The Atomic and Quantum Mechanics Achievements of the Theory of Solar System Wave Packet

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Abstract

In the previous article we discussed about Solar System Wave Packet (SSWP). In that article, we showed that due to many reasons, such as the cleanliness of interplanetary space from other planets, the existence of chondrules and igneous rings around them in the solar system, the existence of super earths and hot Jupiters near the parent star, the existence of Titius-Bode law in the solar system, the many problems of current theories in explaining the formation of the planets of solar system and extrasolar planets, and many other reasons, we need the SSWP theory. Now it makes sense if we want to generalize our theory to the subatomic world. So, based on the theory of SSWP, we will enter into the atomic physics and achieve to a new atomic model with considerable results.

Keywords: Atomic model, Quantum mechanics, Schrodinger equation, Quantum Jumps, Wave-Particle Duality, Spreading of Wave Packet

1. Associated Wave Packet with Sun-Part 1

The equation of SSWP is (Soltani 2024):

$$\begin{cases} \psi(x,t) = C\cos(wt)\cos(\frac{10\pi}{3}x + \frac{\pi}{6})\ e^{-\gamma x^2} & x \geq 0 \\ \psi(x,t) = C\cos(wt)\cos(\frac{10\pi}{3}x - \frac{\pi}{6})\ e^{-\gamma x^2} & x \leq 0 \end{cases} \tag{1}$$

As we said in our previous article (Soltani 2024) it is unacceptable to imagine that the wave packet of solar system is not related with an object. In this section we will show that the equation of SSWP is exactly in the form of real part of the solution of Schrodinger equation and therefore, based on what we have learned from Quantum mechanics, we can attribute it to an object like the sun. In fact, in this section we prove that equation 1 is the result of superposition of a set of infinite number of plane matter waves which each of them is a solution of Schrodinger equation; And so, based on superposition principle, we can consider equation 1 as a solution of Schrodinger equation. But before that we have to talk about the mistakes in wave mechanics.

2. Mistakes in Wave Mechanics

There is a big problem in Schrodinger's mechanics or wave mechanics. Using the equation $E = mc^2$, which holds for Photons, and using the relations $E = hf$ and $c = f\lambda$ De Broglie was able to obtain equation 2 for Photons (e.g., Eisberg & Resnick 1985, Rohlf 1994, Ohanian 1987, Fleisch 2020):

$$hf = mc^2 = (mc)c = pc = pf\lambda \Rightarrow hf = pf\lambda \Rightarrow \lambda = \frac{h}{p} \tag{2}$$

He then assumed that, like light waves, matter waves also exist in nature. He said that matter waves are associated with matter particles and the relations $E = hf$ and $\lambda = h / p$, which hold for photons, are also valid for matter waves. The validity of the relation $\lambda = h / p$ for matter waves has been confirmed by numerous experiments such as Davison-Germer and Thomson experiments. But there is a problem here. If $\lambda = h / p$ is valid for matter waves, we should be able to obtain it for these waves using the calculations like calculations 2. But the energy of matter wave is equal to the energy of its associated particle (e.g., Cohen 1977, Fleisch 2020, Eisberg & Resnik 1985) and for free particles is $E = 1 / 2\, mv^2$ and for bound particles is $E = {}^1/_2\, mv^2 + V$. For free particles and using the $E = 1 / 2\, mv^2$ we can reach the formula $\lambda = h / p$ if we have: $E = {}^1/_2\ hf$ instead of hf.

$$\frac{1}{2}hf = {}^1/_2\, mv^2 \overset{v=f\lambda}{\Longrightarrow} \lambda = h/p \tag{3}$$

We can only change the equation hf and we cannot change the equation $v = f\lambda$. Because $v = f\lambda$ is the equation of phase velocity of wave (here matter wave) and it is obtained by derivative from the following equation (e.g., Walker & Halliday 2007):

$$kx - wt = constant \Rightarrow k\frac{dx}{dt} - w = 0 \Rightarrow \frac{dx}{dt} = \frac{w}{k} \Rightarrow v = f\lambda \tag{4}$$

Therefore, only by changing in the energy equation, we can reach the De Broglie formula for free matter waves. This change does not cause any problems. **The energy of a photon is $E = hf$ and the energy of matter wave is $E = {}^1/_2\ hf$.**

The plane wave equation for matter waves before this change in the energy was equal to $e^{\frac{i}{\hbar}(px \pm Et)}$ (e.g., Cohen 1977, Gasiorowicz 1974, Eisberg 1964), but with the change of the energy formula from hf to ${}^1/_2\ hf$, the form of the plane wave equation is $e^{\frac{i}{\hbar}(px \pm 2Et)}$. Because w is $\frac{2E}{\hbar}$:

$$E = {}^1/_2\, hf = {}^1/_2\, \hbar w \Rightarrow e^{i(kx \pm wt)} = e^{\frac{i}{\hbar}(px \pm 2Et)}$$

The change in the energy formula causes the Schrodinger equation for free waves to change as follows (equation 5). Because the new form of the plane wave namely $e^{\frac{i}{\hbar}(px \pm 2Et)}$ only is the solution of equation 5, not Schrodinger equation.

