizon) of the body. Any observer along the circumference of this circle, regardless of their position on the circle, will measure this constant altitude. This circle is a line of astronomical location with the same altitude. In Figure 1, two lines of location are shown for two independent bodies, and the point of intersection of these two circles can determine the position of the observer. Given that these two circles intersect at two points, two points are calculated for the observer, of which the observer is only located in one point. Using the approximate position of the observer, the actual position can be obtained between the two calculated points.

During the day, only the Sun is visible and other celestial bodies are not seen that can be used to calculate the position of the observer using the intersection of the location lines. Here, a method called the location line transfer method is used. In the location line transfer method, the altitude of the Sun is measured at UTC1 and then another altitude of the Sun is measured with a time difference TΔ that UTC2=UTC1+ΔT. Now, the location circle at UTC1 must be transferred to the distance traveled by the observer in the direction of motion or floating way, resulting in two location lines at UTC2, one directly measured and the other transferred from UTC1. The intersection point of these two circles with each other can determine the position of the observer. In this research, using the location line transfer method of the Sun during the day, the position of the floating unit is calculated from the intersection of the two measured and transferred location lines. In the continuation of the theoretical foundations, the relationships used to transfer the location line on the Earth are discussed.

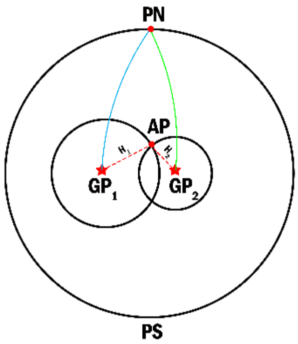

Figure 1. Two lines of position (LOPs) for celestial bodies with geographical positions GP1 and GP2

First, our observer is at position GP1 on the surface of the Earth (Figure 1). Similarly, another celestial body is observed at GP2 (GHA2, Dec2) with observed altitude H2. Two circles are drawn for the two bodies with altitudes H1 and H2 and centered at GP1 and GP2 with radii z1=90°-H1 and z2=90°-H2. Obviously, the two circles intersect each other, and the position of the observer is one of the two intersection points Pos1 and Pos2, and the observer must be able to determine the correct point from other methods.

The LHA angle is the hour angle of the observer located at latitude Lat and longitude Lon, to which the following relationship applies:

$$LHA = GHA + Lon \qquad \overset{if}{GHA + Lon} \leq 180^o \tag{1}$$

$$LHA = GHA + Lon - 360^o \qquad \overset{if}{GHA + Lon} > 180^o \tag{2}$$

The reason for the above relationship is that the local hour angle between the meridian passing through the body and the observer is in the west direction. Therefore, if the hour angle is less than 180 degrees, the body is located in the west of the observer. If the hour angle is greater than 180 degrees, the body is located in the east of the observer. In the case where the body is located in the east of the observer (equation 2), the hour angle is subtracted from 360 degrees so that the east-west hour angle, which is less than 180 degrees, is calculated.

The following conventions are also applied:

Eastern longitude is positive.
Western longitude is negative.
Northern latitude is positive.
Southern latitude is negative.
East meridian angle is negative.
West meridian angle is positive.
North declination is positive.
South declination is negative.
With regard to the above definitions, to solve the position on the Earth, the spherical triangle and the mathematical relationships governing the spherical triangle are used. This triangle is such that one of its corners is the pole (PN) and the other corner (AP) or the hypothetical point that is the floating selection point, and the third corner is the location of the celestial body (GP). This triangle is shown in Figure (2).

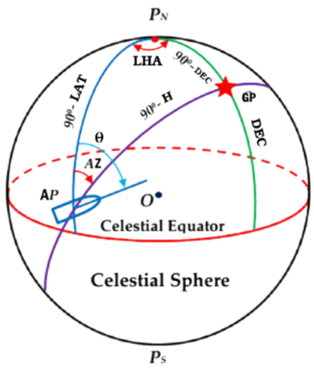

Figure 2: Relationships between the floating unit position and the celestial body in the spherical triangle

The components of this triangle are composed of three sides, namely the complement of the latitude (side APPN in Figure 2), the complement of the altitude of the celestial body (side APGP) and the complement of DEC (side PNGP), as well as three angles, which are respectively as follows:
LHA (same as the local hour angle)
AZ (an arc of the celestial horizon, enclosed between the main north or south vertical if the observer is in northern or southern latitudes on the horizon, which is measured from 0-180 degrees towards the east if the body is in the east and or west if the body is in the west. This angle can be interpreted as a measure of the direction of the body relative to the geographical north)

The third angle is the angle between the observer and the pole with the celestial body as the axis.
This triangle is shown independently in Figure 3. The components of it, including sides and angles, are clearly shown.

