Quantum dark energy in a seven-dimensional universe

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Abstract:

This paper explains the dark energy and acceleration of the universe by quantizing the space in hidden dimensions, which provides the basis and background for the gravitational force through the curvature of space-time. Space-time is considered to be made of a four-dimensional elastic grid in a seven-dimensional universe in which matter also expands along with the universe. Each cube of the grid is considered a quantum of hidden three-dimensional space of Planck volume containing Planck charge, which makes the universe seven dimensional. The dark energy is explained by the electrostatic repulsion between Planck charges in each quantum of the hidden space. Mathematically, this electrostatic repulsion is related to the Hubble constant, which explains the accelerated expansion, dark energy, and increase in the cosmological potential energy of the matter. Expansion of space-time is considered not due to the creation of the new space but due to the stretching of the existing space-time itself like an elastic ruler where the proper length and volume remain constant. The relative values of the Planck constant, gravitational constant, permittivity of free space, and Boltzmann constant are shown to vary owing to the expansion or contraction of space-time in the cosmological, gravitational, and relativistic frameworks. This theory also builds a preliminary framework for the relativistic Newtonian theory of gravity, the relativistic MONDian (Modified Newtonian dynamics) gravity, identifies a valid reason for the transition of Newtonian gravity to MOND at a_0 and explains the dynamics of galaxy clusters without the dark matter.

1. Introduction

In this theory of quantum dark energy in a seven-dimensional universe, Hubble expansion is considered to be due to the stretching of the existing space-time rather than the creation of a new space that is analogous to the markers on an elastic ruler stretching along with the expansion of the ruler, as opposed to the raisin bread model, where matter does not expand along with space. Therefore, matter (e.g., protons, neutrons, electrons, and atoms) is considered to stretch uniformly along with space-time and not become diluted with the expansion of the universe. As the matter also stretches with space, the expansion is non-observable locally but can be observed through the redshift of the light coming from a far-off space of a lower stretch. Therefore, matter exists in different energy states based on the magnitude of the space-time stretch. The matter continues to move to higher potential energies as the space-time expands, but the energy of the photon remains the same. Therefore, in a lower stretch space-time, a blue photon has less energy compared to the same blue photon with the same frequency emitted in a higher stretch space-time. Therefore, when light travels from a lower stretch of space-time to a higher stretch of space-time, it becomes redshifted without violating the law of energy conservation.

The cosmological potential energy of the mass increases with the space-time expansion. In contrast, the energy of the photon does not change with the space-time expansion and it does not gravitate, as it does not resist the space-time expansion because the mass of the photon is zero but is affected by the gravity of the other objects, which means that the energy of a photon does not curve the space-time around it, but takes the path of the curved space around any matter. Thus, electromagnetic fields and electromagnetic-related potential energies, whose forces are meditated by virtual photons, do not curve the space-time in this model of the universe. As photons do not curve the space-time in this theory, the annihilation of matter converts the cosmological potential energy, gravitational potential energy, and the rest mass to photons. Additionally, this theory predicts that the annihilation of matter should release some energy in the form of gravitational waves, but this is not actively considered in this study. Therefore, the mass-energy equivalence principle is only partially satisfied, as all forms of energy do not curve space-time in this theory, but uphold the weak equivalence principle by upholding the equivalence of passive gravitational mass and inertial mass.

This theory proposes an absolute inertial frame, revives the concept of relativistic mass, and considers it to be a part of the inertial mass or passive gravitational mass. However, in this theory, only the rest mass is considered as the active gravitational mass, and no other form of mass or energy is considered as part of the active gravitational mass, without violating Newton's third law of motion. However, a change in the gravitational or the cosmological potential energy is stored as a change in the inertial mass i.e. due to the expansion/contraction of matter. Therefore, a higher stretch space-time consumes more energy than a lower stretch space-time for the same work done because of the relative increase in the inertial mass/potential energy. However, the numerical value of the locally consumed energy will be the same for the work done in both cases, as the local inertial mass is the same in both cases, although the relative inertial mass is different. Hence, energy is considered relative rather than absolute in this theory, although it is conserved.

The proper distance between any two points in space is considered to remain constant despite the stretching of space-time, similar to the markers on an elastic ruler. Therefore, the volume of the universe does not change with space expansion. The accelerated expansion of space-time and matter is considered to be due to electrostatic repulsion between the Planck charges present in the Planck volumes, and hence explains the dark energy.

Space-time is considered to be made of a four-dimensional elastic grid in a seven-dimensional universe. Each cube of the grid is considered to be a quantum of the three-dimensional space of the Planck volume containing the Planck charge. These cubes can become loosely connected to the surrounding cubes to form black holes. As the Hubble constant H_0 decreases with time, the rate of acceleration of the universe is considered to decrease because of the contraction force of the space-time elastic grid opposing the electrostatic repulsion between Planck charges. The space-time membrane acts as a dielectric material between the Planck charges. Matter only exists as a wave function (Ψ) in the four-dimensional space-time grid but not in the hidden three spatial dimensions. The presence of matter in space-time would increase the force required to expand the space-time grid as the matter also expands along with space. The presence of matter increases the permittivity of space-time and hence reduces the electrostatic repulsion within the grid enveloped by the matter compared to the electrostatic repulsion outside the matter. As the expansion of the space-time grid surrounding the matter will be greater than the expansion of the matter owing to the permittivity difference and the net compressing force on the matter from the surrounding Planck charges, space-time becomes naturally curved around any mass. This compression force is considered to be the gravitational force in this theory. Therefore, in this theory, except for photons/electromagnetic fields, the gravitational force is considered a real force rather than a fictitious force of the general theory of relativity.

In this theory, the electrostatic potential energy between the Planck charges in the three-dimensional space is considered to be the same as the dark energy, causing accelerated expansion of the universe. The electrostatic dark energy in three-dimensional space does not gravitate because this energy itself is the cause of the gravitational force in the four-dimensional space-time enveloping the three-dimensional Planck charges. The constancy of the speed of light should not limit the apparent expansion velocity of the universe in this model, as it considers it to be applicable only for objects moving through space but not for the expansion of space-time. Beginning with the birth of the universe, the first law of thermodynamics is strictly followed in this theory to uphold the law of conservation of energy, which includes energy conservation in dark energy and cosmological redshift. The linear momentum and angular momentum are also conserved in this theory.

This theory proposes a flat or zero-curvature, isotropic, and homogenous universe, where the rate of acceleration continues to decrease proportionally to the age of the universe, and its velocity only becomes zero after an infinite amount of time. However, the universe continues to accelerate, but only at a continuously decreasing rate, asymptoting to zero. Invoking the critical density ($\Omega_0 = 1$) or cosmic inflation is not required to explain the flatness in this model of the universe because the uniform expansion of the entire universe due to electrostatic repulsion explains why the universe is flat rather than closed or open.

As there is no increase in the volume of the universe with time, the Big Bang should be replaced with Big Repulsion. This theory makes the gravity electrostatic background dependent to explain the curvature of space-time, dark energy, and cosmological redshift. The electrostatic background provides an additional background to the quantum fields on top of the gravitational background, which mandates an absolute inertial frame and universal time, which is the Hubble time in this model of the universe. Therefore, this theory makes the universe three-layered.

1) Electrostatic or the power layer
2) Space-time or the gravitational layer
3) Quantum fields layer

The first layer acts as the power or energy source for the universe. The gravitational layer creates gravity, and the quantum field layer enables matter, energy, and the rest of the fundamental forces of nature to work on top of the space-time or gravitational layer. This theory also eliminates the cosmic event horizon and the cosmic scale factor (a) because the proper length and volume remain constant despite the expansion of space-time. Therefore, we should be able to see light from any part of the universe, provided we have enough time without any distance limit, such as a cosmic event horizon. The cosmological constant Λ in the gravitational field equations to factor in the dark energy is not required in this model of the universe because dark energy does not gravitate and is isolated to the electrostatic background, which is handled independently of gravity.

The relative values of the Planck constant, gravitational constant, Boltzmann constant, and permittivity of free space are shown to vary owing to the expansion of the space-time grid, proving the existence of an absolute frame of reference. The aforementioned constants also vary with the gravitational potential and in all moving inertial frames. However, the absolute inertial frame cannot be uniquely identified in this theory, as the change in the physical constants is compensated by time dilation or the change in the inertial mass/potential energy. However, this theory requires an absolute inertial frame to explain dark energy, MONDian gravity, and the change in inertial mass/potential energy due to the expansion/contraction of space-time. This theory upholds the invariance of the speed of light in all frames (locally for non-inertial frames), the special theory of relativity, and Lorentz invariance.

In this theory, the constants that do not change with the expansion of the universe, gravitational potential, or velocity of the frame are listed below but are not limited to.

1) Speed of light
2) Planck length
3) Planck time
4) Planck temperature
5) Electric charge
6) Fine-structure constant (α)
7) Rest mass

Black-hole singularities do not exist in this model of the universe as the mass is converted to pure informational entropy at the event horizon, and the space-time terminates at the event horizon and cannot be extended beyond the event horizon, as in the general theory of relativity, as the effective radial length, time, and tangible mass become zero at the event horizon. Additionally, charged black holes do not exist in this model of the universe, as the permittivity of free space becomes infinite at the event horizon. Therefore, stationary black holes can only have two properties, namely informational (entropic) mass and angular momentum, and hence only partially satisfy the no-hair theorem. This theory also builds a preliminary framework for the relativistic Newtonian theory of gravity, which is Lorentz-invariant for a potential full-fledged covariant theory. Relativistic Newtonian gravity proposed in this theory produces similar or even better results in closed form than the general theory of relativity without using weak field approximations for the below listed.

1) Black hole radius (Schwarzschild radius)
2) Photon sphere
3) Gravitational lensing
4) Perihelion precession of Mercury
5) Shapiro time delay

This theory generates relativistic Poisson equations and Maxwell-like equations for gravity or GEM (Gravitoelectromagnetism) equations to enable them to operate in strong gravitational fields and relativistic speeds, which can explain the frame-dragging effect, orbital precession, geodetic effect, and gravitational waves. This theory also generates the equivalent of the Schwarzschild and FLRW (Friedmann–Lemaître–Robertson–Walker) metrics.

The expansion of the universe proposed in this theory identifies a valid reason for the transition of Newtonian gravity to MOND at a_0 ($\sim 1.2 \times 10^{-10}$ m/s^2) and hence makes a case for the absolute reference frame. However, this theory establishes that transitioning to a deep-MOND regime is only possible in the radial space around the black holes, but not around ordinary matter. This theory also builds a framework for the relativistic MONDian gravity and explains the dynamics of galaxy clusters without dark matter by using an updated virial theorem.

In addition, this new model could act as a precursor to theories explaining baryogenesis and primordial nucleosynthesis based on how the initial extremely high expansion energy of space-time interacts with quantum vacuum fluctuations to create matter. However, the cosmic microwave background radiation (CMBR) anisotropy and angular power spectrum still need to be explained in this theory. In addition, this theory explains the early formation of galaxies/objects that were recently observed through the James Webb Space Telescope (JWST) by proving that they were formed much later than calculated.

2. Relativistic acceleration of space-time

Let us consider two points, A and B, on the space-time fabric. We can calculate the apparent outward acceleration of point B when observed from point A based on the redshift of the light coming from point B.

D = Proper distance between points A and B
λ = D (let us consider the wavelength of light to be equal to D)

Thus, by the time light travels from point B to point A, its wavelength expands by D(1+z) based on the cosmological redshift (z) phenomenon. Therefore, point B apparently moved from its original location by D(1+z)–D, which is equal to Dz.

The apparent velocity v of point B due to the redshift is given by $v = \frac{Distance}{time} = \frac{Dz}{t}$, where t is the time taken for the light to travel from point B to point A. As the proper distance between points A and B remains constant, the real velocity of point B is zero. Thus, the apparent acceleration (a) is given by $a = \frac{Dz}{t^2}$. As space stretches like an elastic ruler, the proper distance between points A and B should always remain the same irrespective of apparent acceleration. Therefore, light should take the same amount of time t to travel the apparent distance of D(1+z), which is actually D owing to the constancy of the speed of light. Therefore, the time t is given by t=D/c.

So, $v = \frac{Dz}{\left(\frac{D}{c}\right)} = zc$ and $a = \frac{Dz}{\left(\frac{D}{c}\right)^2} = \frac{zc^2}{D}$

$$v = zc \tag{1}$$

As v=zc (1) has already been established in conjunction with the Hubble law, the above derivation proves that space stretches like an elastic ruler, where the length and volume remain constant, as opposed to the raisin bread model, where length increases and matter is diluted with the expansion of space-time. Therefore, this theory provides a theoretical basis for v=zc, whereas the raisin-bread model does not provide any theoretical reasoning.