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{m}\frac{\partial^2\psi}{\partial x^2} \tag{5}$$

In this new form of the wave equation, the coefficient $1 / 2$ is removed from the right side of the Schrodinger equation for free particle.

Similar to the situation of associated waves with free particles, we must be able to reach the De Broglie equation for associated waves with bound particles. The energy of particle in the bound state is equal to $E = 1 / 2\, mv^2 + V$. In this situation, the equation $E = 1 / 2\, hf$, which is the energy of free wave, must be changed into the equation $E = 1 / 2\, hf + V$ to obtain the De Broglie equation.

$$ {}^{1}/_{2}\, hf + V = {}^{1}/_{2}\, mv^2 + V \xrightarrow{v=f\lambda} \lambda = h/p \tag{6} $$

The equation $E = 1\ /\ 2\ hf\ + V$ is the energy of bound wave. Of course, this is what we expect. Changing the **energy of particle** from $1\ /\ 2\ mv^2$ to $1\ /\ 2\ mv^2 + V$ causes the **energy of associated wave** with particle changes from $1\ /\ 2\ hf$ to $1\ /\ 2\ hf\ + V$. In this situation, the plane wave equation is equal to:

$$ {}^{1}/_{2}\, hf + V = {}^{1}/_{2}\ \hbar w + V \Rightarrow e^{i(kx \pm wt)} = e^{\frac{i}{\hbar}(px \pm 2(E-V)t)} \tag{7} $$

The important point is that in the bound state, like free state, the wave equation is equation 5 (if we calculate the temporal and spatial derivatives of equation 7 and put it in 5, the two sides of equation 5 will be equal to each other). This means that unlike the Schrödinger equation for bound waves, the term $V\psi$ does not exist in the wave equation.

Due to the absence of the term $V\psi$ in the new wave equation (Equation 5), the potential does not affect the wave **directly**; Rather, its effect is **indirect**: Applying potential, for example Coulomb potential, on the quantum entity (particle plus its associated wave) causes the energy of particle of quantum entity to change from $E = 1\ /\ 2\ mv^2$ to $E = 1\ /\ 2\ mv^2 + V$. By changing the energy of the particle, the velocity of the particle changes based on the $E - V = 1\ /\ 2\ mv^2$, and this change in velocity causes the change in the wavelength of the wave through the De Broglie formula. Because in the De Broglie formula $\lambda = h\ /\ mv$, v is the velocity of associated particle with wave. Therefore, as you observed here the effect of V on ψ is indirect.

Point: Because of Schrodinger considered the energy of a quantum entity in the bound state to be equal to $1\ /\ 2\ mv^2 + V$ and hf ($1\ /\ 2\ mv^2 + V$ is the energy of particle and hf is the energy of wave), he had to enter the term $V\psi$ in his wave equation. Because otherwise, $e^{i(kx \pm wt)}$ is not the solution of Schrodinger equation. But we said that considering the energy in the form of $1\ /\ 2\ mv^2 + V$ and hf for the bound state does not lead us to the De Broglie equation. So, we had to adopt a method that would lead us to the De Broglie equation. We assumed the bound wave energy is $1\ /\ 2\ hf\ + V$, which as we showed in equation 6, along with the bound particle energy formula $1\ /\ 2\ mv^2 + V$ leads us to the De Broglie equation. We said that this energy change removes the term $V\psi$ (not the potential itself) from the wave equation. Eliminating $V\psi$ means eliminating the **direct** effect of the potential on ψ. But this does not mean that the potential does not affect ψ. As we said in the previous paragraph, the potential **indirectly** affects the wavelength of ψ by changing the energy of the associated particle with ψ.

Finally, the conclusion of the discussion in this section is that the energy of free and bound matter waves instead of hf should be considered ${}^{1}/_{2}\, hf$ and ${}^{1}/_{2}\, hf + V$ to obtain the De Broglie equation. We showed that these changes in energy lead to the elimination of the term $V\psi$ and the coefficient $1/2$ from the Schrödinger equation and the result of these changes in Schrodinger equation is equation 5. We call equation 5 the *New Wave Equation*. In this article, based on the new wave equation, we find wave functions for the nucleus and electron of the hydrogen atom and we link wave mechanics with Bohr's atomic theory. A link that was not created during the lifetime of Niels Bohr and Schrodinger and even to this day. The new wave mechanics is much more powerful than the Schrodinger mechanics and its atomic model is much more logical than the Schrödinger-Born atomic model and the theory of atomic orbitals.

Moreover, as you know, Schrodinger wave packets spread with time. This is one of the biggest problems of Schrodinger mechanics. This problem is solved in the new wave mechanics. The new wave mechanics does not have the problems of Schrodinger mechanics.