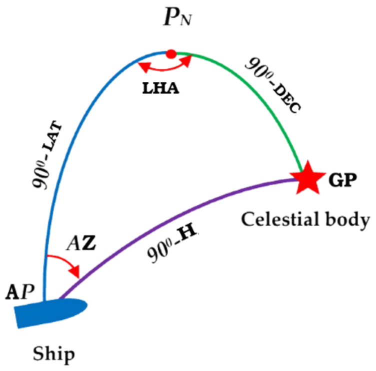

Figure 3: Spherical triangles for determining the position of a ship

The law of cosines is one of the most important relationships governing spherical triangles, which expresses the cosine of one side in terms of the other sides as follows.

$$cos(a) = cos(b)cos(c) + sin(b)sin(c)cos(A) \tag{3}$$

In the above equation, a, b, and c are the sides of the spherical triangle, and A is the angle opposite to side a. The position of a moving observer can be calculated using the law of cosines. To do this, it is assumed that an observer is located at coordinates (Lat1, Lon1) on a moving ship with a constant speed of V knots relative to the Earth and a constant course of θ (from north to east, from 0 to 360 degrees). At UTC1, he measures the altitude H1 corresponding to a celestial body with coordinates GHA1 and Dec1. He measures the altitude H2 corresponding to the same body or another body with coordinates GHA2 and Dec2 at time UTC2 in position Lat2 and Lon2. If the law of cosines is used for side H in the two measurements, the following relationships are obtained:

$$\sin H1 = \cos(90 - Lat1)\cos(90 - Dec1) + \sin(90 - Lat1)\sin(90 - Dec1)\cos(LHA1) \tag{4}$$

$$\sin H2 = \cos(90 - Lat2)\cos(90 - Dec2) + \sin(90 - Lat2)\sin(90 - Dec2)\cos(LHA2) \tag{5}$$

Using equations (1) and (2), we have the following:

$$\sin H1 = \sin Lat1 \sin Dec1 + \cos Lat1 \cos Dec1 \cos(GHA1 + Lon1) \tag{6}$$

$$\sin H2 = \sin Lat2 \sin Dec2 + \cos Lat2 \cos Dec2 \cos(GHA2 + Lon2) \tag{7}$$

Which is calculated as follows for the observer's longitude at the second observation:

$$Lon2 = S \, . \, a\cos\left(\frac{\sin H2 - \sin Lat2 \sin Dec2}{\cos Lat2 \cos Dec2}\right) - GHA2 \tag{8}$$

If the celestial body is observed east of the observer's prime meridian, the value of S is equal to-1 and otherwise it will be +1 (related to the argument of equations 1 and 2 that the hour angle becomes smaller or larger than 180). We also have the following relationships for the movement of a floating unit:

$$Lat1 = Lat2 - \Delta Lat \tag{9}$$

$$\Delta Lat = \frac{V\cos\theta\,(UTC2 - UTC1)}{60} \tag{10}$$

$$Lon1 = Lon2 - \Delta Lon \tag{11}$$

$$\Delta Lon = \frac{1}{60}\int_{UTC1}^{UTC2} \frac{V\sin\theta\ .dt}{\cos LAT} \tag{12}$$

In the above relations, LAT is the latitude, which changes from LAT1 to LAT2 with time. The above relations represent the mathematical relationship of the line-of-sight displacement.