Based on the equations $v=H_0D$ and v=zc from Hubble law, $\frac{z}{D}=\frac{H_0}{c}$, where v is the apparent receding velocity, H_0 is the Hubble constant, D is the distance, z is the cosmological redshift, and c is the speed of light. Therefore, the apparent acceleration $a=\frac{H_0c^2}{c}=cH_0=$ **7.549 × 10^{-10} m/s^2**, which is the current acceleration of the universe for H_0=77.7 (km/s)/Mpc based on H_0 values within the range mentioned in references [1], [2], [3]. As the universe is stretching with a constant volume, there must be length contraction and time dilation in the past, which are given below. As we describe past events from the perspective of the current cosmological reference frame, and to maintain the constancy of the speed of light in all reference frames, both the current length L_0 and the current time T_0 are divided by (1+z).

Cosmological length contraction: $L=\frac{L_0}{(1+z)}$; Cosmological time dilation: $T=\frac{T_0}{(1+z)}$

The cosmological time dilation proposed in this theory is inherent to the fabric of space-time due to the cosmological potential being similar to the gravitational time dilation due to the gravitational potential, and the associated redshift is the cosmological redshift due to the stretching of space-time. Therefore, the relative acceleration of the universe is

$$a=cH_0(1+z) \tag{2}$$

Therefore, point B will always move from point A with an apparent acceleration equal to the above (2), although the proper distance between the two points always remains constant. As the Hubble constant decreases over time, the rate of apparent acceleration of the universe or space-time is also considered to decrease over time. As the proper distance between the two points and the volume of the universe always remains constant, acceleration (2) is only considered apparent.

3. Relativistic Hubble law and the cosmological redshift

Based on the relativistic acceleration of space-time (2), we can calculate the relativistic velocity v of point B from the big repulsion (point A) on the space-time fabric based on the cosmological redshift z.

$$\frac{dv}{dt}=a; \quad dv=a\,dt; \quad dv=cH_0(1+z)dt;$$

$$\int_0^v dv=\int_{t_p}^{t} cH_0(1+z)dt \quad ; \quad \int_0^v dv=c\int_{t_p}^{T_0}\frac{1}{T}(1+\frac{v}{c})dT \text{ using (1)} \quad ; \quad \int_0^v \frac{1}{(c+v)}dv=\int_{t_p}^{T_0}\frac{1}{T}dT$$

$$\ln(c+v)-\ln(c)=\ln(T_0)-\ln\left(t_p\right)$$

Planck time t_p is the minimum age of the universe at the beginning of the big repulsion owing to

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the quantization of space and time to the Planck units. T is the age of the universe, which is considered to be the Hubble time in this universe model. As ln(0) is undefined, the singularity or zero time of the big repulsion is also undefined.

$$\frac{c+v}{c}=\frac{T_0}{t_p} \quad ; \quad 1+\frac{v}{c}=\frac{1}{H_0 t_p} \quad ; \quad 1+z=\frac{1}{H_0 t_p}$$

Therefore, the maximum possible cosmological redshift can be expressed as follows:

$$z=\frac{1}{H_0 t_p}-1 \tag{3}$$

which is **7.36614 × 10⁶⁰** for H_0=77.7 (km/s)/Mpc, and the maximum apparent relativistic recession velocity that is possible is 7.36614×10^{60} times the speed of light.

$$\text{For } H_0 < H_D, \quad 1+z=\frac{H_D}{H_0}=\frac{1}{H_0\left(\frac{1}{H_0}-\frac{D}{c}\right)}=\frac{1}{1-\frac{H_0 D}{c}}=\frac{c}{c-H_0 D} \tag{4}$$

where H_D is Hubble constant when light is emitted at distance D, and $\frac{1}{H_D} \geq t_p$.

Therefore, the relativistic Hubble law based on (1) and (4) is expressed as follows:

$$v=H_0 D(1+z)=\frac{H_0 D}{(1-\frac{H_0 D}{c})} \tag{5}$$

The acceleration at the beginning of the universe (Big Repulsion) is as follows:

$$a=cH_0(1+z)=cH_0\frac{1}{H_0 t_p}=\frac{c}{t_p}$$ = **5.56×10^{51} m/s²** from (2) and (3)

The acceleration of the universe at a distance D from the present is as follows:

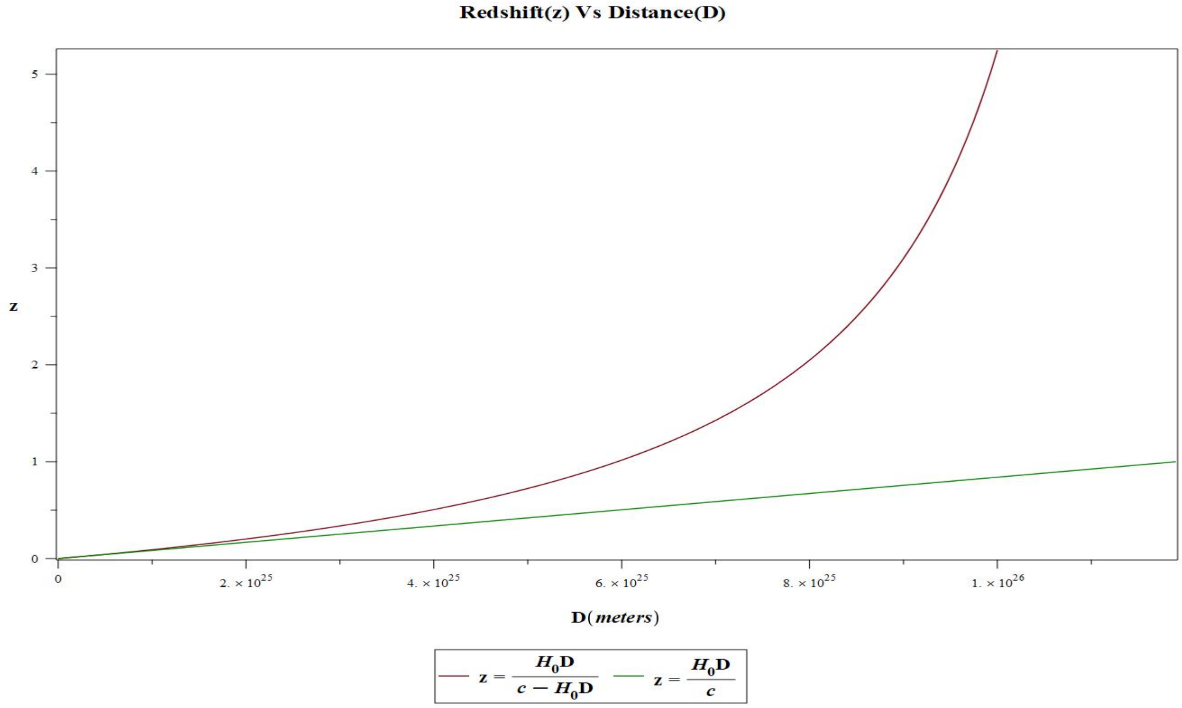
$$a=cH_0(1+z)=cH_0(\frac{c}{c-H_0 D})$$ from (2) and (4)

$$a=cH_0(\frac{c}{c-H_0 D}) \tag{6}$$

We can see in table I that the redshift z values match between the regular Hubble formula and the new relativistic formula for low values of D. The new formula restricts the maximum cosmological redshift to 7.36614×10^{60} owing to the model of the universe that is considered. As shown in Table I, the z values differed between the two formulas as D increased or the time traveled by the light approached the age of the universe. Therefore, the Hubble law $v=H_0D$ is only accurate up to moderate distances, as it is the limiting case of the relativistic Hubble law. We can also see that the new z-values in Table I are in line with the accelerating model of the universe.

Table I (Cosmological redshift values)

D (meters)	$z=\frac{H_0D}{c}$	$z=\frac{H_0D}{c-H_0D}$
1×10^{10}	$8.3994291420\times10^{-17}$	$8.3994291420\times10^{-17}$
2×10^{15}	$1.6798858284\times10^{-11}$	$1.6798858284\times10^{-11}$
3×10^{20}	$2.5198287426\times10^{-6}$	$2.5198350921\times10^{-6}$
$\sim5.9527854\times10^{25}$	0.5	1
1×10^{26}	0.8399429142	5.2477708815
1.19×10^{26}	0.9995320679	2136.0622240085
$\sim1.1905570\times10^{26}$	1	$7.3661442125\times10^{60}$ (Maximum/Big



Graph 1

The age of the universe is 12.58 billion years, which is the Hubble time. As the proper distance and volume remain constant, the maximum observable universe is only 12.58 × 2 billion light years across, which is 25.16 billion light-years for H_0=77.7 (km/s)/Mpc.

Important formulas: based on (4)

Time since big repulsion to the emission of light (T_b)	Time since the emission of light (T_z)	Distance vs Redshift
$T_b=\frac{1}{H_0(1+z)}$	$T_z=\frac{1}{H_0}\frac{z}{(1+z)}$	$D=\frac{zc}{H_0(1+z)}$

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Therefore, the light from galaxy HD1 with cosmological redshift z=13.27 [4] should have been emitted after ~800 million years instead of ~300 million years after the big repulsion, possibly giving enough time for it to form as a galaxy and hence resolving the issue of the early formation of galaxies/objects that were recently observed through the JWST.

4. Variable constants G, h, ε_0, and k_B and the change in inertial mass

As the speed of light is constant, the Planck length and Planck time are considered constants. Therefore, the product of the gravitational constant G and Planck constant h is considered to be constant. As the Planck charge $(q_p = \frac{e}{\sqrt{\alpha}})$ is conserved, alpha (α) or the fine-structure constant is considered constant. Therefore, the product of the Planck constant h and permittivity of free space ε_0 is considered constant. As the Planck temperature is considered to be constant, the product of the gravitational constant G and Boltzmann constant k_B is considered to be constant. Based on the above, we can observe how the constants G, h, ε_0, and k_B change over time, and calculate the relative values in the past and future using the law of conservation of energy.

Cosmological redshift $z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}}$ or $\lambda_{obs} = \lambda_{emit}(1+z)$ **(7)**

As the energy is conserved in cosmological redshift, $E = \frac{hc}{\lambda_{obs}} = \frac{h_{t_p} c}{\lambda_{emit}}$ **(8)**

Here, h is the current Planck constant and h_{t_p} is the old Planck constant when the age of the universe is Planck time t_p.

$z = \frac{1}{H_0 t_p} - 1$ from (3), and $h = h_{t_p}(1+z)$ based on (7) and (8) **(9)**

As $hG = h_{t_p} G_{t_p}$, $h\varepsilon_0 = h_{t_p}\varepsilon_{t_p}$, and $k_B G = k_{B_{t_p}} G_{t_p}$

$$h = h_{t_p}(1+z) = h_{t_p}(\frac{1}{H_0 t_p}) \quad \textbf{(10)}$$

$$G = \frac{G_{t_p}}{(1+z)} = G_{t_p}(H_0 t_p) \quad \textbf{(11)}$$

$$\varepsilon_0 = \frac{\varepsilon_{t_p}}{(1+z)} = \varepsilon_{t_p}(H_0 t_p) \quad \textbf{(12)}$$

$$k_B = k_{B_{t_p}}(1+z) = k_{B_{t_p}}(\frac{1}{H_0 t_p}) \quad \textbf{(13)}$$

Below are the relative values of G_{t_p}, h_{t_p}, ε_{t_p}, and $k_{B_{t_p}}$ in MKS units when the age of the universe is Planck time t_p, based on the current values of G, h, ε_0, and k_B for H_0=77.7 (km/s)/Mpc. Therefore, these constants vary proportionally with relativistic Hubble rate.