3. Associated Wave Packet with Sun-Part 2

We said that we want to prove in this section that equation 1 is the real part of the solution of Schrodinger's equation. But considering that the Schrodinger equation has been changed to the new wave equation, we should prove that equation 1 is the solution of the new equation. Base of calculations in this section is superposition principle. Consider a set of infinite number of plane matter waves $Ae^{i(kx-wt+\emptyset_0)}$, $Ae^{i(kx+wt+\emptyset_0)}$, $Ae^{i(kx-wt-\emptyset_0)}$ and $Ae^{i(kx+wt-\emptyset_0)}$ which move in the positive and negative directions of the x-axis (These four groups of waves are all of possible form of plane matter waves). It is clear these four groups of waves with general equation $Ae^{i(kx\pm wt\pm\emptyset_0)}$ are the solution of the new wave equation. Therefore, based on superposition principle, the superposition of these waves must be the solution of the new wave equation. In this section we prove that equation 1 is the superposition of a set of mentioned matter waves; And so, we can consider equation 1 as a solution of new wave equation. And therefore, based on what we have learned from Quantum mechanics, we can attribute it to an object like the sun.

From infinite numbers of mentioned plane waves we will show that the superposition of *a part of* these waves, which have three following properties, is in the form of equation 1: **1)**- their wave number is around the median of k_0 and between $k_0 + {}^{\Delta k}/_2$ and $k_0 - {}^{\Delta k}/_2$, **2)**- the amplitude of these waves is on the bell shaped function $A(k) = (\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$(which is a Gaussian function), and **3)** - their angular frequency is equal to w_0 [1]

First consider a set of infinite number of these waves with equation: $Ae^{i(kx-w_0t+\emptyset_0)}$ which move in the *positive direction of x-axis*. In such a case, the resultant of these waves, using the superposition principle, is a wave packet with equation 8 (Branson 2003, Gasiorowicz 1974, Cohen 1977).

$$\psi_{total}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{k_0-{}^{\Delta k}/_2}^{k_0+{}^{\Delta k}/_2} A(k)e^{i(kx-w_0t+\emptyset_0)}dk \tag{8}$$

Where k means k_x. The equation 8 is the equation of wave packet. In this equation $A(k)$ is $(\frac{2\alpha}{\pi})^{1/4}e^{-\alpha(k-k_0)^2}$ and is amplitude of wave packet, α is a constant with a positive value and shows the width of the bell-shaped function $A(k)$. Coefficient $(\frac{2\alpha}{\pi})^{1/4}$ is a normalization coefficient which is obtained by normalize of $A(k)$ (note: You may ask why a Gaussian function is chosen for $A(k)$ in formula 8. Since a Gaussian function is very localized, it is a suitable choice to be considered as the amplitude of the associated wave packet with a particle.

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[1] In the Electromagnetic (EM) waves we cannot consider one w_0 for two or many waves in which their k is different from each other, because for all of the EM waves we have: $w = ck$ where c is the velocity of light. But for matter waves the issue is different. In the matter waves, based on our new wave mechanics, we have $w = \hbar k^2/m$ for both free and bound wave. As you can see w is the function of k and m. Therefore, it is possible to choose one value of w_0 for the waves in which their k is different from each other.

So, Schrödinger put it into the quantum wave packet equation, equation 8.). Since equation 8 is derived from the superposition principle, it is the solution of the new wave equation.

To solve the integral of equation 8, we calculate the superposition of all of the waves in one moment, which we consider to be the origin of time ($t = 0$), and then we can obtain the net wave at any other time. We have:

$$\psi(x,0) = \frac{1}{\sqrt{2\pi}} \int A(k) e^{i(kx + \emptyset_0)} dk \tag{9}$$

The above equation is the momentary image of the net wave. Multiply equation 9 by $e^{ik_0x - ik_0x}$. We have:

$$\psi(x,0) = \frac{1}{\sqrt{2\pi}} e^{i(k_0x + \emptyset_0)} \int A(k) e^{i(k-k_0)x} dk \tag{10}$$

Considering $\acute{k} = k - k_0$, we have:

$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0x + \emptyset_0)} \int e^{-\alpha \acute{k}^2} e^{i\acute{k}x} d\acute{k} \tag{11}$$

Using the variable transformation $\acute{k} - \frac{ix}{2\alpha} = q$ (Branson 2003, Gasiorowicz 1974) we can change this integral to the familiar Gaussian integral $\int_{-\infty}^{\infty} dq\, e^{-\alpha q^2} = \sqrt{\frac{\pi}{\alpha}}$ and solution it. After replacement and simplification, we reach the following final solution (Branson 2003 & Gasiorowicz 1974):

$$\psi(x,0) = (\frac{\alpha}{2\pi^3})^{1/4} \sqrt{\frac{\pi}{\alpha}}\, e^{i(k_0x + \emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} = (\frac{1}{2\pi\alpha})^{1/4} e^{i(k_0x + \emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \tag{12}$$

Now, how is the time variation of equation 12? Let's go back to equation 8:

$$\psi(x,t) = \frac{1}{\sqrt{2\pi}} \int A(k) e^{i(kx - w_0t + \emptyset_0)} dk = (\frac{\alpha}{2\pi^3})^{1/4} \int e^{-\alpha(k-k_0)^2} e^{i(kx - w_0t + \emptyset_0)} dk$$

Substituting $e^{ik_0x - ik_0x}$; we will have:

$$\psi(x,t) = (\frac{\alpha}{2\pi^3})^{1/4} e^{i(k_0x + \emptyset_0) - iw_0t} \int e^{-\alpha \acute{k}^2} e^{i\acute{k}x} d\acute{k}$$

This integral is similar to integral 11. So, we can solution it with the similar way. Therefore, we have:

$$\psi(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{i(k_0x - w_0t + \emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \tag{13}$$

We call ψ here ψ_1.

$$e^{i\theta} = cos\theta + isin\theta \Longrightarrow Re\, \psi_1(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0x - w_0t + \emptyset_0)\, e^{-\frac{x^2}{4\alpha}} \tag{14}$$

Due to the presence of the factor $k_0x - w_0t$, equations 13 and 14 represent a traveling wave packet that propagates in the positive direction of the x-axis (Walker & Halliday 2007). This means that the location of the nodes is not known. Due to the absence of t in $e^{-\frac{x^2}{4\alpha}}$ in equations 13 and 14, **the wave packets in these equations does not spread**.