Assuming V and θ to be constant within the time interval UTC1 to UTC2, the integral of equation (12) can be solved analytically as follows (Daub,1979):

$$\Delta Lon = \frac{V\sin\theta\,(UTC2 - UTC1)}{60\cos Lat2} \tag{13}$$

If , the error caused by the approximation is less than 0.1 (Pierros, 2018). Dividing equation (6) by equation (7) will lead to the following relationship

$$\sin Lat1 \sin Dec1 - \sin Lat2 \sin Dec2 + \cos Lat1 \cos Dec1 \cos\left(GHA1 + Lon1\right)$$

$$-\cos Lat2 \cos Dec2 \cos\left(GHA2 + Lon2\right) - \sin H1 + \sin H2 = 0 \tag{14}$$

By substituting Lon2 from equation (8), Lat1 from equations (9) and (10), and Lon1 from equations (11) and (12), we rewrite the expression of equation (14) as a function of F based on Lat2, as follows:

$$F\left(Lat2\right) = 0$$

Indicates:

$$\text{Sin}\left(\text{Lat2}-\frac{V\cos\theta(UTC2-UTC1)}{60}\right)\text{ sinDec1- sinLat2 sinDec2+}$$
$$\text{cos}\left(\text{Lat2}-\frac{V\cos\theta(UTC2-UTC1)}{60}\right)\text{ cosDec1 cos(GHA1}\pm S.\,Arc\,cos\left(\frac{sinH2-sinLat2\ sinDec2}{cosLat2\ cosDec2}\right)-GHA2-$$
$$\left(\frac{V\sin\theta(UTC2-UTC1)}{60\ cos\ Lat2}\right))\text{ - cosLat2 cosDec2}\left(\frac{sinH2-sinLat2\ sinDec2}{cosLat2\ cosDec2}\right)\text{- sinH1+ sinH2=0} \tag{15}$$

We can find the numerical root (x = Lat2) using the secant method iteration, as follows:

$$x_n = x_{n-1} - F\left(x_{n-1}\right)\frac{x_{n-1} - x_{n-2}}{F(x_{n-1}) - F(x_{n-2})} \tag{16}$$

The iteration terminates when the relative error, , is less than 0.0001 degrees. Then, the longitude Lon2 can be calculated from equation (16). The algorithm and flowchart of the method are shown in the appendix A.

For calculation with the secant method, two initial values, and , ideally close to the root, are required. This initial guess does not compromise the need for an assumed position (or only the latitude if we are precise), because:

- In the secant method, the choice of reasonable initial values far from the true root does not affect the accuracy of the result, but only requires more iterations.
- The first initial value can be found without using approximate navigation, using the method of intersection between lines of sight and considering the observer as fixed. Then, the second initial value can be chosen near the first.

Also, determining s for equation (8) does not require an assumed position because:

- A navigator is likely to have at least an estimate of the local magnetic field deviation, and therefore has the relative position of the local meridian relative to east and west.
- Even if not, this can be inferred in a variety of ways (for example, by observing the sun crossing the observer's meridian).
- If the navigator chooses a wrong s, the method will either not converge or will yield an obviously wrong position that is easily detectable as wrong. Then, the navigator can repeat the method with the correct s.

However, the observer should avoid celestial bodies close to his local meridian (local hour angle of the body close to zero), as this may lead to a wrong or no solution, as pointed out by Kotlaric (1981).

In the next section, the relationships introduced for the transfer of the sun's line of sight are evaluated using the accurate relationships for calculating the sun's position.

Discussion and Conclusion:

In this study, due to the importance of celestial navigation in ocean navigation, the positioning of floating units during the day has been studied. In this method, the position of the observer can be determined using two consecutive measurements of the sun's altitude. The basis of this method is the transfer of the line of sight and the calculation of the intersection point of this line with the second line of sight. This algorithm was first introduced in the study of Pierros (2018), but it has not been analyzed for errors. The error caused by positioning is one of the most important points to consider in position calculation. Positioning methods in the ocean that have a lot of error are not welcomed and are abandoned. Therefore, in this study, the Pierros (2018) method has been used and evaluated with various conditions to reduce error. At first, it is necessary to test the numerical scheme to ensure the accuracy of the results.

To test the scheme used, the example solved in the Pierros (2018) article is used. In this example, a floating unit observes two consecutive altitudes of the sun at a half-hour interval in the afternoon and the position of the floating unit is calculated. The example is as follows:

Example 1) On May 18, 2016, a vessel with a speed of 20 knots and a course of 225° at UTC 1:18:00:00 was in position 45° 00.0'N, 045° 00.0'W and the height of sun (GHA1=90° 53.0', Dec1= 19° 45.5') H1=44° 36.6'. At UTC 2:18:30:00, the sun is in the west of the observer's meridian (S=+1) and the sun related values are GHA2=98° 23.0', Dec2= 19° 45.8') and H2=39° 38.0'. Using the stated relations, we estimate that at the time of the second observation, the ship should be in position 44° 52.9'N, 045° 10.0'W.

