Upper Limits for Velocity, Momentum, and Energy of Motion

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Abstract: The velocity of a moving object with its non-zero mass or a moving particle with its non-zero mass is less than the speed of light because light has no mass. So, the speed of light is the upper limit for all moving objects in the Universe. In this article, the upper limits for momentum and energy of motion are derived for scientific problem-solving.

Keywords: classical mechanics, electromagnetic energy, light waves, speed of light

1. Introduction

The Sun is the main source of natural light on the Earth. Light waves are generally electromagnetic and do not require a medium for their propagation. Thus, the light can travel in a vacuum. Quanta of light are the smallest discrete packets of electromagnetic energy. A quantum of light that has no mass is called a photon. The relativistic motion [1-5] of a particle is concerned with the motion of the particle whose velocity approaches the speed of light.

2. Velocity-Momentum Relation

In classical mechanics, a variable-mass object is an object whose mass varies with time. A rocket, a moving object that burns completely at regular time intervals, and an object in motion that loses its mass at every unit of time are examples of variable-mass. In this section, the author introduces definitions of momentum-velocity equivalence [4-9].

Definition for constant-mass: The momentum (P) of a moving object with its constant-mass (m) is directly proportional to its velocity (v).

$$P \ \alpha \ v \ \Rightarrow P = mv, \text{where } m \text{ is constant.}$$

$$\text{Here,} \frac{dP}{dt} = F = \frac{d(mv)}{dt} = m\frac{dv}{dt}$$

Definition for variable-mass: Suppose a moving object loses its mass (m) at every unit of time and increases its velocity (v) proportionately with its mass (m). Then, the momentum (P) of the moving object is directly proportional to its velocity (v).

$$P \ \alpha \ v \ \Rightarrow P = mv = x_i v_i \text{ at the unit } t_i \text{of time for } i = 1, 2, 3, 4, \cdots, \text{where } 0 < x < m.$$

$$\text{Then,} \frac{dP}{dt} = F = v\frac{dx}{dt} + x\frac{dv}{dt} = \frac{d(mv)}{dt}$$

The above equation can be written as follows for our understanding:

$$\frac{dP}{dt} = F = v\frac{dm}{dt} + m\frac{dv}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} \tag{1}$$

3. Kinetic Energy

Kinetic energy is a fundamental concept in classical mechanics that quantifies the mechanical energy performed by an object due to its motion.

The equation of kinetic energy is given below:

$$E = \frac{1}{2}mv^2, \tag{2}$$

where m is the mass and v is the velocity.

By differentiating the equation of kinetic energy with respect to time t, we obtain

$$\frac{dE}{dt} = \frac{1}{2}\left(m2v\frac{dv}{dt}\right)$$

By simplifying the above differential equation, we receive

$$\frac{dE}{dt} = mv\frac{dv}{dt}$$

where the mass (m) of an object in motion is constant.
From the equation (1), we conclude that

$$dE = mvdv = vd(mv). \tag{3}$$

4. Derivation of the Lorentz Factor

The Lorentz factor is the factor by which time, length, and mass change for an object moving at a speed close to the speed of light. By the following experiment, the author of this article derives the equation of the Lorentz factor [5, 9].

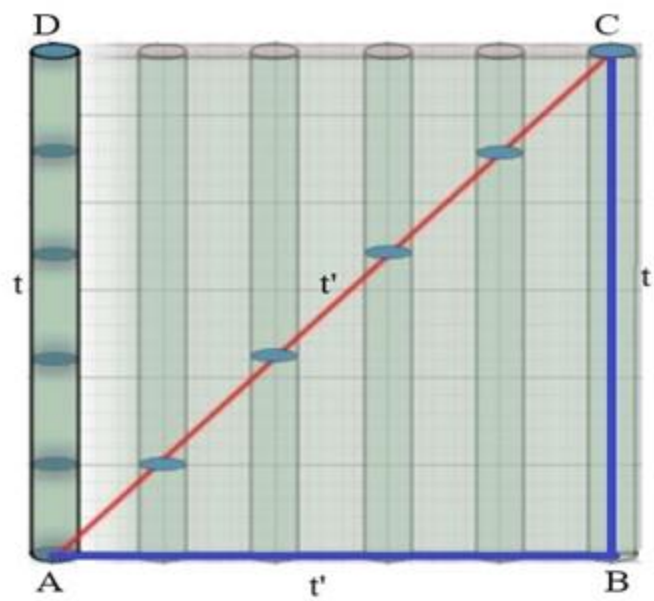

Figure 1. Relativistic speed of the metal tube.

Figure 1. depicts the relativistic speed of the metal tube (AD). Let's consider that the metal tube shown in the Figure 1 is at a stationary position and the light takes time t to travel from the point

A to the point D. The length of the metal tube is measured as follows: AB = BC = h = ct, where c is the speed of light and t is the time taken to move from the point A to the point D.