$h = 6.62607015\times10^{-34}$	$G = 6.67430\times10^{-11}$	$\varepsilon_0 = 8.8541878128\times10^{-12}$	$k_B = 1.380649\times10^{-23}$
$h_{t_p} = 8.99530332\times10^{-95}$	$G_{t_p} = 4.91638563\times10^{50}$	$\varepsilon_{t_p} = 6.52212243\times10^{49}$	$k_{B_{t_p}} = 1.87431709\times10^{-84}$

As [$E=h(1+z)\nu=m(1+z)c^2$] because of the constancy of the speed of light, it follows that $m_0 = m_z(1+z)$, where m_0 is the current mass and m_z is the original mass when the light was emitted in the past at cosmological redshift z. However, the mass increase due to the expansion of space-time should only be seen as an increase in the cosmological potential energy U_c or an increase in inertial mass, but not as a change in rest mass. This is analogous to the relativistic mass of an object moving at a certain velocity with a constant rest mass.

$$|U_c| = (m_z(1+z)-m_z)c^2 = m_z zc^2 = m_z\left(\frac{1}{1-\frac{H_0 D}{c}}-1\right)c^2 \approx m_z H_0 Dc \text{ using (4)}$$

Changes in inertial mass can be observed by converting m_0 and m_z to energy, which is the observed energy difference in the cosmological redshift. Therefore, the energy of a photon does not increase with space-time expansion, whereas the potential energy/inertial mass increases with expansion, which perfectly explains the observed energy difference in the cosmological redshift. Similarly, for gravitational redshift, the change in inertial mass is manifested as gravitational potential energy U. Similar to cosmological redshift, the change in inertial mass in gravitational redshift is associated with the change in physical constants G**,** h**,** ε_0**,** and k_B. Therefore, the energy of a photon does not change when moving against gravity, but the gravitational redshift z, which is the decrease in the frequency of the photon, is due to an increase in the relative Planck constant [$E=h(1+z)\nu$]. This proves that photons do not curve space-time by themselves but take the path of any curved space. As photons do not curve the space-time in this theory, the annihilation of matter would convert the cosmological potential energy, gravitational potential energy, and the rest mass to photons. Therefore, the gravitational potential energy U of mass m in the gravitational field of mass M can be expressed as follows:

$U = -(m(1+z)-m)c^2 = -mzc^2$, where m(1+z) is the inertial mass at infinity **(14)**

$$U = -mzc^2 \quad \textbf{(15)}$$

$$\frac{U}{m} = \phi = -zc^2 \quad \textbf{(16)}$$

where z is the gravitational redshift, and ϕ is the gravitational potential.

However, after factoring in the time dilation or change in inertial mass, the physical constants would appear to be locally constant in the cosmological, gravitational, and relativistic frameworks.

Table II (Relative change in the values of the physical constants)

Gravitational (From infinity)	$G(1+z)$	$\frac{h}{(1+z)}$	$\varepsilon_0(1+z)$	$\frac{k_B}{(1+z)}$
Transverse relativistic	$G(1+z)$ or $G\gamma$	$\frac{h}{(1+z)}$ or $\frac{h}{\gamma}$	$\varepsilon_0(1+z)$ or $\varepsilon_0\gamma$	$\frac{k_B}{(1+z)}$ or $\frac{k_B}{\gamma}$
Cosmological (From the big repulsion)	$\frac{G_{t_p}}{(1+z)}$	$h_{t_p}(1+z)$	$\frac{\varepsilon_{t_p}}{(1+z)}$	$k_{B_{t_p}}(1+z)$

where z is the gravitational, transverse relativistic, and cosmological redshifts, respectively.

For example, the Planck constant due to the cosmological redshift ten years from now is h(1+δz) (10), where δz is the change in the cosmological redshift value ten years from now.

$(1+\delta z)=\frac{H_0}{H_1}$ using (4), where H_1 is the Hubble constant after ten years.

δz values:

After 10 years: $7.9463103517\times10^{-10}$
After 50 years: $3.9731551758\times10^{-9}$
After 100 years: $7.9463103517\times10^{-9}$

Similar changes in the relative values of the physical constants can be observed and calculated in the gravitational field and relativistic frames using the factor (1+z), where z is the gravitational and transverse relativistic redshifts, respectively.

In Table III, the local inertial mass increases by a factor of (1+z) in all three cases owing to space expansion. Similarly, for space contraction, the local inertial mass decreases by a factor of (1+z). Here, m_0 is the local inertial mass, and m_z is the original mass at the point of photon emission, where the inertial mass is equal to the rest mass. The change in the inertial masses in the gravitational and cosmological cases should only be seen as a change in their respective potential energies, rather than a change in their rest masses, which always remain constant.

However, the absolute inertial frame cannot be uniquely identified in this theory based on the change in the physical constants, as any change will be compensated for by the time dilation or the change in the inertial mass; hence, the local physical laws remain unchanged. In the transverse relativistic case in Table III, although there is no difference between the rest and moving inertial frames according to the special theory of relativity, the change in the inertial mass can still be determined based on the assumed absolute reference frame. Refer to Section 6 for more details regarding the absolute reference frame.

Table III (Inertial mass change formulas)

Cosmological	Gravitational	Transverse relativistic
Space expansion: $m_0 = m_z(1+z) = m_z \dfrac{1}{1-\dfrac{H_0 D}{c}}$	Space expansion: $m_0 = m_z(1+z) = m_z \dfrac{1}{\sqrt{1-\dfrac{2GM}{Rc^2}}}$	In the rest inertial frame: $m_0 = m_z(1+z) = m_z \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$
Space contraction: $m_0 = \dfrac{m_z}{(1+z)} = m_z\left(1-\dfrac{H_0 D}{c}\right)$	Space contraction: $m_0 = \dfrac{m_z}{(1+z)} = m_z\sqrt{1-\dfrac{2GM}{Rc^2}}$	In the moving inertial frame: $m_0 = \dfrac{m_z}{(1+z)} = m_z\sqrt{1-\dfrac{v^2}{c^2}}$

5. Acceleration of the universe due to electrostatic repulsion

In this theory, the expansion of space-time is due to electrostatic repulsion between the Planck charges in the Planck volumes in the hidden three-dimensional space of the seven-dimensional universe.

The acceleration at the beginning of the big repulsion is $a = cH_0(1+z) = \dfrac{c}{t_p}$ based on (2) and (3), which can be reformulated as $\sqrt{\rho_{t_p} G_{t_p}}$, where ρ_{t_p} is the Planck energy density and G_{t_p} is the gravitational constant when the age of the universe is Planck time t_p. As acceleration a is directly related to the Planck energy density, it proves that the proposed model of the universe has Planck charges in Planck volumes, which requires the Planck energy density ρ_{t_p} in the hidden three-dimensional space. As we are determining the acceleration of the universe in the past with respect to our local cosmological reference frame, the relative values of the physical constants should be used. Therefore, $\dfrac{c}{t_p} = \sqrt{\rho_{t_p} G_{t_p}}$.

Planck energy $E_p = \dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{\alpha(l_p)} = \sqrt{\dfrac{\hbar c^5}{G}}$, where l_p is Planck length.

When the age of the universe is Planck time t_p, the energy density of the hidden three-dimensional space ρ_{t_p} of the universe is $\dfrac{E_{p(t_p)}}{(l_p)^3}$.

$$\rho_{t_p} = \frac{1}{4\pi\varepsilon_{t_p}}\frac{e^2}{\alpha(l_p)^4} = \frac{\sqrt{\dfrac{\hbar_{t_p} c^5}{G_{t_p}}}}{(l_p)^3} \quad ; \quad \rho_{t_p} G_{t_p} = \frac{1}{4\pi\varepsilon_{t_p}}\frac{e^2}{\alpha(l_p)^4} G_{t_p} = \left(\frac{c}{t_p}\right)^2 = a^2$$

$$G_{t_p} = \frac{G}{H_0 t_p} \text{ from (11)} \quad ; \quad \rho_{t_p} G_{t_p} = \frac{\rho_{t_p} G}{H_0 t_p} = \left(\frac{c}{t_p}\right)^2 \quad ; \quad \rho_{t_p} = \frac{H_0 c^2}{G t_p} = \mathbf{6.29 \times 10^{52}\ J/m^3}$$

The relative energy density ρ_{t_p} is determined with respect to the current energy scale of the universe. As the energy scale increases with the expansion of the universe, the relative energy density ρ_{t_p} decreases, although there is no real change in the energy density.

The current electrostatic energy density ρ responsible for the acceleration of the universe can be determined by setting D = 0 or z = 0 in the following generalized equation:

$$\rho G(1+z) = (cH_0(1+z))^2 \quad ; \quad \rho = \frac{(cH_0)^2(1+z)}{G} = \frac{(cH_0)^2(\frac{c}{c-H_0D})}{G} \text{ from (4)}$$

$$\rho = \rho_{t_p} H_0 t_p = \frac{\rho_{t_p}}{1+z} = \frac{(cH_0)^2}{G} = \mathbf{8.54 \times 10^{-9}\ J/m^3}.$$

$$a^2 = \rho G \tag{17}$$

Therefore, the product of the net electrostatic energy density of the universe in the hidden three dimensions and gravitational constant is equal to the square of the acceleration of the universe.

ρ = Current electrostatic energy density of the universe responsible for the acceleration
G = Local gravitational constant, which always remains constant (Refer to Table V)
a = Current acceleration of the universe, which is cH_0 (2)

As the universe accelerates owing to electrostatic repulsion, the electrostatic potential energy in the hidden three-dimensional space is gradually transferred to the four-dimensional space-time grid and stored as potential energy. As the elastic space-time grid resists expansion, the rate of acceleration gradually decreases to follow $a=cH_0$ (2), which asymptotes to zero. As the total energy is conserved, the potential energy density of the four-dimensional space-time grid ρ_g is as follows:

$$|\rho_g| = \rho_{t_p} - \rho$$

As the energy density ρ decreases faster than the relative energy density ρ_{t_p} with time, the potential energy density of the four-dimensional space-time grid ρ_g continues to increase until infinity.

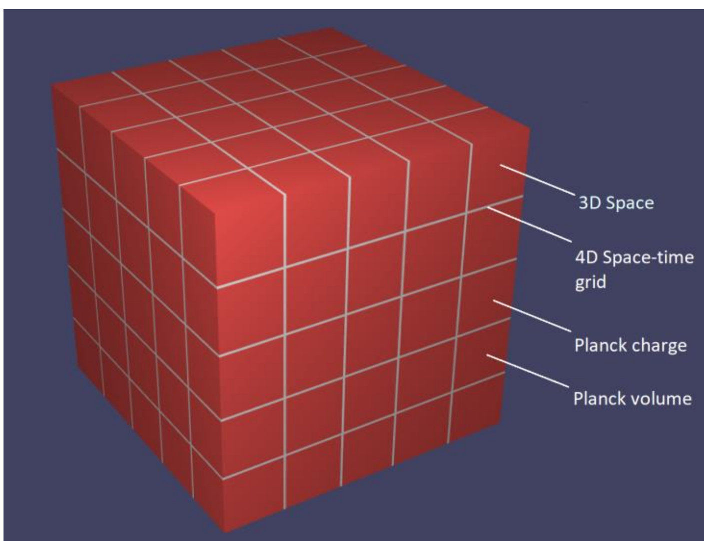
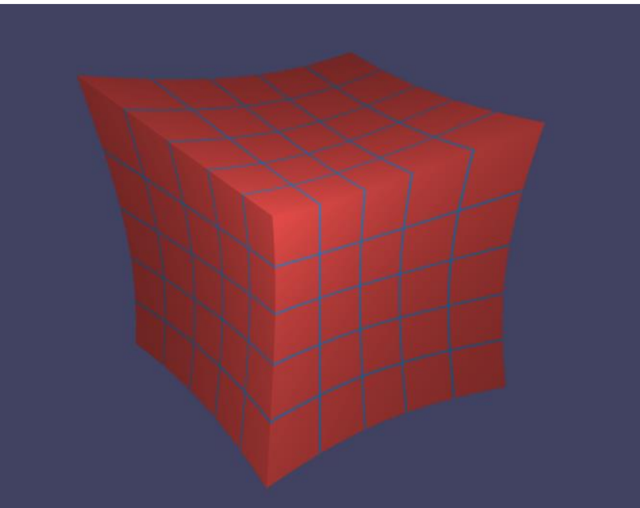
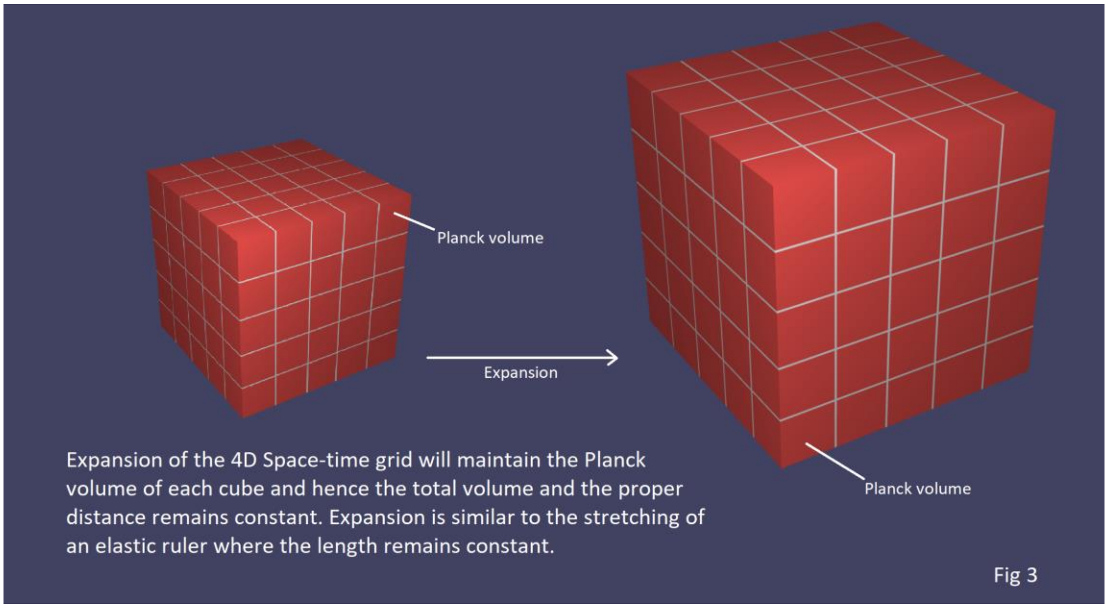


Fig 1


Matter in the 4D Space-time grid curves the space-time around it due to the electrostatic repulsion outside the matter being greater than inside as mass also increases/expands along with space-time.
Fig 2



6. The absolute frame of reference

The four-dimensional space-time grid proposed in this theory acts as an absolute inertial frame. The physical constants change with the cosmological expansion, gravitational field, and velocity, according to the formulas given in Table II. Therefore, in the gravitational field, the absolute reference frame is the one at an infinite distance from the mass generating the gravitational field, and the values of the physical constants change as we move from infinity toward the mass owing to the contraction of space-time. For example, the relative gravitational constant would be higher and the Planck constant would be lower near the mass than farther away from it.