Therefore, in order to compare the written program (the algorithms of the Michalsky and Jinn-Moss algorithms) with the Pierros method, we solve the virtual example 1 of the article again:

By setting the initial values Lat20=45N and Lat21=46N and S=+1, we use the described secant method along with the mentioned algorithms and proceed to execute the software. The output of the information is shown in Table 1.

Table 1: Calculation of the latitude and longitude of the ship along with the declination and Greenwich hour angle of the sun by the written program

$Long_2$	Lat_2	GHA_2	GHA_1	Dec_2	Dec_1	Method
045°9.9' W	44° 53' N	98° 23.0'	90° 53.0'	19° 45.8' N	19° 45.5' N	(2018)Pierros
045° 9.8' W	44° 53.04' N	98° 23.01'	90°53.02'	19°45.88' N	19° 45.62' N	(1988)Michalsky
045° 10.13' W	44° 52.9' N	98° 23.18'	90°53.20'	19°45.73' N	19°45.46' N	Jean Meeus (1998)

The results of solving the Pierros 2018 example are shown in Table (1). According to the table, the numerical solution converged after 3 iterations and the Lat2 point (latitude at 18:30) was calculated. Using Lat2, the Long2 point (longitude at 18:30) can also be obtained.

The actual point of the floating unit equal to the example mentioned at 18:30 is 44° 52.9'N 045° 10.0'W, which is the result of calculations using the Pierros2018 method, 0.1 minutes in longitude and latitude.

In this study, two very accurate algorithms, Michalsky(1988) and Jean Meeus (1998), were used to calculate the Greenwich hour angle (GHA) and declination (Dec) of the sun.

The results of the simulation of the Pierros 2018 example showed that the rewritten numerical scheme in this study can accurately calculate the position of the sun in space. The results are in full compliance with the results of the Pierros 2018 algorithm.

The position of the sun, namely the GHA and Dec, play a very important role in increasing the accuracy of the calculations for determining the position of a ship at sea. The discussion of the position of the sun and its role in the accuracy of the floating position has not been discussed in the Pierros 2018 article.

The solution to the mentioned example using the Michalsky (1988) algorithm for calculating the sun is presented in Table (1). According to this table, it is observed that the sun's Dec and GHA have a difference of about 0.12 and 0.02 minutes, respectively, with Pierros 2018. Also, the Jean Meeus (1998) algorithm also shows the sun's Dec with a difference of about 0.07 minutes and its GHA with a difference of about 0.02 minutes with Pierros 2018. Regarding the discussion of the difference between the position calculated by the Michalsky (1988) and Jinn-Moss (1998) algorithms with the expected position of 44° 52.9'N 045° 10.0'W, a difference of 0.19 and 0.13 miles is observed, respectively.

Example 2)

To perform the real test of the numerical scheme, a car was used on a highway (Hemmat Expressway). The experimental scenario was as follows: On May 27, 2023, at 06h44'47.3" UTC at the entrance of Hemmat Expressway, the Astro Calculator app from DBG Calculator Marine Calculations was used to calculate the geographic position of the observer (the geographic position was recorded for validation, which this app also receives the position from the phone's satellite tracker). This app, which is written for the purposes of celestial navigation at sea, uses the algorithms of Jean Meeus (1998), Urban and Seidelmann (2023), and Bertagne and Franco (1988) to calculate the position of the sun in space, so this app has good accuracy in calculations and its only weakness is in the output data with two decimal places.

A very important point for using this app is that it calculates the two very important components for conducting this experiment, namely position and time, with the desired accuracy, and then uses the time and position received from the mobile device to obtain the position of the sun in space and provides it to the user. Therefore, the combination of the app and mobile sensors is a suitable tool for practical testing of this scheme.

