Equations of the Energy-Work Relation: Right and Wrong

Chinnaraji Annamalai
School of Management, Indian Institute of Technology, Kharagpur, India
Email: anna@iitkgp.ac.in
https://orcid.org/0000-0002-0992-2584

Abstract: In classical mechanics, the energy of an object in motion is the form of energy that quantifies the work performed by the object due to its motion and mass. The energy is also transformed into sound energy, thermal energy including heat and light, and material deformation. This paper points out how to use the equations of motion to derive the energy-work equations.

Keywords: motion, deformation, heat, light, sound

I. Variation in Energy-Work Equation

In Newtonian mechanics, kinetic energy, force, and work [1-15] are measured by the mass, displacement, velocity, and, acceleration of a moving object over time.

Displacement is quantified by the linear distance between the initial and final positions of an object in motion. The displacement (s)-uniform velocity (v) equation is given below:

$$v = \frac{\text{displacement}}{\text{time}} = \frac{s}{t} \Rightarrow s = vt. \quad (1)$$

$$\text{Acceleration } (a) = \frac{v - u}{t} = \frac{v}{t}, \text{where the initial velocity } (u) = 0. \quad (2)$$

$$v^2 = u^2 + 2as \Rightarrow v^2 = 2as, \text{where the initial velocity } (u) = 0. \quad (3)$$

Now, let us prove the variations on the equations of relation between the energy (E) and the work (W) of a moving object given below:

$$(i)\ E = W = \frac{1}{2}mv^2 \text{ and } (ii)\ E = \frac{1}{2}W = \frac{1}{2}mv^2.$$

First, let us prove the equation (i).
The energy of motion, also called kinetic energy, is denoted by the following equation.

$$E = \frac{1}{2}mv^2 \quad (4)$$

By substituting the equation (3) in the equation (4), we get

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m \times 2as = ma \times s = F \times s = W, \text{where } F = ma. \quad (5)$$

From the equation (5), we conclude that

$$E = W = \frac{1}{2}mv^2.$$

Hence, the equation (i) is proved.

Next, let us prove the equation (ii).

Substituting the equations (1) and (2) in the equation (4) as follows:

$$E = \frac{1}{2}mv^2 = \frac{1}{2}m(v \times v) = \frac{1}{2}m\left(\frac{s}{t} \times v\right) = \frac{1}{2}m\left(s \times \frac{v}{t}\right) = \frac{1}{2}ma \times s = \frac{1}{2}F \times s. \quad (6)$$

From the equation (6), we conclude that

$$E = \frac{1}{2}W = \frac{1}{2}mv^2.$$

Hence, the equation (ii) is proved.

Both equations $E = W = \frac{1}{2}mv^2$ and $E = \frac{1}{2}W = \frac{1}{2}mv^2$ are not equal.

II. Error Correction in Energy-Work Equation

There is an error in the equation (5) by substituting $v^2 = 2as$. Let us find and correct the error in the equation (5) by mathematical derivation shown below:

$$a = \frac{v - u}{t} \Rightarrow \frac{ds}{dt} = u + at \Rightarrow \int_0^s ds = \int_0^t (udt + atdt) \Rightarrow s = ut + a\frac{t^2}{2}. \quad (7)$$

$$v^2 = (u + at)^2 \Rightarrow v^2 = u^2 + 2a\left(ut + a\frac{t^2}{2}\right) \Rightarrow v^2 = u^2 + 2as. \quad (8)$$

If acceleration $(a) = 0$ and v is the uniform velocity in the equation (7), then $s = ut = vt$. (9)

Displacement (s), initial velocity (u), final velocity (v), and acceleration (a) of a moving object with mass (m) over time (t) are shown in the above equations.

If initial velocity $(u) = 0$, then $v^2 = 0^2 + 2a\left(0t + a\frac{t^2}{2}\right)$, that is, $v^2 = a^2t^2$. (10)

The initial velocity $(u) = 0$ should not be substituted in the equation (3) directly.

Velocity (v) in the equation (1) and the equation (9) denotes uniform velocity and the equations (1) and (9) should not be substituted in the equation of energy-work relation.

Definition of Energy-Work Relation: let us consider an object of mass (m) moving with a velocity (v), the energy (E) of the object in motion, also called kinetic energy, is defined as work (W) done to accelerate the object from rest to its current velocity.

$$E = W = F \times s, \text{where } F = ma \text{ and } s = ut + a\frac{t^2}{2}. \quad (11)$$

$$\text{If initial velocity } (u) = 0, \text{then } s = (0)t + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}at^2. \quad (12)$$

By substituting the equations (10) and (12) in (11), we get

$$E = W = \frac{1}{2}mv^2, \text{which is the correct equation.}$$

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