As the Hubble constant decreases owing to space-time expansion, the relative values of the physical constants of the absolute inertial frame need to be updated to keep up with the expansion. Similarly, an inertial frame moving with velocity will have a higher gravitational constant and a lower Planck constant compared to the absolute inertial frame. However, a source of light emitting green photons will still emit green photons of the same frequency in both frames with different Planck constants owing to the time dilation in the moving frame; hence, the local value of the Planck constant will appear to be constant. In this theory, the universal time is the time in the absolute inertial frame, which is the Hubble time.

However, the absolute inertial frame cannot be uniquely identified in this theory based on the change in the physical constants as any change will be compensated by the time dilation or the change in the inertial mass/potential energy as shown in Table V. Therefore, the local values of all physical constants remain constant owing to the change in the inertial mass or time dilation. As the absolute frame of reference cannot be uniquely identified, but is still mandated by this theory, it can be assumed that the CMBR is static with respect to the absolute inertial frame. Therefore, we can calculate the changes in the physical constants and inertial mass relative to the CMBR.

In this theory, the acceleration due to Newtonian gravity is locally invariant in all frames, except for time dilation or gravitomagnetic effects as shown in Table IV. Consider two point masses, M and m ($M \gg m$), which are active and passive gravitational masses, respectively, separated by distance r and moving at velocity v in a stationary reference frame. The time dilation of the moving masses in the stationary reference frame compensates for the time dilation of the observer moving with the masses. In the moving reference frame, the inertial mass decreases to $\frac{M}{\gamma}$ owing to the space contraction. Therefore, the effective local gravitational constant G remains constant in all frames of four-dimensional space-time, although the relative G varies, and only the rest mass M contributes to the active gravitational mass, although the inertial mass changes, which means that the product GM remains constant or invariant in all frames of reference.

Table IV (Invariant acceleration due to Newtonian gravity)

	WRT stationary reference frame (without time dilation)	WRT moving reference frame
Masses aligned perpendicular to the direction of motion	$\gamma ma = F = \frac{GM(\gamma m)}{r^2}$; $a = \frac{GM}{r^2}$	$a = \frac{(G\gamma)\frac{M}{\gamma}}{r^2} = \frac{GM}{r^2}$
Masses aligned parallel to the direction of motion	$\gamma^3 ma = F = \frac{GM(\gamma m)}{\left(\frac{r}{\gamma}\right)^2}$; $a = \frac{GM}{r^2}$	$a = \frac{(G\gamma)\frac{M}{\gamma}}{r^2} = \frac{GM}{r^2}$

In this theory, local physical laws, physical constants, and inertial mass remain constant owing to the contraction/expansion of space-time, although the relative values of non-local constants and inertial mass vary in the cosmological, gravitational, and relativistic frameworks.

Table V (Invariant local physical laws and constants)

Physical constant	Physical constant due to space contraction	Physical constant due to space expansion	Invariant law, constant, and mass in the local inertial frame due to space expansion (z is the redshift due to expansion)
G	$G(1+z)$	$\frac{G}{(1+z)}$	$F = m(1+z)a = \frac{G}{(1+z)}\frac{M(1+z)m(1+z)}{r^2} \equiv \left(F = \frac{GMm}{r^2}\right)$
h	$\frac{h}{(1+z)}$	$h(1+z)$	$E = m(1+z)c^2 = h(1+z)\nu \equiv \left(E = h\nu = mc^2\right)$
ε_0	$\varepsilon_0(1+z)$	$\frac{\varepsilon_0}{(1+z)}$	$F = m_e(1+z)a = \frac{(1+z)}{4\pi\varepsilon_0}\frac{e^2}{r^2} \equiv \left(F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)$
k_B	$\frac{k_B}{(1+z)}$	$k_B(1+z)$	$\langle E\rangle(1+z) = \frac{3}{2}Nk_B(1+z)T \equiv \left(\langle E\rangle = \frac{3}{2}Nk_B T\right)$

As the inertial mass increases with the expansion of the universe, a point mass m moving with velocity v in an inertial reference frame slows down to $\frac{v}{(1+z)}$ to conserve the non-relativistic linear momentum in the absence of external forces. $mv = m(1+z)\frac{v}{(1+z)}$, where z is the cosmological red shift. Therefore, the respective non-relativistic linear kinetic energy decreases as some part of the kinetic energy is converted into cosmological potential energy or an increase in the inertial mass, along with the energy contribution from the expansion of the universe. $\frac{1}{2}mv^2 \neq \frac{1}{2}m(1+z)\left(\frac{v}{1+z}\right)^2$.

Therefore, the linear kinetic energy KE of an object decreases continuously to $\frac{KE}{(1+z)}$ in an expanding universe without violating the law of energy conservation. Similarly, the non-relativistic kinetic energy KE_o of a point mass in a circular orbit continuously decreases to $\frac{KE_o}{(1+z)}$ in an expanding universe. As this kinetic energy is not sufficient to maintain the point mass in its initial circular orbit, it expends its potential energy until it finds a stable orbit. Therefore, in an expanding universe, any orbiting mass continuously loses its orbit and spirals down to the mass by which it is gravitationally bound, thereby providing falsifiable predictions for this theory. However, it still needs to be determined whether this phenomenon is connected to the spiral shape of galaxies, especially during the early period of galaxy formation.

7. The relativistic Newtonian theory of gravity

In this theory, the presence of mass increases the permittivity of the four-dimensional space-time grid and hence reduces the electrostatic repulsion within the grid enveloped by the mass compared to the electrostatic repulsion outside the mass. As the contraction of the space-time grid within the mass will be greater than the contraction outside owing to the permittivity difference and the net compressing force on the matter from the surrounding Planck charges, space-time becomes naturally curved around any mass. This compression force is considered to be the gravitational force in this theory. Therefore, the attractive gravitational force between the two masses is considered to be a pushing-type force rather than a pulling-type force, as the net pulling force between the two masses is zero because of Newton’s third law of motion. Hence, in this theory, gravitational force is considered a real force rather than a fictitious force of the general theory of relativity.

The change in gravitational constant G in the gravitational field is similar to the change in the gravitational constant due to cosmological space-time expansion. Therefore, we can use Newton's law of gravitation and introduce variable G and the curvature of space to derive a relativistic law. In this theory, only the rest mass contributes to the active gravitational mass M, and inertial/relativistic mass to the passive gravitational mass m_R.

$F = \frac{G(1+z)Mm_R}{\left(\frac{R}{1+z}\right)^2} = (1+z)^3\frac{GMm_R}{R^2}$ (using Table II), where G is the gravitational constant at infinity,

and $m_R = \frac{m}{(1+z)}$ is the passive gravitational/inertial mass at radius R and m is the rest mass at infinity,

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where the rest mass is equal to the inertial mass. As the space-time grid contracts and becomes curved as we move from infinity towards mass M, R is divided by (1+z) to factor in the local curvature of space, which is also the local length contraction factor, as observed from infinity. As the inertial mass is equal to the passive gravitational mass, gravitational acceleration $g=\frac{G(1+z)M}{\left(\frac{R}{1+z}\right)^2}$.

$$\frac{d\phi}{dR}=g=(1+z)^3\frac{GM}{R^2}=\left(1-\frac{\phi}{c^2}\right)^3\frac{GM}{R^2} \text{ using (16)}$$

$$\int_0^{\phi}\left(\frac{1}{c^2-\phi}\right)^3 d\phi=\frac{GM}{(c^2)^3}\int_{\infty}^{R}\frac{1}{R^2}dR \quad ; \quad \frac{1}{2(c^2-\phi)^2}-\frac{1}{2(c^2)^2}=-\frac{GM}{(c^2)^3}\frac{1}{R}$$

$$\frac{1}{2\left(1-\frac{\phi}{c^2}\right)^2}-\frac{1}{2}=-\frac{GM}{c^2}\frac{1}{R} \quad ; \quad \frac{1}{2\left(1+z\right)^2}-\frac{1}{2}=-\frac{GM}{c^2}\frac{1}{R}$$

$$1+z=\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}} \qquad \textbf{(18)}$$

As the above gravitational redshift matches the redshift from the Schwarzschild solution of the Einstein field equations without using the weak field approximation, it validates the concept of variable physical constants. Additionally, the formulas below for the gravitational force and gravitational potential energy are not approximations but complete solutions that work in both strong and weak gravitational fields for non-spinning stationary spherical masses without using any weak field approximation of the general theory of relativity.

The gravitational redshift z becomes infinite at $R=\frac{2GM}{c^2}$ in (18), which yields the Schwarzschild radius R_S without using a weak-field approximation.

Gravitational force F:

$$F=(1+z)^3\frac{GMm_R}{R^2}=\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}\right)^3\frac{GMm_R}{R^2} \qquad \textbf{(19)}$$

Here, the product GM remains invariant in all frames of reference to uphold Lorentz invariance.

Gravitational potential energy U:

$$U=-m_Rzc^2=-m_R\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}-1\right)c^2 \qquad \textbf{(20)}$$

where m_R is the inertial mass at radius R. Equating the relativistic gravitational potential energy U with the relativistic kinetic energy $m_R\left(\gamma-1\right)c^2$ directly yields the Schwarzschild radius $R_S=\frac{2GM}{c^2}$

for escape velocity c. For weak gravitational fields, (20) reduces to the regular Newtonian gravitational potential energy and hence validates this theory.

$$U = -m_R\left(\left(1-\frac{2GM}{Rc^2}\right)^{-\frac{1}{2}}-1\right)c^2 \approx -m_R\left(1+\frac{GM}{Rc^2}-1\right)c^2 = -\frac{GMm_R}{R}$$

Gravitational potential ϕ:

$$\phi = \frac{U}{m_R} = -zc^2 = -\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}-1\right)c^2 \qquad \textbf{(21)}$$

The maximum energy E_{max} (rest + potential) at infinity from mass M is expressed as follows:

$E_{\max} = m_R(1+z)c^2 = mc^2$ (14), where $m = m_R(1+z)$ is the inertial mass at infinity.

Therefore, the increase in gravitational potential energy is equal to the energy equivalent ($E=mc^2$) of the increase in inertial mass. As the energy is conserved in the free-fall motion of mass m in the gravitational field, the velocity of mass m can be derived from the maximum energy E_{max}. Through dimensional analysis, the energy at any point during freefall must be of the form $m\left(\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}\right)c^2$.

Therefore, $v=\sqrt{\frac{2GM}{R}}$, which is the velocity of mass m in a freefall motion from the point of maximum potential energy or infinity, which is also equal to the escape velocity v_e. As the freefall motion is inertial, the Lorentz factor γ is directly realized from energy conservation, which is $\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}$, hence, this theory upholds the special theory of relativity in all inertial frames. However, if $v > v_e$ in either direction of the gravitational field, the relativistic mass $\delta\gamma$m must be used in (19), where $\delta\gamma$ is the Lorentz factor for the velocity $v-v_e$. For any velocity v perpendicular to the gravitational field, γm must be used in (19) to obtain accurate results.

Let us consider the point rest masses M and m ($M \gg m$), where mass m is the rest mass equal to the inertial mass at infinity. The formula for the orbital velocity is invariant at all radii greater than R_S for the gravitational/inertial mass orbiting around mass M in a circle.