In Table 2, the information of the sun's coordinates (GHA angle and Dec) from this app for points from east to west of Hemmat Expressway with a difference of approximately 10 minutes in time from each other is shown. Although this app provides the user with the value of the sun's coordinates at the test locations in Table 2, in this study, two algorithms of Michalsky (1988) and Jean Meeus (1998) were used to calculate the position of the sun and the coordinates of the sun from the DBG Calculator app were not used. Only time, altitude, and position were taken from this app, and the purpose of using this software package was only to receive the output of the phone sensors.

Table (2): Information on the position of the sun from the Asrto Calculator online software

time	GPS coordinates	Altitude	Dec	LHA	point
UTC1:06:44:47.3	051°29.4'E‹35°45.3'N	H1=62°38.6'	Dec1=21°15.9'N	LHA1=333°24.3'	1
UTC2:06:54:48.2	051°28.3'E‹35°45.4'N	H2=64°27.0'	Dec2=21°15.9'N	LHA2=335°53.4'	2
UTC3:07:04:9.6	051°27.5'E‹35°45.4'N	H3=66°05.4'	Dec3=21°16.0'N	LHA3=338°12.9'	3
UTC4:07:14:12.1	051°23.9'E‹35°45.0'N	H4=67°45.3'	Dec4=21°16.1'N	LHA4=340°40.0'	4
UTC5:07:23:51.4	051°18.6'E‹35°45.2'N	H5=69°15.0'	Dec5=21°16.1'N	LHA5=342°59.5'	5
UTC6:07:33:45.4	051°15.2'E‹35°43.7'N	H6=70°43.4'	Dec6=21°16.2'N	LHA6=345°24.6'	6
UTC7:07:43:14.7	051°12.4'E‹35°41.7'N	H7=72°01.7'	Dec7=21°16.3'N	LHA7=347°44.1'	7
UTC8:08:01:12.6	051°10.7'E‹35°43.3'N	H8=74°01.3'	Dec8=21°16.4'N	LHA8=352°11.8'	8

In Table 3, the calculations of the numerical scheme for the time and points of Table 2 are presented. In this table, in each row, 2 starting and destination points are required for calculation, which are entered in the route column. The calculation method is as follows: By moving from the starting point to the destination, the geographic coordinates of the destination point are calculated by the numerical scheme and compared with the actual value.

Table 3: Calculation of the observer's longitude and latitude using the Jean Meeus (1998) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.015000'	35.754581°	51.468615°	0.125122'	0.183110'	0.194390'
2 to 3	9.356667'	35.753827°	51.454778°	0.170361'	0.213298'	0.243032'
3 to 4	10.041667'	35.744537°	51.392774°	0.327777'	0.333586'	0.243032'
4 to 5	9.655000'	35.755637°	51.311178°	0.138239'	0.070651'	0.149759'
5 to 6	9.900000'	35.722599°	51.246283°	0.344052'	0.423049'	0.486459'
6 to 7	9.488333'	35.691734°	51.202628°	0.195965'	0.242349'	0.277931'
7 to 8	17.965000'	35.724651°	51.183445°	0.179041'	0.306678'	0.306874'

For example, if we move from point 1 to point 2 in Table (2), the time difference between these two points is approximately 10 minutes. The latitude and longitude calculated by the Jean Meeus (1998) algorithm for the position of the sun in this route are 51.468615 ° and 35.754581°, respectively. The error of the scheme for calculating latitude and longitude is 0.183110' and 0.125122', respectively. As a result, the distance between the point calculated by the scheme and the actual point is approximately 0.194390'. The same procedure is used for the rest of the cases, i.e., the routes between two points sequentially (for example, from 2 to 3, from 3 to 4, etc., to 7 and 8). It is worth noting that the minimum error in the distance between the actual point and the calculated point is 0.149759' for the route from 4 to 5, and the maximum error is 0.486459' for the route from 5 to 6.

In Table (4), the latitude and longitude of the points in Table (2) are calculated using the Michalsky(1988) algorithm, as in Table (3). The maximum error is for the route from 5 to 6 with a value of 0.617557', and the minimum error is for the route from 4 to 5 with a value of 0.018699'.