Previous calculations were about superposition of the waves $e^{i(kx-w_0t+\emptyset_0)}$. Similarly, we use the recent trend to obtain the superposition of plane waves traveling *in the negative direction of the x-axis*, i.e. $Ae^{i(kx+w_0t+\emptyset_0)}$. If we do this, we get to equation 15:

$$\psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{i(k_0x+w_0t+\emptyset_0)}\, e^{-\frac{x^2}{4\alpha}} \tag{15}$$

$$Re\,\psi_2(x,t) = (\frac{1}{2\pi\alpha})^{1/4} cos(k_0x + w_0t + \emptyset_0)\ e^{-\frac{x^2}{4\alpha}} \tag{16}$$

This equation shows a traveling wave packet that propagates in the negative direction of the x-axis.

Now we sum up the two equations 14 and 16 together to get the final wave.

$$Re\,\psi_{total}(x,t) = Re\,\psi_1 + Re\,\psi_2$$

Thus:

$$Re\,\psi_{total}(x,t) = (\frac{1}{2\pi\alpha})^{1/4}\, e^{-\frac{x^2}{4\alpha}}[cos(k_0x - w_0t + \emptyset_0) + cos(k_0x + w_0t + \emptyset_0)] \tag{17}$$

Using $cos\alpha + cos\beta = 2cos\frac{1}{2}(\alpha+\beta)cos\frac{1}{2}(\alpha-\beta)$ and $\cos(\theta) = \cos(-\theta)$ we obtain the equation of a standing wave packet.

$$\begin{cases}\alpha = k_0x - w_0t + \emptyset_0 \\ \beta = k_0x + w_0t + \emptyset_0\end{cases} \Rightarrow Re\,\psi_{total}(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4}\cos(k_0x + \emptyset_0)cos\,(w_0t)e^{-\frac{x^2}{4\alpha}} \tag{18}$$

There is not the structure of $kx \pm wt$ in equation 18 so the ψ_{total} is a standing wave. As you observe, equation 18, which is the real part of a solution of the new wave equation, is exactly the same as equation 1 for $x \geq 0$. Is this similarity coincidental? No. *Therefore, equation 1 is the real part of a solution of the new wave equation. It means that the new wave equation and quantum mechanics are valid for astronomical objects.* Comparing equation 18 and equation 1, we have

$$\gamma = \frac{1}{4\alpha} \quad and \quad C = 2(\frac{1}{2\pi\alpha})^{1/4}$$

If we put these values in equation 1, then we get the final equation of SSWP for $x \geq 0$:

$$Re\,\psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0t)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}} \qquad x \geq 0 \tag{19}$$

Equation 19 is obtained by calculating the superposition of a set of infinite number of waves $Ae^{i(kx-w_0t+\emptyset_0)}$ and $Ae^{i(kx+w_0t+\emptyset_0)}$ that move in opposite directions to each other (pay attention to the sign + behind $\emptyset_0$). Now if we sum a set of infinite number of plane wave functions with the equations $Ae^{i(kx-w_0t-\emptyset_0)}$ and $Ae^{i(kx+w_0t-\emptyset_0)}$ (pay attention to the sign $-$ behind $\emptyset_0$) together, by following the path we have taken from equation 8 to equation 19, we reach the following relation;

$$Re\ \psi_t(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0 t)\, cos(\frac{10\pi}{3}x - \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}}$$

Which is the same as equation 1 for $x \leq 0$. Therefore, the final form of SSWP (equation 1) is as follows:

$$\begin{cases} Re\ \psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0 t)\, cos(\frac{10\pi}{3}x + \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}} & x \geq 0 \\ Re\ \psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0 t)\, cos(\frac{10\pi}{3}x - \frac{\pi}{6})\ e^{-\frac{x^2}{4\alpha}} & x \leq 0 \end{cases} \qquad (20)$$

In this equation, the larger the α is, the more the width of wave packet, along the x-axis. We drew Fig. 2 and Fig. 3 in our previous article by $\alpha = 10$ (Soltani 2024). Due to the absence of t in $e^{-\frac{x^2}{4\alpha}}$ in equation 20, **the wave packet in this equation does not spread**.