$(1+z)^3\frac{GMm_R}{R^2} = \frac{m_R v^2}{\left(\frac{R}{1+z}\right)}$; $v=(1+z)\sqrt{\frac{GM}{R}}$, where z is the gravitational redshift at radius R and m_R is the relativistic gravitational/inertial mass due to translational, vibrational, and rotational kinetic energy (for example, due to orbital motion, temperature, pressure, and angular momentum) and contraction of space. $\frac{R}{(1+z)}$ signifies the curvature of space at radius R.

The length contraction, time dilation, and mass change in the gravitational field are given by $L_R = \frac{L}{(1+z)}$, $T_R = \frac{T}{(1+z)}$, and $m_R = \frac{m}{(1+z)}$, where L, T, and m are the length, time, and the rest mass, respectively, at infinity **(22)**

As the radial lengths L_R, T_R, and inertial mass m_R become zero at the Schwarzschild radius, the space-time terminates at the event horizon of a black hole, unlike the general theory of relativity, in which the space-time is extended into the black hole. Blackhole singularities do not exist in this universe model as the mass becomes zero and is converted to the equivalent pure informational entropy at the event horizon. Additionally, charged black holes do not exist in this model of the universe as the permittivity of free space ε_0 becomes infinity (from Table II) at the event horizon. Therefore, stationary black holes can only have two properties, namely informational (entropic) mass, and angular momentum, and hence only partially satisfy the no-hair theorem.

7.1. Space-time interval (ds^2)

The invariant line element (ds^2) in the Minkowski space-time in spherical coordinates is given by $ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$. The presence of a gravitational field dilates the relativistic proper time $d\tau$ to $\frac{d\tau}{(1+z)}$. Therefore, $d\tau_g = \frac{d\tau}{(1+z)}$, where $d\tau_g$ is the relativistic proper time with gravity included and z is the gravitational redshift. We can substitute $d\tau = d\tau_g(1+z)$ in the line element (ds^2) of the Minkowski space-time to find the line element with gravity ($ds^2 = c^2 d\tau_g{}^2$).

$c^2\left(d\tau_g(1+z)\right)^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$, where dt, dr, and r are the coordinate time and lengths, respectively, as measured from the absolute reference frame or reference frame at infinity.

$$ds^2 = -\frac{c^2dt^2}{(1+z)^2} + \frac{dr^2}{(1+z)^2} + \frac{r^2}{(1+z)^2}(d\theta^2 + \sin^2\theta d\phi^2) \quad \textbf{(23)}$$

By applying gravitational length contraction and time dilation (18) (16), we obtain the Schwarzschild metric equivalent for this theory.

Schwarzschild metric equivalent:

$$ds^2 = -c^2dt^2\left(1-\frac{2GM}{Rc^2}\right) + dr^2\left(1-\frac{2GM}{Rc^2}\right) + r^2\left(1-\frac{2GM}{Rc^2}\right)(d\theta^2 + \sin^2\theta d\phi^2)$$

OR

$$ds^2 = -c^2dt^2\left(1-\frac{\phi}{c^2}\right)^{-2} + dr^2\left(1-\frac{\phi}{c^2}\right)^{-2} + r^2\left(1-\frac{\phi}{c^2}\right)^{-2}(d\theta^2 + \sin^2\theta d\phi^2)$$

Once the (1+z) factor $\left(1-\frac{\phi}{c^2}\right)$ is identified by solving the relativistic Poisson equation (28), it can be substituted into (23) to obtain the space-time interval equation for this theory, which is equivalent to the equation $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ in the general theory of relativity [5]. For any inertial or non-inertial reference frame except the absolute reference frame, the respective coordinate length, time, and gravitational redshift z up to the reference point in the frame must be used in (23) to make the line element (ds^2) invariant in all reference frames. However, the non-relativistic or the classical Doppler redshift, if any, must be excluded from the gravitational redshift z because the relativistic component is already part of the line element (23).

In this theory, space-time ends at the event horizon of a black hole as the time and length become zero and hence eliminates singularity, unlike in the general theory of relativity, where the length dr becomes infinity and custom coordinate systems are used to avoid infinities and to extend the space-time up to the singularity at the center of a black hole. Additionally, the slope of the light ray of the light cone is always equal to 1 (45°) in this theory, even near the event horizon, unlike the general theory of relativity, where the light cone gradually becomes narrow and the slope becomes infinity at the event horizon. Setting the line element $ds^2=0$, which is light-like, and c=1 in (23), ignoring dθ and dϕ, we obtain the slope $\frac{dt}{dr} = 1$.

Applying the cosmological length contraction and time dilation (4), we obtain the equivalent of the FLRW metric of the flat space-time for this theory.

FLRW metric equivalent:

$$ds^2 = -c^2 dt^2 \left(1 - \frac{H_0 r}{c}\right)^2 + dr^2 \left(1 - \frac{H_0 r}{c}\right)^2 + r^2 \left(1 - \frac{H_0 r}{c}\right)^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

where dt, dr, and r are the coordinate time and lengths, respectively, measured from the current cosmological reference frame in which the Hubble constant is H_0.

7.2. Photon sphere

As the photon takes a curved path around the black hole owing to the space-time curvature, matching the fictitious acceleration due to gravity for the photon with the centripetal acceleration gives the radius of the photon sphere for a non-spinning black hole, as predicted by the general theory of relativity. Here, the forces are considered to be fictitious, as the photon does not curve space-time by itself, but only takes the path of curved space-time owing to optical refraction. The path taken by a photon due to refraction in curved space-time is equivalent to the null geodesic path of the general theory of relativity. In this theory, the refraction of photons owing to the curvature is considered equivalent to the centripetal acceleration. Therefore, the fictitious centripetal acceleration of the photon due to refraction in the curved space-time around mass M is $\frac{c^2}{\left(\frac{R}{1+z}\right)}$, where the factor (1+z) accounts for the local curvature of the space.

$(1+z)^3 \frac{GM}{R^2} = \frac{c^2}{\left(\frac{R}{1+z}\right)}$ (19), by solving using (18), we obtain the radius R_{ph} of the photon sphere.

$$\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}\right)^2 \frac{GM}{Rc^2} = 1 \tag{24}$$

$$R_{ph} = \frac{3GM}{c^2} \tag{25}$$

The probability wave function (ψ) of any matter particle $(m \ll M)$ also takes the path of the curved space-time and is refracted like a photon, but curves the space-time unlike a photon and causes a wobble in mass M; hence, the forces are considered real. A photon does not cause a wobble in mass M, because it is massless. As matter particles move with velocities less than c, the centripetal acceleration experienced by a matter particle owing to the refraction of its wave function is $\frac{v^2}{\left(\frac{R}{1+z}\right)}$.

7.3. Gravitational lensing

As the angle of deflection θ for the photon sphere is π rad owing to the symmetry of the sphere, as shown in Figure 4, we can calculate the angle of deflection for a non-spinning stationary spherical mass for any radius R within the relativistic Newtonian regime by replacing 1 in (24) with $\mathrm{Tan}\left(\frac{\pi}{4}\right)$, which is the ratio of the gravitational acceleration to the centripetal acceleration of the photon at its closest approach to mass M. As this ratio should be proportional to the angle of deflection θ, we can generalize this equation by replacing $\mathrm{Tan}\left(\frac{\pi}{4}\right)$ with $\mathrm{Tan}\left(\frac{\theta}{4}\right)$ to determine the angle of deflection for any radius R.

$$\left(\sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}\right)^2 \frac{GM}{Rc^2} = \mathrm{Tan}\left(\frac{\pi}{4}\right)$$

$$\theta = 4\,\mathrm{Tan}^{-1}\left((1+z)^2 \frac{GM}{Rc^2}\right) \tag{26}$$

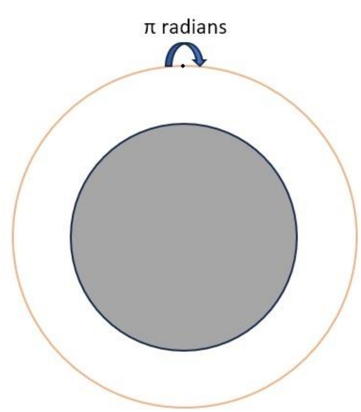

Fig 4

For θ=2π, we obtain $R=\frac{2GM}{c^2}$, which is the Schwarzschild radius and hence validates the above equation (26). Therefore, light bends onto itself at the event horizon of a black hole and does not travel beyond it as in the general theory of relativity and hence provides another proof that space-time ends at the event horizon and the non-existence of a singularity at the center of a black hole. For weak gravitational fields, the deflection angle in (26) decreases to $\theta=\frac{4GM}{Rc^2}$, which is the same as the deflection predicted by the general theory of relativity. Therefore, the equation (26) predicts the angle of deflection in a closed form that works in both strong and weak gravitational fields better than the general theory of relativity without using any approximations or higher-order terms.

7.4. Perihelion precession of Mercury

From Kepler's second law, $dA=\frac{1}{2}r^2 d\theta$, where dA is the change in the area swept out by the orbiting mass, dθ is the change in the angle, and r is the radius. Let dA' and dθ' be the local changes in the area and angle, respectively. Divide the above equation on both sides by the local time dt' and apply length contraction (22) to factor in the local curvature of space-time.

$$\frac{dA'}{dt'}=\frac{1}{2}\left(\frac{r}{1+z}\right)^2\frac{d\theta'}{dt'}$$

Apply time dilation using (22), where dt is the local time without a space-time curvature. The curvature stretches out the space-time fabric, and hence, the local time dilates to dt'=dt(1+z).

$$\frac{dA'}{dt'}=\frac{1}{2}\left(\frac{r}{1+z}\right)^2\frac{d\theta'}{dt(1+z)}=\frac{1}{2}r^2\frac{d\theta'}{dt}\frac{1}{(1+z)^3}$$

As the angular momentum is conserved, the above equation can be compared with the non-relativistic equation without the space-time curvature to obtain the following equation:

$$d\theta'=d\theta(1+z)^3=d\theta\left(1-\frac{2GM}{Rc^2}\right)^{-\frac{3}{2}}\approx d\theta\left(1+\frac{3GM}{Rc^2}\right)$$, ignoring higher-order terms.

The polar equation of the ellipse is $R=\frac{a(1-\varepsilon^2)}{1-\varepsilon\cos\theta}$.

$$\theta'=\int_0^{2\pi}\left(1+\left(\frac{3GM}{c^2}\frac{1-\varepsilon\cos\theta}{a(1-\varepsilon^2)}\right)\right)d\theta \quad ; \quad \theta'=2\pi+\frac{6\pi GM}{c^2 a(1-\varepsilon^2)}$$, the second term provides the precession angle Δϕ of perihelion per revolution, where M is the mass of the sun, a is the semi-major axis of Mercury, and ε is the orbital eccentricity.

$$\Delta\phi=\frac{6\pi GM}{c^2 a(1-\varepsilon^2)}$$

Solving the above, we obtain a perihelion precession of 43"/century, which is the same as that predicted by the general theory of relativity [6].

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7.5. Shapiro time delay

We can calculate the gravitational time delay of light passing near a mass such as the sun. As the gravitational length contraction and gravitational time dilation go together in this theory, we can calculate the effective speed of light as observed by an observer on Earth (B) as the light from a distant object (A) passes near the sun and reaches Earth. The speed of light is constant at every point in space in the gravitational field, as the length contraction is compensated by time dilation.

Therefore, $\dfrac{\dfrac{\text{Distance}}{(1+z)}}{\dfrac{Time}{(1+z)}} = \dfrac{\text{Distance}}{Time} = c$ (the speed of light) remains constant. However, a stationary observer from the local reference frame would see a change in the speed of light as it passes through the gravitational field. Here, time expansion is applied instead of contraction, as the time delay is observed from the observer's frame of reference, whose local time is faster than the time near the sun. Therefore, the length contraction and time expansion do not cancel out in the observer's reference frame, and hence produce a time delay according to this theory, which is called the Shapiro time delay.

The effective speed of the light experienced by a stationary observer is $\dfrac{\dfrac{\text{Distance}}{(1+z)}}{\text{Time}(1+z)} = \dfrac{c}{(1+z)^2}$.

$\dfrac{dx}{dt} = \dfrac{c}{(1+z)^2} = c\left(1 - \dfrac{2GM}{Rc^2}\right)$ using (18), where R is the distance between the sun and the traveling photon. Neglecting the deflection of light near the sun, the path of light from A to B is a straight line.