Table (4): Calculation of the observer's longitude and latitude using the Michalsky(1988) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.015000'	35.752768°	51.468015°	0.233916'	0.219079'	0.294009'
2 to 3	9.356667'	35.751889°	51.454097°	0.286653'	0.254169'	0.353389'
3 to 4	10.041667'	35.742496°	51.392016°	0.450234'	0.379013'	0.545652'
4 to 5	9.655000'	35.753491°	51.310331°	0.009463'	0.019855'	0.018699'
5 to 6	9.900000'	35.720355°	51.245337°	0.478700'	0.479776'	0.617557'
6 to 7	9.488333'	35.689361°	51.201526°	0.338354'	0.308410'	0.421263'
7 to 8	17.965000'	35.72204°	51.182236°	0.032219'	0.234150'	0.192938'

The time difference between points in Table (2) is approximately 10 minutes. To calculate the role of the time difference and the accuracy of the scheme, another experiment with a time difference of 10 to approximately 80 minutes is required, as shown in Table (5). Therefore, the following table is derived from the same points as Table (2), with the difference that to create different time differences, the first point is used with the next points (for example, 1 to 2, 1 to 3, ... to 1 to 8) for each route.

Table (5): Calculation of the longitude and latitude of the observer using the Jean Meeus (1998) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.015000'	35.754581°	51.468615°	0.125122'	0.183110'	0.194390'
1 to 3	19.371667'	35.754185°	51.455047°	0.148929'	0.197166'	0.218736'
1 to 4	29.413333'	35.746320°	51.394335°	0.220800'	0.239922'	0.294593'
1 to 5	39.068333'	35.751597°	51.307044°	0.104181'	0.177339'	0.177789 '
1 to 6	48.968333'	35.725430°	51.249728°	0.174222'	0.216333'	0.247545'
1 to 7	58.456667'	35.692042°	51.203080°	0.177500'	0.215181'	0.249260'
1 to 8	76.421667'	35.721672°	51.176351°	0.000299'	0.118939'	0.096628'

In Table (5), the longitude and latitude calculated by the Jean Meeus (1998) algorithm for the route from point 1 to 3, which is equal to the points in Table (2), are 51.455047 ° and 35.754185 °, respectively. The error of the scheme for calculating the longitude and latitude is 0.197166' and 0.148929', respectively. As a result, the distance between the point calculated by the scheme and the actual point is approximately 0.218736'. It is worth noting that the minimum error in the distance between the actual point and the calculated point is 0.096628' for the route from 1 to 8, and the maximum error is for the route from 1 to 4 with a difference of 0.294593'.

Similarly, in Table (6), the longitude and latitude of the points in Table (2) are calculated using the Michalsky (1988) algorithm, as in Table (5). The maximum error for the route from 1 to 4 is 0.407213', and the minimum error is for the route from 1 to 8 with a value of 0.195966'.

Table (6): Calculation of the longitude and latitude of the observer using the Michalsky(1988) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.015000'	35.752768°	51.468015°	0.233916'	0.219079'	0.294009'
1 to 3	19.371667'	35.752306°	51.454411°	0.261661'	0.235357'	0.324173'
1 to 4	29.413333'	35.744376°	51.393663°	0.337439'	0.280249'	0.407213'
1 to 5	39.068333'	35.749587°	51.306337°	0.224754'	0.219759'	0.287112'
1 to 6	48.968333'	35.723350°	51.248982°	0.299022'	0.261061'	0.366759'
1 to 7	58.456667'	35.689879°	51.202290°	0.307239'	0.262628'	0.374273'
1 to 8	76.421667'	35.719367°	51.175481°	0.137987'	0.171163'	0.195966'

Determining the position of a stationary observer is another case that this scheme is capable of calculating. In this case, the stationary observer measures two altitudes of the sun at a time interval of less than one hour from the sun and finally calculates its position. This mode is suitable for cases where the observer is stationary and has no movement. Therefore, in the following, using the introduced scheme, the determination of the position for a stationary observer is studied.

Example 3)

To study the determination of the position of a stationary observer, it is assumed that an observer is stationary at the position of 27° 10.5'N,056° 12.9'E on August 03, 2023. In Table (7), the information extracted from the Astro Calculator application is presented, which, assuming that the sun is in the west of the observer's meridian (S=+1), the two Michalsky (1988) and Jean Meeus (1998) algorithms can be used to calculate the position of the observer. Given that the observer is stationary, it is expected that the longitude and latitude of the point calculated by the two algorithms will show the same initial position, i.e. 12.9'E 056°,27° 10.5'N.