Here we demonstrated that equation of SSWP (equation 1) is the real part of a solution of the new wave equation. So, based on what we have learned from Quantum mechanics, we can attribute it to an object in Solar system. The biggest and heavyset object in solar system is sun. Therefore, the wave packet of solar system can only belong to the sun. **The Davisson–Germer experiment showed that the wave nature is a part from the reality of subatomic entities (like electrons, protons), and here we showed that wave nature is a part from the reality of celestial entities (like sun) too. Neither of these two conclusions is strange. Rather, they are truths that we should become accustomed to.**

In this section, we proved that the new wave equation is valid for astronomical objects. The new wave equation is based on de Broglie equation ($\lambda = \frac{h}{mv}$). Therefore, the de Broglie equation is valid in astronomical scale. But, according to the very large mass of sun, by using the de Broglie relation the wavelength of SSWP (namely 0.6 AU) will not obtain. So, instead of Planck constant we must choose another value for celestial objects, which is larger than h. We call this new value the Planck constant in Astronomy ($h_{Astronomy}$) abbreviated as h_A and we have: $\lambda_A = \frac{h_A}{P}$. In such a case, the new wave equation in the astronomical scale can be written as follows:

$$i\hbar_A \frac{\partial}{\partial t}\psi(x,t) = -\frac{{\hbar_A}^2}{m}\frac{\partial^2\psi(x,t)}{\partial x^2} \qquad (21)$$

4. How Was the SSWP Formed?

In section 3 we did not do anything strange. Rather, we have used only the superposition principle. There have been numerous numbers of plane matter waves in the early solar system which the superposition of a specific set of them made the wave packet in Fig. 2 in our previous article (Soltani 2024). Our calculations showed that only the interference of this special group of waves is constructive. In the protoplanetary disk and *before the formation of the sun*, there were countless plane matter waves with the general equation $e^{i(kx \pm wt \pm \emptyset_0)}$. *After the formation of sun* because of gravitational collapse, a special group of these waves interfered constructively with each other and created the SSWP.

5. Atomic Physics

5.1. New Atomic Model

It makes sense if we want to generalize our theory, which was about solar system, to the subatomic world. As Niels Bohr used the planetary model to describe the atom in 1913. In this section, we present a new atomic model based on the model of the SSWP. *Our atomic model explains why the Bohr atomic orbits are quantized.* Niels Bohr could not explain this issue. Here, based on this model we will explain the main and secondary spectrum of the hydrogen atom. Some of the atomic models are shown in Fig. 1.

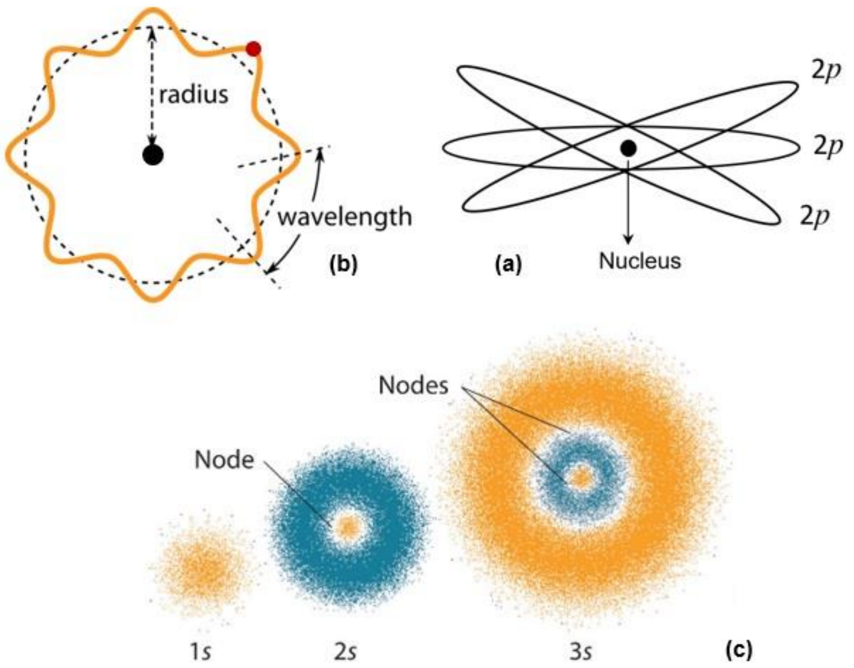

Fig. 1. Some of the atomic models. This figure shows three atomic models: **a). Bohr-Sommerfeld atomic model**: In this model, as you can see, subshells have space orientation. In this model, the horizontal $2P$ subshell is in the same plane of the $1S$ and $2S$ subshells. $1S$ and $2S$ subshells are not shown in this figure. **b). De Broglie atomic model.** De Broglie believed in standing waves around the nucleus. **c). Born-Schrodinger atomic model.** In this figure S orbitals are shown. According to Max Born probability theory, the electron can be found in the distance between the nodes in the Fig. 1c. Other atomic models, such as the Rutherford or Thomson atomic models, are not shown in this figure. Fig. 1c is plotted using equation $P(r) = r^2(R_{nl})^2$, where R is the solution of Radial Schrodinger Equation (Gasiorowicz 1974).