$dt = \dfrac{1}{c}\left(\dfrac{1}{1-\dfrac{2GM}{Rc^2}}\right)dx \approx \dfrac{1}{c}\left(1+\dfrac{2GM}{\sqrt{x^2+b^2}c^2}\right)dx$ by ignoring higher-order terms, where b is the impact parameter. Integrating the left side from T_A to T_B and the right side from X_A to X_B.

$$\int_{T_A}^{T_B} dt = \int_{X_A}^{X_B} \frac{1}{c}\left(1+\frac{2GM}{\sqrt{x^2+b^2}c^2}\right)dx \quad ; \quad T_B - T_A = \frac{X_B - X_A}{c} + \frac{2GM}{c^3}\ln\left(\frac{X_B+\sqrt{X_B{}^2+b^2}}{X_A+\sqrt{X_A{}^2+b^2}}\right)$$

The second term gives the additional one-way time delay Δt of the light coming from point A, as observed by an observer at point B, which fits the experimental data better than the Schwarzschild metric using Schwarzschild coordinates [7].

$$\Delta t = \frac{2GM}{c^3}\ln\left(\frac{X_B+\sqrt{X_B{}^2+b^2}}{X_A+\sqrt{X_A{}^2+b^2}}\right)$$

7.6. Relativistic Poisson and gravitoelectromagnetism (GEM) equations

Relativistic Poisson and Maxwell-like GEM equations can be generated to make them work in strong gravitational fields and relativistic speeds as well by applying the Laplace operator (∇^2) on the relativistic gravitational potential. The relativistic GEM equations can explain the frame-dragging effect, orbital precession, geodetic effect, and gravitational waves. The relativistic GEM and Poisson equations together can form the equivalent of Einstein field equations for this theory with the scope for further generalization.

As $z=-\frac{\phi}{c^2}$ (16), $(1+z)=\left(1-\frac{\phi}{c^2}\right)$.

7.6.1. Relativistic Poisson equations

The relativistic gravitational potential ϕ in Cartesian coordinates based on (21) is as follows:

$$\phi=-\left(\frac{1}{\sqrt{1-\frac{2GM}{\left(\sqrt{x^2+y^2+z^2}\right)c^2}}}-1\right)c^2$$

The relativistic Poisson equation outside the point mass can be generated by applying the Laplace operator to the gravitational potential.

$$\nabla^2\phi=-\frac{3G^2M^2x^2(1+z)^5}{(x^2+y^2+z^2)^3c^2}-\frac{3G^2M^2y^2(1+z)^5}{(x^2+y^2+z^2)^3c^2}-\frac{3G^2M^2z^2(1+z)^5}{(x^2+y^2+z^2)^3c^2}$$
$$-\frac{3GMx^2(1+z)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}}-\frac{3GMy^2(1+z)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}}-\frac{3GMz^2(1+z)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}}+\frac{3GM(1+z)^3}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$\nabla^2\phi=-2\pi G\rho\left[(1+z)^5-(1+z)^3\right]$, where $\rho=\frac{M}{\frac{4}{3}\pi R^3}$ is the density of the point mass.

Outside the point mass: $$\nabla^2\phi=-2\pi G\rho\left[\left(1-\frac{\phi}{c^2}\right)^5-\left(1-\frac{\phi}{c^2}\right)^3\right]\approx 4\pi G\rho\frac{\phi}{c^2} \quad \textbf{(27)}$$

The relativistic Poisson equation at the point mass can be generated by considering only the positive terms and ignoring the negative terms in the Laplacian ($\nabla^2\phi$) because the positive terms represent the positive divergence or source of the gravitational flux.

$\nabla^2\phi=\frac{3GM(1+z)^3}{(x^2+y^2+z^2)^{\frac{3}{2}}}=4\pi G\rho(1+z)^3$, where ρ is the density of the point mass.

At the point mass: $$\nabla^2\phi=4\pi G\rho\left(1-\frac{\phi}{c^2}\right)^3 \quad \textbf{(28)}$$

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For weak gravitational fields, where the gravitational redshift z is negligible, the above equations (27) and (28) are reduced to Laplace and non-relativistic Poisson equations, respectively.

Outside the point mass: $\nabla^2\phi = -2\pi G\rho\left[(1+z)^5-(1+z)^3\right] \approx -2\pi G\rho[1-1] = 0$

At the point mass: $\nabla^2\phi = 4\pi G\rho(1+z)^3 \approx 4\pi G\rho[1] = 4\pi G\rho$

7.6.2. Relativistic GEM equations

Relativistic Maxwell-like GEM equations can be generated using (28) and (27) at the point mass/current and outside point mass/current, respectively, where E_G is the gravitoelectric field, B_G is the gravitomagnetic field, ϕ is the relativistic gravitational potential due to the gravitoelectric effect, and J is the mass current density.

At the point mass/current:

$$\nabla . E_G = -4\pi G\rho\left(1-\frac{\phi}{c^2}\right)^3$$

$$\nabla \times B_G = -\frac{4\pi G\left(1-\frac{\phi}{c^2}\right)^3}{c^2} J + \frac{1}{c^2}\frac{\partial E_G}{\partial t}$$

$$\nabla . B_G = 0$$

$$\nabla \times E_G = -\frac{\partial B_G}{\partial t}$$

Outside the point mass/current:

$$\nabla . E_G = 2\pi G\rho\left[\left(1-\frac{\phi}{c^2}\right)^5 - \left(1-\frac{\phi}{c^2}\right)^3\right]$$

$$\nabla \times B_G = \frac{2\pi G\left[\left(1-\frac{\phi}{c^2}\right)^5 - \left(1-\frac{\phi}{c^2}\right)^3\right]}{c^2} J + \frac{1}{c^2}\frac{\partial E_G}{\partial t}$$

$$\nabla . B_G = 0$$

$$\nabla \times E_G = -\frac{\partial B_G}{\partial t}$$

The gravitational force equivalent to the Lorentz force of electromagnetism is given below:

$F = m_i(E_G + v_{mi} \times B_G)$, where m_i is the passive inertial/relativistic mass and v_{mi} is the mass velocity.

8. MOND (Modified Newtonian dynamics)

We can generate a relativistic MOND formula using the (1+z) factor, similar to relativistic Newtonian theory of gravity. However, the relativistic effect in the deep-MOND regime is not significant because of lower acceleration. This theory proposes that the MOND regime can only be present around a black hole but not around ordinary masses such as gas and stars. In the absence of black holes, the gravitational field around ordinary masses is always Newtonian at any acceleration. Therefore, the cutoff acceleration for the MOND regime, which is a_0 (~1.2×10^{-10} m/s^2) [8], is only applicable for the area around a central black hole but not for ordinary masses.

Each cell in Figure 6 of the curved space around the black hole represents the Planck volume with Planck charge. The radial length becomes zero at the event horizon but continues to expand to follow relativistic Newton's law of gravitation as we move away from the black hole. In addition, the number of Planck volumes on the circumference remains constant at any radius from the event horizon.

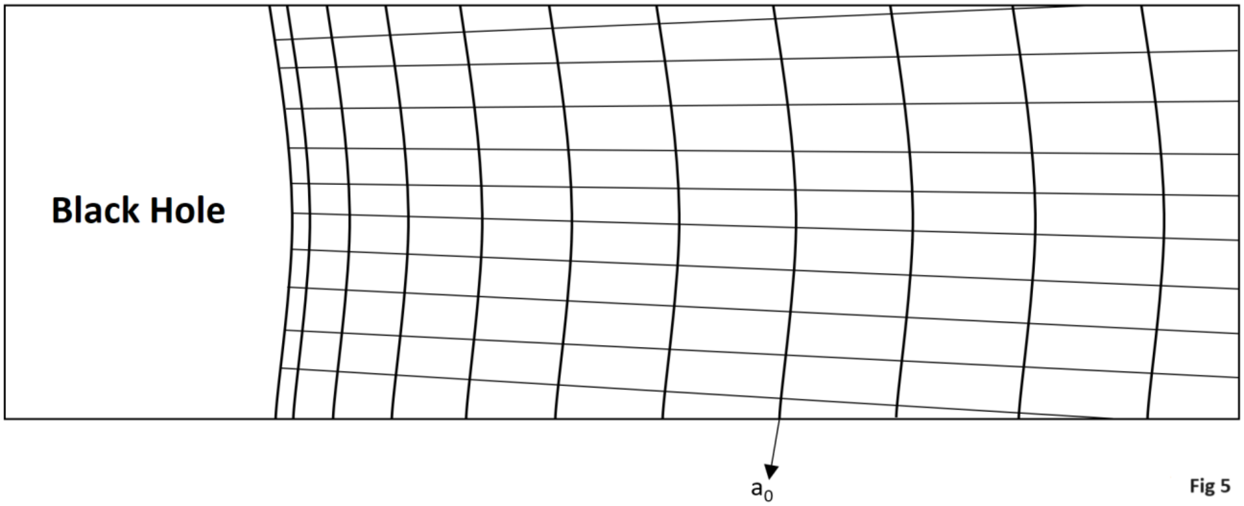

Fig 5

To understand the reason for the transition of the Newtonian regime to the MOND regime at a_0, we can flatten the above curved space, as depicted in Figure 7. As the relative Planck volume cannot increase beyond the maximum allowed by the expansion of the universe, the relative size of the Planck volume should remain constant after the a_0 radius, as shown in Figure 7.

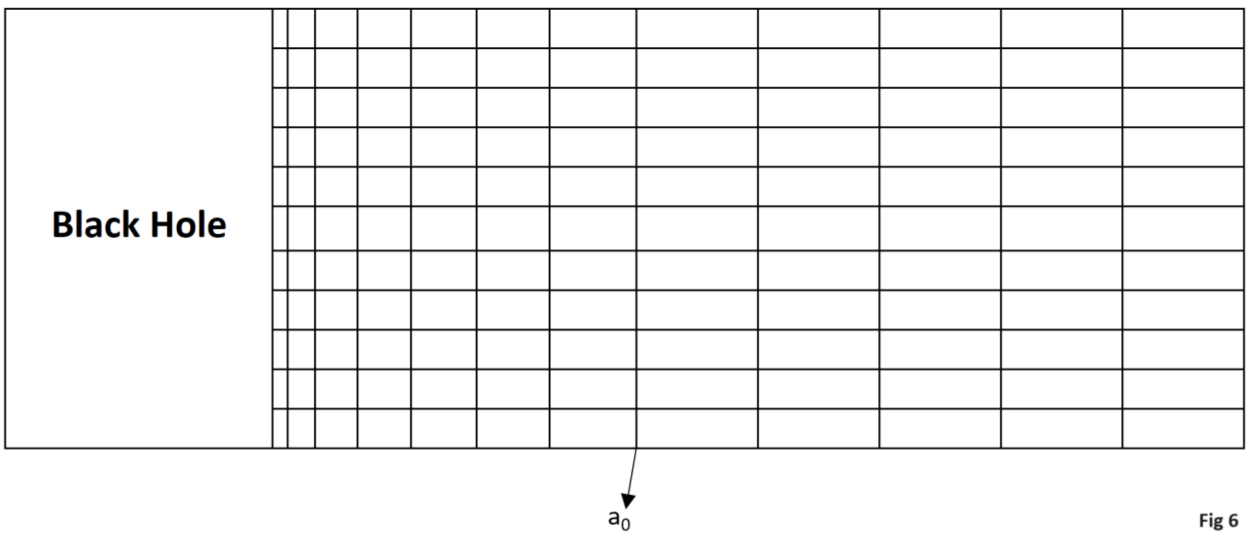

Fig 6

Once the space becomes curved, as is the case around the black hole, the Planck volumes beyond the a_0 radius also become curved; hence, the relative Planck volumes also gradually increase as the stretching of space-time continues to increase beyond the a_0 radius owing to the curvature, as the circumference farther from the a_0 radius should stretch more than the one closer to it. However, the rate of change in the Planck volume for $a < a_0$ is less than the rate of change for $a > a_0$. Therefore, the Newtonian law of gravitation switches to the MOND law at a_0 radius around a black hole.

Let us consider an elastic rubber band stretched around a cylinder; the radial force exerted by the rubber band on the cylinder is $\frac{1}{2\pi}$ times the tension in the rubber band, owing to the circumference of the cylinder given by $2\pi r$. As the current acceleration of the universe is cH_0 (2) per this theory, the radial acceleration at a_0 should be $\frac{cH_0}{2\pi}$ [9], which is ~1.2×10^{-10} m/s^2 per the above analogy. Here, the whole universe acts like a rubber band wrapped around the black hole at a_0 radius. Additionally, we can see that the space-time grid ends at the event horizon of the black hole and does not extend into it, as in the general theory of relativity.