Table (7): Sun position information extracted from the Astro Calculator astronomical navigation application

time	GPS coordinates	Altitude	Dec	LHA	point
UTC1:08:45:48.0	27°10.5'N،056°12.9'E	H1=78°49.7'	Dec1=17°31.6'N	LHA1=006°06.2'	1
UTC1:08:56:28.2	27°10.5'N،056°12.9'E	H2=77°24.2'	Dec2=17°31.5'N	LHA2=008°46.2'	2
UTC2:09:07:56.8	27°10.5'N،056°12.9'E	H3=75°33.5'	Dec3=17°31.4'N	LHA3=011°38.4'	3
UTC3:09:19:40.4	27°10.5'N،056°12.9'E	H4=73°26.7'	Dec4=17°31.3'N	LHA4=014°34.3'	4
UTC4:09:31:45.8	27°10.5'N،056°12.9'E	H5=71°06.7'	Dec5=17°31.1'N	LHA5=017°35.7'	5
UTC5:09:42:51.6	27°10.5'N،056°12.9'E	H6=68°52.3'	Dec6=17°31.0'N	LHA6=020°22.1'	6
UTC6:09:55:42.4	27°10.5'N،056°12.9'E	H7=66°12.2'	Dec7=17°30.9'N	LHA7=023°34.8'	7
UTC7:10:25:40.2	27°10.5'N،056°12.9'E	H8=59°46.6'	Dec8=17°30.6'N	LHA8=031°04.3'	8

In Table (8), the numerical scheme calculations in determining the position of the observer for the times and points of Table (7) are presented. The longitude and latitude calculated by the Jean Meeus (1998) algorithm for time 1 to 2 are 56.219970 ° and 27.171436°, respectively, and the distance between the point calculated by the scheme and the initial point is approximately 0.340988'. The minimum distance of the calculated point is related to time 3 to 4 with a time difference of 11.726667', which is 0.091553', and the maximum distance of the calculated point is related to time 7 to 8 with a time difference of 29.963333', which is 0.618941'.

Table (8): Calculation of the longitude and latitude of the observer using the Jean Meeus (1998) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.670000'	27.171436°	56.219970°	0.213848'	0.298215'	0.340988'
2 to 3	11.476667'	27.181208°	56.207690°	0.372455'	0.438583'	0.539759'
3 to 4	11.726667'	27.175166°	56.213296°	0.009946'	0.102235'	0.091553'
4 to 5	12.090000'	27.177341°	56.211729°	0.140472'	0.196242'	0.224227'
5 to 6	11.096667'	27.171708°	56.214966°	0.197499'	0.002040'	0.197640'
6 to 7	12.846667'	27.172716°	56.214487°	0.137050'	0.030798'	0.139856'
7 to 8	29.963333'	27.164934°	56.217498°	0.603983'	0.149864'	0.618941'

Similarly, in Table (9), the longitude and latitude of the points in Table (7) are calculated using the Michalsky (1988) algorithm, as in Table (8). The maximum error for time 7 to 8 is 0.689683', and the minimum error is related to time 3 to 4 with a value of 0.225694'.

Table (9): Calculation of the longitude and latitude of the observer using the Michalsky(1988) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.670000'	27.169260°	56.218161°	0.344400'	0.189660'	0.383770'
2 to 3	11.476667'	27.179094°	56.205803°	0.245626'	0.551826'	0.549296'
3 to 4	11.726667'	27.173137°	56.211330°	0.111801'	0.220185'	0.225694'
4 to 5	12.090000'	27.175416°	56.209688°	0.024984'	0.318704'	0.284814'
5 to 6	11.096667'	27.169901°	56.212858°	0.305937'	0.128542'	0.326831'
6 to 7	12.846667'	27.171042°	56.212315°	0.237488'	0.161108'	0.277573'
7 to 8	29.963333'	27.163515°	56.215228°	0.689111'	0.013651'	0.689683'

The time difference between the points in Tables (8 and 9) is about 10 minutes. To calculate the role of the time difference and the accuracy of the scheme, another experiment with a time difference of 10 to approximately 100 minutes, such as Tables (10 and 11), is required.