Fig. 2 shows our atomic model, which we call the "New Atomic Model". To achieve to Fig. 2, we used the same method, which we used to obtain the equation and shape of SSWP in our previous article (Soltani 2024). The Bohr atomic model is a successful model which its orbits are at $x_1 = 0.53$Å, $x_2 = 4x_1$, $x_3 = 9x_1$, etc. In order to be able to attribute a standing wave to Bohr model, our wavelength should be equal to $2x_1 = 1.06$ Å. Suppose the first orbit of the new atomic model is the first orbit of the Bohr atomic model, namely $x_1 = 0.53$ Å. The first orbit in our model is equivalent to the first node and the first node means $\emptyset = \frac{\pi}{2}$ because $Cos\ \frac{\pi}{2} = 0$. In such a case we have:

$$x_1 = 0.53 \text{ Å} \Rightarrow \psi(x_1) = 0 \Rightarrow cos(kx_1 + \emptyset_0) = \cos\frac{\pi}{2} = 0 \Rightarrow kx_1 + \emptyset_0 = \frac{\pi}{2}$$

$$\xRightarrow{k=\frac{2\pi}{1.06}} \emptyset_0 = -\frac{\pi}{2}$$

In the same way of the reasonings in our previous article (Soltani 2024), the final form of "hydrogen atom wave packet" is equal to:

$$\begin{cases} Re\,\psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0' t)\, cos(5.92x - \frac{\pi}{2})\ e^{-\frac{x^2}{4\alpha}} & x \geq 0 \\ Re\,\psi(x,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0' t)\, cos(5.92x + \frac{\pi}{2})\ e^{-\frac{x^2}{4\alpha}} & x \leq 0 \end{cases} \quad (22)$$

The equation 22 in cylindrical coordinate:

$$x = r\cos\theta \Rightarrow\ Re\,\psi(r,\theta,t) = 2(\frac{1}{2\pi\alpha})^{1/4} cos(w_0' t)\, cos(5.92r\cos\theta - \frac{\pi}{2})\ e^{-\frac{(r\cos\theta)^2}{4\alpha}} \quad (23)$$

In equations 22 and 23, we consider the angular frequency equal to w_0' that not to be confused with w_0 in equation 20. Fig. 2 is drawn in cylindrical coordinate. In the same way as described in section 3, we can show that the equation 22 is the real part of the solution of the new wave equation.

The first node of the associated wave packet with the hydrogen nucleus is at r_1 = 0.53Å and the second node is at $r_2 = 2r_1$, and third at $r_3 = 3r_1$, etc. In the new atomic model, we have: $r_n = nr_1$, and in the Bohr model we have $r_n = n^2 r_1$. This means that the orbit number n in the Bohr model is the orbit number n^2 in the new atomic model. For example, the second orbit in the Bohr model is the fourth orbit in the new atomic model (Fig. 3). In Fig. 2 the electron rotates on the first node of the associated wave packet with the nucleus (just like rotation of planets in Fig. 2 in our previous article (Soltani 2024). This orbit is completely circular due to the presence of the wave packet of nucleus. This orbit never turns into an ellipse under the effect of the inverse_square force of the nucleus; Because unlike SSWP, the nucleus's wave packet of an atom is always present and active and its oscillations always keep the electron in the circular orbits. (As we said in section 7 of the previous article (Soltani 2024), the SSWP probably disappeared around 4.5 billion years ago; and now it is gone and not active).
As mentioned in the previous article (Soltani 2024), the role of the oscillation of the wave packet is to prevent particles from being placed between the nodes. In the new atomic model, the formation of an atom (for example, a hydrogen atom) is such that the electron approaches the proton and is placed in the first node of the associated wave packet with proton. The proton’s associated wave packet prevents the electron from being placed in the distance between the nodes. In this atomic model we use Bohr's assumptions and assume that the electron does not radiate while it orbits about the nucleus (Bohr 1913). In this model, like the Bohr model, the spectral lines of the hydrogen atom are caused by the jump of electrons from the higher orbits to the lower orbits (Bohr 1913), which we will discuss in next section about it.