Figure 8 shows the distortion of the space-time grid around ordinary matter. As the matter is present only in the four-dimensional space-time grid, it is depicted in red in the middle of Figure 8. Each cube in this figure is a Planck volume with a Planck charge. As the number of circumferential Planck volumes and the respective area increase radially as we move away from the matter, cutoff acceleration such as a_0 is not applicable, and Newton's law of gravitation can be applied at any acceleration without using any modification, such as MOND, in the absence of black holes. However, the gravity of ordinary matter will still switch to MOND at a_0 if it is present within the a_0 radius of a black hole owing to the geometry of the space-time as shown in Figure 6 and 7. This could explain why the expected gravitational lensing is not observed around the gaseous part of the Bullet cluster (1E 0657-56) [10], but around the galactic matter as it is subjected to MONDian gravitational lensing because of the presence of black holes at the centers of the galaxies.

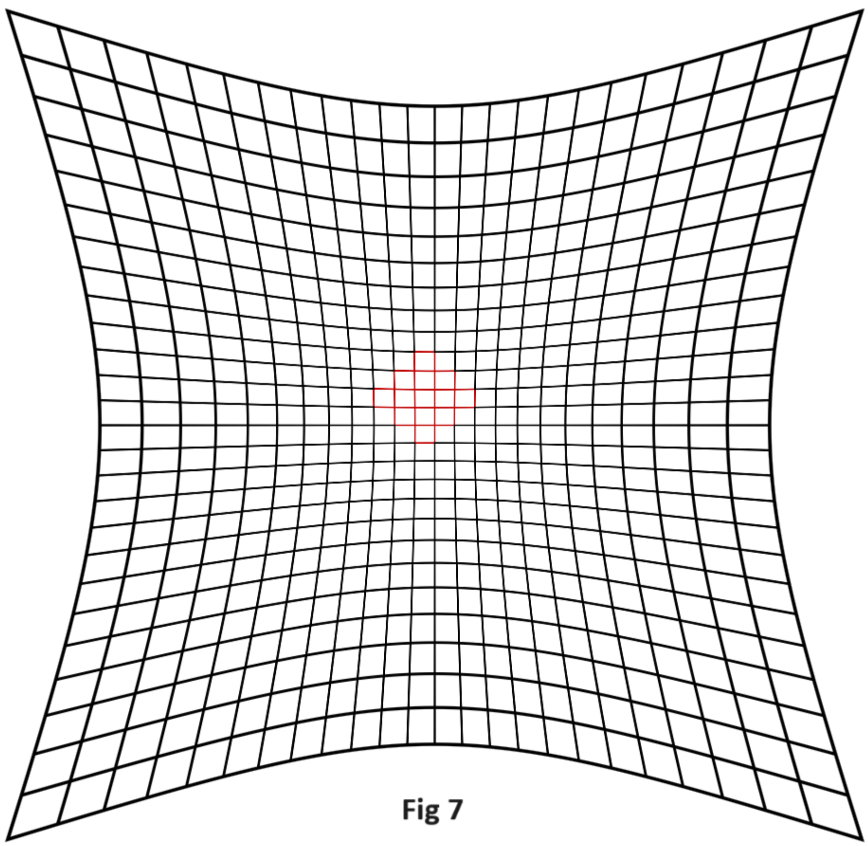

Fig 7

As dark matter has not been detected in the galaxy NGC 1052-DF2 [11], this theory predicts that galaxies that do not need dark matter to explain the rotation curves beyond the a_0 orbit should not have black holes at their centers or anywhere in the galaxies.

8.1. The relativistic MONDian gravity

The MONDian gravitational law can be expressed as follows:

$$F = \frac{GMm}{r^2} f\left(\frac{r}{r_0}\right) \text{ [12]}$$

where f(x) → 1 for $x \ll 1$, f(x) → x for $x \gg 1$ and r_0 is the radius at which $a=a_0$.

We can use the above MOND formula and introduce variable G and gravitational length contraction to develop a relativistic law around a non-spinning stationary black hole, similar to the relativistic Newtonian theory of gravity.

$F = \dfrac{G(1+z)Mm_R}{\dfrac{R_0}{(1+z_0)}\dfrac{R}{(1+z)}} = (1+z_0)(1+z)^2\dfrac{GMm_R}{R_0 R}$, where z_0 is the gravitational redshift (18) at R_0, z is the gravitational redshift (29) at R, M is the uniformly distributed total active gravitational mass within a_0 radius ($R \le R_0$), m is the rest mass at infinity, where the rest mass is equal to the inertial mass, and $m_R = \dfrac{m}{(1+z)}$ is the gravitational/inertial mass at radius R.

$$\frac{d\phi}{dR} = g = (1+z_0)(1+z)^2\frac{GM}{R_0 R} = (1+z_0)\left(1-\frac{\phi}{c^2}\right)^2\frac{GM}{R_0 R} \text{ using (15)}$$

$$\int_{\phi}^{\phi_0}\left(\frac{1}{c^2-\phi}\right)^2 d\phi = (1+z_0)\frac{GM}{R_0(c^2)^2}\int_R^{R_0}\frac{1}{R}dR$$

$$\frac{1}{(c^2-\phi_0)} - \frac{1}{(c^2-\phi)} = (1+z_0)\frac{GM}{R_0(c^2)^2}\ln\left(\frac{R_0}{R}\right)$$

$$\frac{1}{\left(1-\frac{\phi_0}{c^2}\right)} - \frac{1}{\left(1-\frac{\phi}{c^2}\right)} = (1+z_0)\frac{GM}{R_0 c^2}\ln\left(\frac{R_0}{R}\right) \quad ; \quad \frac{1}{(1+z_0)} - \frac{1}{(1+z)} = (1+z_0)\frac{GM}{R_0 c^2}\ln\left(\frac{R_0}{R}\right)$$

$$1+z = \frac{(1+z_0)}{1-(1+z_0)^2\dfrac{GM}{R_0 c^2}\ln\left(\dfrac{R_0}{R}\right)} \qquad \textbf{(29)}$$

MONDian gravitational force F:

$$F = \left[1+z_0\mu\left(\frac{z}{z_0}\right)\right](1+z)^2\frac{GMm_R}{R^2}\mu\left(\frac{R}{R_0}\right) \qquad \textbf{(30)}$$

where μ(x) → 1 for $x \ll 1$, μ(x) → x for $x \gg 1$

The relativistic MONDian gravitational formula above is reduced to the relativistic Newtonian formula for gravity (19) for $R \le R_0$ and $z \ge z_0$.

MONDian gravitational potential energy U:

$$U = -m_R z c^2$$

MONDian gravitational potential ϕ:

$$\phi = \frac{U}{m_R} = -zc^2 \quad \textbf{(31)}$$

According to this theory, any mass beyond the a_0 radius of a black hole should follow the regular relativistic Newtonian theory of gravity. Therefore, any spherical and uniformly distributed mass beyond the a_0 radius (R_0) can be considered as a point mass located at the center of the galaxy exerting relativistic Newtonian gravity at $R > R_0$ for any acceleration without requiring MOND. Therefore, the total gravitational force F_T experienced at radius R beyond a_0 radius of a galaxy with a central black hole is as follows:

$$F_T = F_M + F_N \quad \textbf{(32)}$$

where F_M is the MONDian force due to the mass present within R_0 and F_N is the Newtonian force due to the mass present between R_0 and R, where $R > R_0$, ignoring the external field effect (EFE). Therefore, according to this theory, even for accelerations greater than a_0 at $R > R_0$ owing to the gravitational force F_T, there can still be a contribution from MONDian gravity as opposed to the current understanding of MOND, where it is applicable only for accelerations less than a_0.

8.2. MONDian gravitational lensing

In the deep MOND regime, the fictitious gravitational force experienced by a photon can be equated with the fictitious centripetal acceleration of the photon to determine the lensing formula around a stationary black hole, which is similar to the formula (26) of the relativistic Newtonian regime.

$a = (1+z_0)(1+z)^2 \dfrac{GM_0}{R_0 R} = \dfrac{c^2}{\left(\dfrac{R}{1+z}\right)}$; $(1+z_0)(1+z)\dfrac{GM_0}{R_0 c^2} = 1$, which is the ratio of the gravitational acceleration to the centripetal acceleration of the photon at its closest approach to mass M_0 present within a_0 radius ($R \le R_0$). By replacing the ratio 1 with $\mathrm{Tan}\left(\dfrac{\pi}{4}\right)$, we obtain the gravitational lensing equation in the deep MOND regime. As this ratio should be proportional to the angle of deflection θ, we can generalize this equation by replacing $\mathrm{Tan}\left(\dfrac{\pi}{4}\right)$ with $\mathrm{Tan}\left(\dfrac{\theta}{4}\right)$ to determine the angle of deflection for any radius *R*.

$$\theta = 4\,\mathrm{Tan}^{-1}\left((1+z_0)(1+z)\frac{GM_0}{R_0 c^2}\right) \approx \frac{4GM_0}{R_0 c^2} = \frac{4\sqrt{GM_0 a_0}}{c^2} \quad \textbf{(33)}$$

Therefore, in the absence of EFE, the gravitational deflection angle θ remains almost constant in the deep-MOND regime. If a spherically distributed mass M_N is present between R_0 and R ($R > R_0$) of a black hole, the total deflection angle at $R > R_0$ can be obtained by summing the MONDian (33) and Newtonian (26) deflection angles, which can be applied to both individual galaxies and galaxy clusters.

$$\theta = 4\,\mathrm{Tan}^{-1}\left((1+z_0)(1+z_m)\frac{GM_0}{R_0c^2}\right) + 4\,\mathrm{Tan}^{-1}\left((1+z_n)^2\frac{GM_N}{Rc^2}\right) \approx \frac{4\sqrt{GM_0a_0}}{c^2} + \frac{4GM_N}{Rc^2} \quad \textbf{(34)}$$

where z_m is the MONDian gravitational redshift (29) at radius R due to mass M_0, z_n is the Newtonian gravitational redshift (18) at radius R due to mass M_N, and M_0+M_N is the total mass.

8.3. MONDian Poisson equation

The relativistic MONDian gravitational potential ϕ in Cartesian coordinates based on (31) and (29) is expressed as follows:

$$\phi = -\left(\frac{(1+z_0)}{1-(1+z_0)^2\frac{GM_0}{R_0c^2}\ln\left(\frac{R_0}{\sqrt{x^2+y^2+z^2}}\right)} - 1\right)c^2$$

The Poisson equation at $R > R_0$ for the central point mass located within the a_0 radius ($R \leq R_0$) of a non-spinning stationary black hole can be generated by applying the Laplace operator to the gravitational potential. Here, the hypothetical point mass located at the center is the sum of all masses, including the central black hole located within the a_0 radius.

$$\nabla^2\phi = -\frac{2G^2M^2x^2(1+z_0)^2(1+z)^3}{(x^2+y^2+z^2)^2R_0{}^2c^2} - \frac{2G^2M^2y^2(1+z_0)^2(1+z)^3}{(x^2+y^2+z^2)^2R_0{}^2c^2} - \frac{2G^2M^2z^2(1+z_0)^2(1+z)^3}{(x^2+y^2+z^2)^2R_0{}^2c^2}$$
$$-\frac{2GMx^2(1+z_0)(1+z)^2}{(x^2+y^2+z^2)^2R_0} - \frac{2GMy^2(1+z_0)(1+z)^2}{(x^2+y^2+z^2)^2R_0} - \frac{2GMz^2(1+z_0)(1+z)^2}{(x^2+y^2+z^2)^2R_0} + \frac{3GM(1+z_0)(1+z)^2}{(x^2+y^2+z^2)R_0}$$

Ignoring the first three terms in the above Laplacian ($\nabla^2\phi$) as they are negligible and setting $z \approx z_0 \approx 0$ as the redshift is negligible beyond a_0 radius, we obtain the following non-relativistic equation:

$$\nabla^2\phi = \frac{GM_0}{(x^2+y^2+z^2)R_0} = \frac{GM_0}{R^2R_0} = \frac{a_0R_0}{R^2}, \text{ where } a_0 = \frac{GM_0}{R_0{}^2}$$

As any point mass present beyond the a_0 radius ($R > R_0$) or in the deep MOND regime exerts Newtonian gravity in this theory, we can sum the MONDian and Newtonian Poisson equations based on (32) to obtain the final non-relativistic MONDian Poisson equation.

MONDian Poisson equation:

$$\nabla^2\phi = \mu\left(\frac{a_0R_0}{R^2}\right) + 4\pi G\rho \quad \textbf{(35)}$$

where $\mu(x) \to x$ for $R \gg R_0$, $\mu(x) \to 0$ for $R \ll R_0$, and ρ is the density of the point mass located anywhere outside of the black hole. The Poisson equation for MOND (35) is equivalent to the following Poisson equation [13] based on AQUAL ("A QUAdratic Lagrangian").

$$\nabla.\left[\mu\left(\frac{|\nabla\phi|}{a_0}\right)\nabla\phi\right]=4\pi G\rho$$, where μ(x) → 1 for $x \gg 1$, μ(x) → x for $x \ll 1$.