Suppose the metal tube with mass m and velocity v moves from the point A to the point B horizontally and the light travels from the point A along with the moving metal tube. When the light reaches the point C at time t', the metal tube moves to point B at time t'.

$$AB = vt' \text{and } AC = ct'$$

According to the definition of the Pythagoras theorem, we can derive the Lorentz factor as follows:

$$AC^2 = AB^2 + BC^2 \tag{4}$$

By substituting $AB = vt'$and $AC = ct'$ in the equation (4), we obtain

$$(ct')^2 = (vt')^2 + (ct)^2$$

$$t'^2\left(1 - \frac{V^2}{C^2}\right) = t^2$$

By simplifying the above equation, we get

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{t'}{t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5}$$

Hence, γ is the Lorentz factor.

5. Mass-Energy Relation

The equation of relativistic mass is m=$m_0\gamma$,

where the Lorentz factor $is\ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$\text{Now, m} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{6}$$

Squaring both sides of the equation (6):

$$m^2 = \frac{{m_0}^2}{1 - \frac{v^2}{c^2}} \Longrightarrow m^2(c^2 - v^2) = {m_0}^2c^2$$

By simplifying the above equation, we obtain

$$m^2c^2 - m^2v^2 = {m_0}^2c^2 \tag{7}$$

By differentiating the equation (7) with respect to time, we get

$$c^2 2m\frac{dm}{dt} - \left[2mv^2\frac{dm}{dt} + 2m^2 v\frac{dv}{dt}\right] = 0$$

By simplifying the above equation, we receive

$$c^2\frac{dm}{dt} = v^2\frac{dm}{dt} + mv\frac{dv}{dt} = v\left[v\frac{dm}{dt} + m\frac{dv}{dt}\right], \text{where } v\frac{dm}{dt} + m\frac{dv}{dt} = \frac{d(mv)}{dt}.$$

$$c^2 dm = vd(mv) \Rightarrow cd(cm) = vd(mv) \tag{8}$$

Substituting the equation (8) in the differential equation (2) of kinetic energy:

$$dE = cd(cm) = vd(mv), \tag{9}$$

where c is a numeric constant and also one of the values of v.
Therefore, $v = c$. (10)

By substituting the equation (10) in the equation (1), we conclude that

$$E = \frac{1}{2}mc^2. \tag{11}$$

6. Energy-Work Relation

The equations of displacement (s), initial velocity (u), final velocity (v), acceleration (a), force (F), work (W), and energy (E) of a moving object with mass (m) and time (t) are shown below:

$$a = \frac{v-u}{t} \Rightarrow v = u + at \Rightarrow \frac{ds}{dt} = u + at \Rightarrow \int_0^s ds = \int_0^t (udt + atdt) \Rightarrow s = ut + a\frac{t^2}{2}$$

$$v^2 = (u+at)^2 \Rightarrow v^2 = u^2 + 2a\left(ut + a\frac{t^2}{2}\right) \Rightarrow v^2 = u^2 + 2as \text{ and } F = ma.$$

Let us derive the energy-work equivalence [6-10] from the velocity equation.

$$v^2 = u^2 + 2as, \tag{12}$$

where u is the initial velocity, v is the final velocity, a is the acceleration, and s is the displacement of a moving object.

Now, let us consider the velocity of a moving particle approaching the speed of light. Then,

$$u = 0 \text{ and } v = c. \tag{13}$$

By substituting the equation (13) in the equation (12), we obtain

$$c^2 = 0 + 2as \Rightarrow c^2 = 2as \Rightarrow as = \frac{1}{2}c^2 \Rightarrow mas = \frac{1}{2}mc^2 \Rightarrow F \times s = W = \frac{1}{2}mc^2, \tag{14}$$

where W denotes the work.
From the equation (11) and (14), we conclude that

$$E = \frac{1}{2}mc^2, \tag{15}$$

The equation (15) denotes the mass-energy equivalence.

7. Upper Limits

The relativistic motion is concerned with an object whose velocity approaches the speed of light. The relativistic momentum (P) of a particle with its non-zero mass (m) cannot be equal to mc.
$i.e.$ $P \neq mc$ because c is the speed of light with zero mass.
Therefore, $P = mc_v, (c_v < c)$, is the upper limit for momentum. (16)

We know that the speeds of subatomic particles with mass are less than the speed of light. For example, neutrino speed is not faster than the speed of light because it has mass.

Since $P \neq mc$, the relativistic energy $(E) \neq mc^2$.

If $P = mc_v$, then $E = \frac{1}{2}mc_v^2 < \frac{1}{2}mc^2$.

Hence, $E = \frac{1}{2}mc^2$ is the upper limit for the energy of motion. (17)

8. Conclusion

In this article, the upper limits for momentum and the energy of motion have been proved mathematically for scientific problem-solving in the relativistic mechanics. The relativistic mechanics is concerned with the motion of bodies whose velocity approaches the speed of light. This idea can enable the researchers for further involvements in the scientific research and development.

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