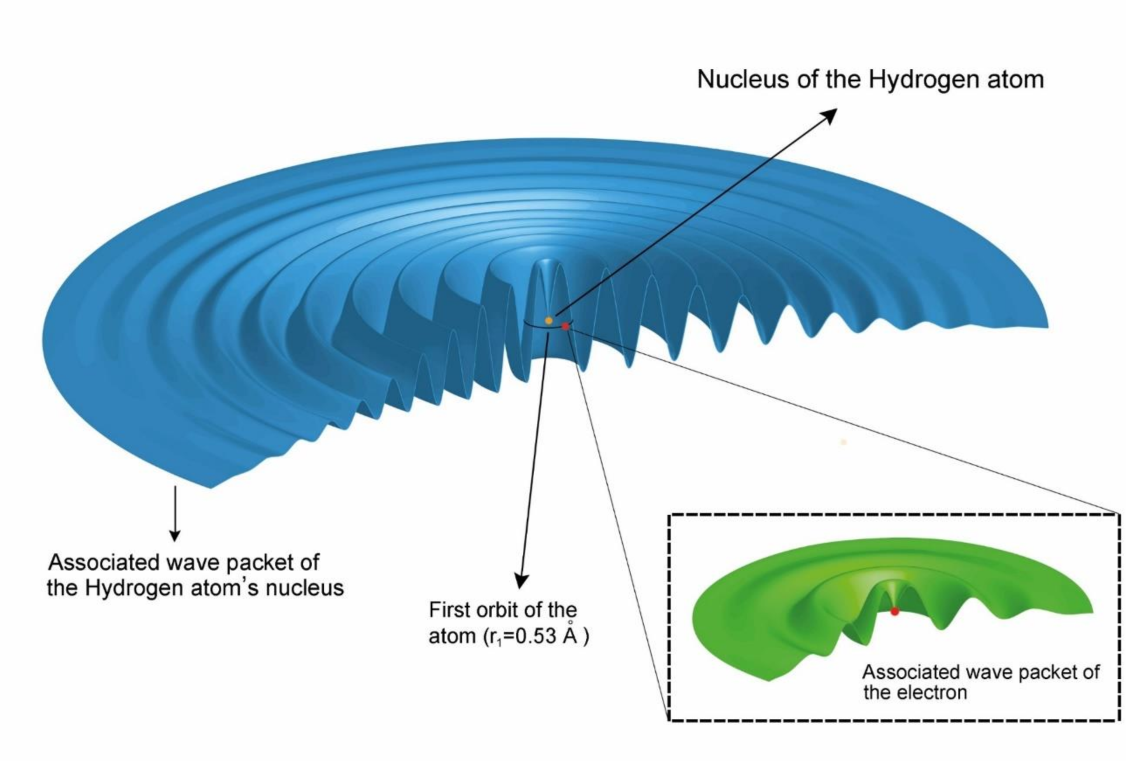

Fig. 2. The New atomic model. Blue wave packet is the diagram of $\psi(x,t)$ at the moment t = 0 based on equation 23. In this figure, associated wave packet with nucleus and associated wave packet with the electron are shown at t = 0. In this figure, the electron and its associated wave rotate about the nucleus of hydrogen atom in first orbit. The second and the other orbits are not shown in this figure. The wavelength of the blue wave packet is $2r_1$ ($r_1 = 0.53$Å). If we consider the first orbital velocity of electron based on the Bohr model $v_1 = \frac{1}{137}c$, in such a case the wavelength of associated wave packet with electron in the first orbit of hydrogen atom, based on de Broglie equation ($\lambda = \frac{h}{mv}$), is 3.3 Å (Although, no one has ever measured the velocity of the electron in the first orbit of the hydrogen atom with a laboratory device; But $v_1 = \frac{1}{137}c$ is an empirical value. If v_1 does not have this value, the Rydberg constant obtained from the Bohr atomic model was not equal to its experimental value). There is no matter if here the wavelength of associated wave with electron is larger than the wavelength of the associated wave with the nucleus.

In Fig. 2, the nucleus's wave packet is plotted using equation 23 with α = 7 and $\lambda = 2r_1 = 1.06$ Å or k = 5.92. And the electron wave packet with λ = 3.3 Å or k = 1.9 and α = 7. **The new atomic model includes both the wave mechanics and the Bohr atomic model inside.** Fig. 2 is probably the final model of Hydrogen atom.

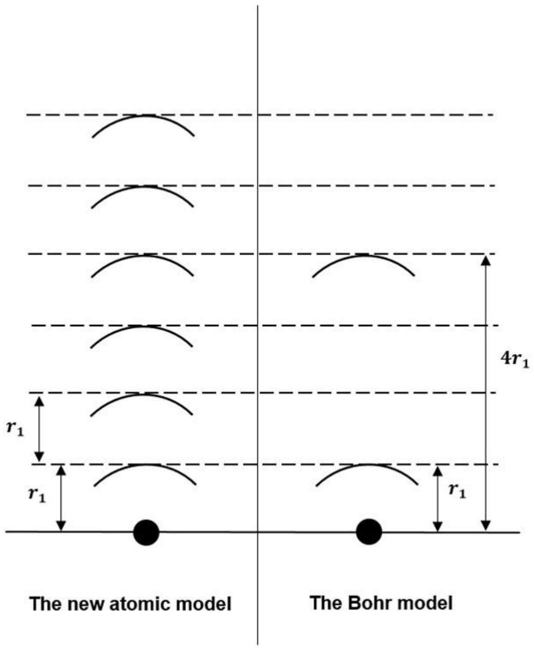

Fig. 3. The new atomic model compared to the Bohr model. For example, the second orbit in the Bohr model is the fourth orbit in the new atomic model. The black circles are hydrogen nucleus.