8.4. MONDian Lagrangian density

The non-relativistic Lagrangian density $\mathcal{L}$ for this theory, which is equivalent to AQUAL, and the respective action $\mathcal{S}$ based on (35) are given below.

$$\mathcal{L}=-\frac{(\nabla\phi)^2}{8\pi G}-\frac{\mu\left(\frac{a_0R_0}{R^2}\right)}{4\pi G}\phi-\rho\phi \quad \textbf{(36)}$$

where μ(x) → x for $R \gg R_0$, μ(x) → 0 for $R \ll R_0$

$$\mathcal{S}=\iint \mathcal{L}\, d^3r dt$$

8.5. MOND in galaxy clusters

The need for dark matter is not completely eliminated when MOND is applied to galaxy clusters [14], as it is applied only for accelerations less than a_0. As there can be a contribution from MONDian gravity, even for accelerations greater than a_0, as per this theory (32), it can be applied to galaxy clusters to eliminate the need for dark matter. For a simple gravitationally bound system such as a point mass m rotating around mass M with $M \gg m$, the total kinetic energy T is given by $T=\frac{1}{2}mv^2$, Newtonian potential energy U is given by $U=-\frac{GMm}{R}$, and rotational velocity v is given by $v=\sqrt{\frac{GM}{R}}$. By substituting v into T, we obtain the regular Newtonian-based virial theorem: $\langle T\rangle=-\frac{1}{2}\langle U\rangle$. However, when both Newtonian and MONDian dynamics are involved, the regular virial theorem cannot be applied to a system that is gravitationally bound or in hydrostatic equilibrium with the binding potential, such as a galaxy cluster, and hence the need for dark matter to fit the cluster dynamics into the regular virial theorem. We can eliminate the need for dark matter in galaxy clusters using the updated virial theorem with both Newtonian and MONDian gravity included.

$F_T=F_M+F_N=\frac{GM_0}{RR_0}+\frac{GM_N}{R^2}$ (32), where M_0 is the mass within R_0, M_N is the spherically distributed mass between R_0 and R (R > R_0), and M_0+M_N is the total baryonic mass.

$\frac{GM_0}{RR_0}+\frac{GM_N}{R^2}=\frac{v^2}{R}$; $v=\sqrt{\frac{GM_0}{R_0}+\frac{GM_N}{R}}$; $T=\frac{1}{2}m\left(\frac{GM_0}{R_0}+\frac{GM_N}{R}\right)$, which is the updated virial theorem based on this theory for a simple gravitationally bound system, with contributions from both Newtonian and MONDian gravity. The updated virial theorem can be applied to a galaxy cluster using two point masses located at the center of the cluster: the first point mass M_N exerting Newtonian gravity, and the other point mass M_0 exerting MONDian gravity. The sum of all baryonic masses within the a_0 radius of the individual galaxies in the cluster would become the mass of the point mass M_0 exerting MONDian gravity, and the sum of the rest of the baryonic masses in the cluster would become the mass of the point mass M_N exerting Newtonian gravity in the absence of intergalactic black holes.

In the galaxy clusters, let us consider that approximately one-tenth of the total baryonic mass is present within the a_0 radius of the individual galaxies with central black holes, that is, $M_N=9M_0$, and consider a gravitationally bound point mass m rotating around the cluster at its very edge. The additional dynamic mass (dark matter) predicted by the regular Newtonian virial theorem can be identified by comparing it with the updated (Newtonian + MONDian) virial theorem.

$T = \frac{1}{2} m \left(\frac{GM_0}{R_0} + \frac{GM_N}{R} \right) = x \frac{1}{2} m \frac{G(M_0 + M_N)}{R}$, where x is the dark-matter factor.

For $M_N=9M_0$, $T = \frac{1}{2} m \left(\frac{GM_0}{R_0} + \frac{9GM_0}{R} \right) = x \left(\frac{1}{2} m \frac{10GM_0}{R} \right)$; $a_0 = \frac{GM_0}{R_0^{\ 2}}$; $R_0 = \sqrt{\frac{GM_0}{a_0}}$

For the Coma cluster, solving the above quadratic equation for the dark matter factor x with the Newtonian gravitational mass $x(10M_0)\approx 6.2\times 10^{15}\ M_\odot$, the radius $R\approx 6.17\times 10^{23}$ (~20 Mpc) [15], and $a_0=1.2\times 10^{-10}$ in MKS units, we get $x\approx 7.24$. Therefore, the regular Newtonian virial theorem predicts that the total gravitational mass of the Coma cluster is ~7.24 times the total baryonic mass which means that approximately 86 percent of the total gravitational mass in the Coma cluster is identified as dark matter.

For the Virgo cluster, with the Newtonian gravitational mass $x(10M_0)\approx 7.4\times 10^{14}\ M_\odot$ and the radius $R\approx 2.4\times 10^{23}$ (~7.4 Mpc) [16] in MKS units, we get $x\approx 8.74$. Therefore, approximately 88 percent of the total gravitational mass in the Virgo cluster is identified as dark matter.

For the Fornax cluster, with the Newtonian gravitational mass $x(10M_0)\approx 3.32\times 10^{14}\ M_\odot$ and the radius $R\approx 1.59\times 10^{23}$ (~5.18 Mpc) [17] in MKS units, we get $x\approx 8.66$. Therefore, approximately 88 percent of the total gravitational mass in the Fornax cluster is identified as dark matter.

As the percentages of the dark matter predicted in the galaxy clusters (Coma, Virgo, and Fornax) are in line with the standard model of cosmology and completely accounted for with the updated virial theorem using only the baryonic mass, this theory does not require dark matter to explain the missing mass in galaxy clusters. Therefore, according to this theory, the generalized virial theorem for any gravitationally bound system with both MONDian and Newtonian dynamics based on (29) and (31) for $z_0\approx 0$ is given below, where R is the virial radius.

Virial theorem:

$$\langle T \rangle = -\frac{1}{2} \left(\frac{\langle U_0 \rangle}{\ln\left(\frac{R_0}{R}\right)} + \langle U_N \rangle \right)$$

9. The temperature of the universe

The internal energy U of black-body photon gas is given by $U = \left(\frac{8\pi^5 k^4}{15h^3c^3}\right)VT^4$,

where k = Boltzmann constant, h = Planck constant, c = Speed of light, V = Volume, and T = Temperature. [18]

As the number of photons $N = \left(\frac{16\pi k^3 \zeta(3)}{h^3c^3}\right)VT^3$, $U = \frac{\pi^4 NkT}{30\zeta(3)}$.

The maximum possible temperature of the universe, when the age of the universe is t_p is given below by upholding the law of conservation of energy and considering CMBR as black-body radiation. The energy U and the number of photons N remain constant with the expansion of the universe. However, owing to cosmological time dilation in the past, photons appear to be more energetic (blue-shifted) owing to the apparent increase in frequency ν. Therefore, the energy U would appear to be U(1+z) in the past, as the energy is relativistic in this theory.

$U(1+z) = \frac{\pi^4 NkT_{t_p}(1+z)}{30\zeta(3)}$; $T_{t_p} = T(1+z)$, which can be generalized as follows:

$$T = T_0(1+z) \tag{37}$$

As $T_0 = 2.725$ K and the maximum possible cosmological redshift z is 7.4×10^{60} from (3), $T_{tp} = 2 \times 10^{61}$ K. As the T_{tp} value is greater than the Planck temperature, the maximum possible temperature of the universe is the Planck temperature, which is 1.416784×10^{32} K. However, this does not mean that this was the temperature at the time of the big repulsion, but only the maximum possible temperature. As the CMBR was emitted after the initial t_p of the big repulsion, the original temperature of the CMBR when it was first emitted should be less than the Planck temperature. The future temperature of the CMBR can also be calculated using the above equation. For example, the CMBR temperature after 10^7 years is 2.722 K.

As the entropy of a photon gas $S = \frac{4U}{3T}$, $S = \frac{S_0}{(1+z)}$ (37), where S_0 is the current entropy of the CMBR and S is the entropy at the cosmological redshift z.

As the interaction of the electrostatic expansion energy with the quantum vacuum fluctuations to create matter at the time of big repulsion will be the same throughout space-time, the created primordial elementary particles, atoms, and their attributes, such as temperature and density, should be uniform throughout space-time without having the particles interact with one another or without being causally connected, eliminating the need for cosmic inflation and solving the horizon problem, the flatness problem, and explaining the uniformity of the CMBR. However, minor fluctuations in the density of the created particles due to the randomness of the quantum vacuum fluctuations and the subsequent concentration of matter due to gravity could explain temperature anisotropy. However, the angular power spectrum of the CMBR still needs to be explained in this theory, possibly through baryon acoustic oscillations (BAO) and primordial black holes as the a_0 radius will be closer to the black holes due to the high cutoff MONDian acceleration $\left(\frac{CH_{cmb}}{2\pi}\right)$, where H_{cmb} is the Hubble constant at the time of recombination. According to this theory, MONDian gravity around primordial black holes, if any, in CMB, should behave like dark matter. In addition, black holes can act as seeds for galaxy formation, as per a recent study using JWST [19].

10. Atomic structure

In this theory, the local Bohr radius a_0 does not change with an increase in the gravitational potential energy or cosmological potential energy owing to the expansion of matter.

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{4\pi\frac{\varepsilon_0}{(1+z)}\left(\hbar(1+z)\right)^2}{m_e(1+z)e^2}$$, where $m_e(1+z)$ is the relative inertial mass of the electron owing to the gravitational and/or cosmological redshift z.

However, the relative energy of the Bohr atom increases to $E_{nz}=E_n(1+z)$ as the relative Planck constant increases by the same factor [$\Delta E=h(1+z)\nu$].

$$E_n = -\frac{Z^2e^4m_e}{32\pi^2\varepsilon_0{}^2\hbar^2n^2} \quad ; \quad E_{nz} = -\frac{Z^2e^4m_e(1+z)}{32\pi^2\left(\frac{\varepsilon_0}{1+z}\right)^2\left(\hbar(1+z)\right)^2n^2} = -(1+z)\frac{Z^2e^4m_e}{32\pi^2\varepsilon_0{}^2\hbar^2n^2}$$

Therefore, $E_{nz}=E_n(1+z)$. We observe a gravitational and/or cosmological redshift z as ΔE increases with an increase in gravitational and/or cosmological potential energy. However, the local numeric value of the energy of the Bohr atom remains unchanged owing to time dilation, although the relative energy changes.

In an inertial reference frame moving with velocity v with respect to the absolute reference frame, the local Bohr radius a_0 does not change, and the relative energy of the Bohr atom decreases to $E_{nz} = \frac{E_n}{(1+z)}$, where $\frac{m_e}{(1+z)}$ is the relative inertial mass of the electron. Here, the relative decrease in ΔE corresponds to the transverse relativistic redshift z.

11. Conclusions

This theory proves that both space-time and matter expand at the Hubble rate like an elastic ruler with constant proper distance and volume, establishes the relativistic acceleration of space-time by proposing cosmological time dilation similar to gravitational time dilation, and proves the relativistic Hubble law. The relative values of the fundamental constants have been shown to vary in proportion to the Hubble rate of the universe, and a relativistic Newtonian theory of gravity and relativistic MONDian gravity have been proposed. MONDian gravity is shown to work only around black holes but not around ordinary matter. The derived Schwarzschild radius, photon sphere, gravitational lensing, perihelion precession of mercury, and Shapiro time delay agree with the general theory of relativity without using any weak-field approximations or higher-order terms. Relativistic GEM equations are derived to prove the existence of gravitational waves and gravitomagnetic effects. This theory predicts gravitational/cosmological time dilation, gravitational/cosmological length contraction, and cosmological potential energy; eliminates black hole singularities; and clearly defines the relativistic scalar gravitational potential, whereas the general theory of relativity does not have a clear definition. The general theory of relativity does not explain the mechanism of dark energy that accelerates the expansion of the universe and requires dark matter that has not yet been discovered, whereas this theory successfully explains dark energy and dark matter in galaxy clusters. Overall, this theory proposes a hidden extradimensional electrostatic background to gravity to explain the dark energy, relativistic cosmology, relativistic gravity, and dark matter in galaxy clusters, which eliminates cosmic inflation, cosmic event horizon, cosmic scale factor (a), critical density (Ω), cosmological constant (Λ), and black hole singularities.

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Methods:
The computations in this paper were performed by using Maple™. Maple 2020.2. Maplesoft, a division of Waterloo Maple Inc., Waterloo, Ontario. Maple is a trademark of Waterloo Maple Inc.

Data availability statement:
Data sharing is not applicable to this article as no new data were created or analyzed in this study.