Table (10): Calculation of the longitude and latitude of the observer using the Jean Meeus (1998) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.670000'	27.171436°	56.219970°	0.213848'	0.298215'	0.340988'
1to 3	22.146667'	27.174999°	56.213450°	0.000033'	0.092981'	0.082773'
1 to 4	33.873333'	27.175030°	56.213394°	0.001828'	0.096389'	0.085826 '
1 to 5	45.963333'	27.175298°	56.213382°	0.017878'	0.125778'	0.113389'
1 to 6	57.060000'	27.175037°	56.213643°	0.002192'	0.097056'	0.086429'
1 to 7	69.906667'	27.174894°	56.213643°	0.006356'	0.081404'	0.072745'
1 to 8	99.870000'	27.173976°	56.215323°	0.061418'	0.019389'	0.063838 '

In the table above, for time 1 to 3 equal to the times of Table (7), the longitude and latitude calculated by the Jean Meeus (1998) algorithm are 56.213450 ° and 27.174999°, respectively, and the scheme error for calculating longitude and latitude is 0.092981' and 0.000033', and as a result, the distance of the point calculated by the scheme from the initial point is approximately 0.082773'. The minimum distance of the calculated point is related to time 1 to 8, which is 0.063838', and the maximum distance of the calculated point is related to time 1 to 2, which is 0.340988'.

Similarly, in Table (11), the longitude and latitude of the times of Table (7) are calculated using the Michalsky(1988) algorithm, as in Table (10). The maximum error for time 1 to 2 is 0.383770', and the minimum error is related to time 1 to 8 with a value of 0.205636'.

Table (11): Calculation of the longitude and latitude of the observer using the Michalsky(1988) method

Route	Time difference	Calculated latitude of destination	Calculated longitude of destination	Error in latitude of destination	Error in longitude of destination	Distance between destination and calculated point
1 to 2	10.670000'	27.169260°	56.218161°	0.344400'	0.189660'	0.383770'
1to 3	22.146667'	27.172846°	56.211600°	0.129229'	0.204024'	0.222960'
1 to 4	33.873333'	27.172900°	56.211500°	0.125979'	0.209975'	0.225462'
1 to 5	45.963333'	27.173192°	56.210967°	0.108503'	0.241976'	0.241229'
1 to 6	57.060000'	27.172952°	56.211406°	0.122882'	0.215646'	0.227978'
1 to 7	69.906667'	27.172835°	56.211621°	0.129918'	0.202763'	0.222448'
1 to 8	99.870000'	27.171976°	56.213193°	0.181436'	0.108449'	0.205632 '

Conclusion:

Using the algorithms provided in the books of Jean Meeus (1998), Michalsky (1988), and the Astro Calculator software (to extract the position of the sun), the input data required to determine the position of the sun were compared several times with the British astronomical calendar, and the mentioned software. The existing errors were identified and after reviewing and making the necessary corrections in the written programs, feedback was taken again. Then, by entering the time and day of the Gregorian calendar, as well as the height of the sun to the program written in Matlab (i.e., using trigonometric relations and the secant method), the intersection point of the two lines of location on the Earth's surface, which indicates the real position of the observer, can be determined without the need for the internet.

The results of Table (12 and 13) show that the minimum average error between the destination point and the calculated point in the road test (moving observer), the Jean Meeus (1998) method with different time differences is equal to 0.204946', and also the minimum average error between the initial point and the calculated point when the observer is stationary, the Jean Meeus (1998) method with different time differences is equal to 0.120855'. Therefore, if the observer can take the heights of the objects with the appropriate accuracy, an accurate position is obtained.

Table (12) Average error in the distance between the destination point and the calculated point in the road test

The average error in the distance between the destination point and the calculated point.	Method
0.271639'	Jean Meeus (1998) with a time difference of approximately 10 minutes.
0.349072'	Michalsky(1988) with a time difference of approximately 10 minutes.
0.204946'	Jean Meeus (1998) with different time differences
0.321357'	Michalsky (1988) with different time differences

Table (13): Average error in the distance between the initial point and the calculated point by the stationary observer

The average error in the distance between the destination point and the calculated point.	Method
0.307566'	Jean Meeus (1998) with a time difference of approximately 10 minutes.
0.391094'	Michalsky(1988) with a time difference of approximately 10 minutes.
0.120855'	Jean Meeus (1998) with different time differences
0.247068'	Michalsky (1988) with different time differences

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APPENDIX A

Flowchart of the method of transferring the line of position