5.2. Quantum jumps and Spectral Lines of the Hydrogen Atom

When an electric spark is passed through Hydrogen lamp (gaseous hydrogen), the H_2 molecules are broken and the excited H atoms are produced. These atoms emit light with discrete frequencies (namely Ballmer, Lyman, …. series) (e.g., Atkins & Paula 2018). In the hydrogen spectrum, there are other lines besides the *main spectrum lines* (like Ballmer, Lyman, …. series.) (Merton & Barratt 1922, Allan 1924, Nicholson 1920, Richardson 1926), which are called *secondary spectral lines*. These lines are not predicted by the Bohr atomic model. By 1922, 750 number of these lines were discovered between H_α and H_β (Allan 1924) (H_α and H_β are the first and second frequencies of the Ballmer series). Some physicists have linked secondary lines to impurities in the hydrogen lamp (Merton & Barratt 1922), and some have attributed them to hydrogen molecules in the lamp (Merton & Barratt 1922). But all these was just speculation. Today, these lines are known as the *molecular spectrum of hydrogen*, which this is based on Merton's article (Merton & Barratt 1922). Merton in his article and in the section "Experimental Results" proved in a very vague way that two groups of the secondary spectrum lines are related to hydrogen molecules and finally concluded that: "it is *probable* that the whole of the secondary spectrum is due to the hydrogen molecule". But it can be shown that this conclusion is wrong. And only part of the secondary spectrum may be related to the hydrogen molecules inside the lamp. In this part of the article, based on the new atomic model, we predict many wavelengths for the spectrum of the hydrogen atom. If it is proven that these wavelengths exist in the list of wavelengths observed from the hydrogen lamp. This means that the origin of these wavelengths is atomic, not molecular.

In the Bohr atomic model, the lines of the emission and absorption spectrum of hydrogen atom are the result of *quantum jumps*. We use from the idea of quantum jumps in the new atomic theory. Consider Fig. 3. As we said, if the Bohr model has n orbits, the new atomic model has n^2orbits. For example, the second Bohr orbit is the fourth orbit in the new atomic model (Fig. 3). Based on this, for example, the spectrum line Ly_α in the Lyman series, which equals ν_{21} in the Bohr model, is equal to ν_{41} in the new atomic model. Since the number of orbits in the new atomic model is more, we should expect more quantum jumps and consequently *more emission lines* in the hydrogen emission spectrum. For example, according to Fig. 3, in the new atomic model, we should be able to observe $\nu_{21}, \nu_{32}, \nu_{31}, \nu_{51}, \ldots$ and many other frequencies, in addition to the lines predicted by the Bohr atomic model. For example, ν_{31} in the new atomic model equals

$$h\nu_{31} = E_3 - E_1 \xRightarrow{E_H = \frac{-\mu k_e e^2}{2m_e r}} \nu_{31} = \frac{1}{1 + \frac{m_e}{M_P}} \frac{k_e e^2}{2h}\left(\frac{1}{r_1} - \frac{1}{3r_1}\right) = 3.35 \times 10^{15} \times \frac{2}{3}$$

$$\therefore \Rightarrow \nu_{31} = 2.23 \times 10^{15}\ Hz \ \Rightarrow \lambda_{31} = 1340\ \text{Å}$$

In the above calculations we used from equation $E_H = \frac{\mu}{m_e} E_\infty = \frac{-\mu k_e e^2}{2m_e r} = \frac{1}{1+\frac{m_e}{M_P}} \frac{k_e e^2}{2r}$ instead of $E_\infty = \frac{-k_e e^2}{2r}$ (Ohanian 1987, Eisberg & Resnick 1985). The equation $E_\infty = \frac{-k_e e^2}{2r}$ is the equation of energy of electron when the mass of nucleus is infinite. In the above equation, μ is reduced mass. The Bohr atomic model does not predict the wavelength $\lambda_{31} = 1340$ Å. The observation of this wavelength in the hydrogen spectrum will be a strong confirmation for the new atomic theory. As another example, based on the new atomic model, we have:

$$h\nu_{114} = E_{11} - E_4 = \frac{1}{1 + \frac{m_e}{M_P}} \frac{k_e e^2}{2}\left(\frac{1}{4r_1} - \frac{1}{11r_1}\right) \Rightarrow \nu_{114} = 0.53 \times 10^{15}\ Hz \ \Rightarrow \lambda_{114}$$
$$= 4640\ \text{Å}$$

Other examples are:

$$h\nu_{104} = E_{10} - E_4 \Rightarrow \lambda_{104} = 4925\ \text{Å} \qquad and \qquad \lambda_{84} = 5980\ \text{Å}$$

These three wavelengths are in the visible spectrum region. It is possible to calculate dozens of other wavelengths from the secondary spectrum using this method. I do not have the list of wavelengths observed from the hydrogen lamp, but it is very likely that the four mentioned wavelengths, and dozens of others that can be calculated using the new atomic model, are among the hundreds of observed wavelengths of Hydrogen Lamp spectrum. If so, it would be an important achievement for the new atomic model.

6. The Efforts of Bohr and De Broglie

The content you read in this section, like the previous sections, is the result of reading many books and articles and the result of deep thinking and analysis. Here, we analyze the working methods, thoughts and results of Bohr, De Broglie's theories, not review. And we reach important results. Consider an electron orbiting a proton in a circular orbit. we have:

$$m\frac{v^2}{r} = k\frac{e^2}{r} \Rightarrow v = \sqrt{\frac{ke^2}{mr}} \qquad (24)$$

The angular momentum of the electron is equal to:

$$L = r \times P = rmv = rm\sqrt{\frac{ke^2}{mr}} = e\sqrt{mrk} \tag{25}$$

It is clear that if we have r in equation 25, we can get L. So, Bohr used equation 26:

$$E = \frac{1}{2}mv^2 + k\frac{e^2}{r} \Rightarrow r = -\frac{ke^2}{2E} \tag{26}$$

The first ionization energy of electron in hydrogen atom is equal to $13.6\,eV$. Using 26, the radius of the first orbit is equal to:

$$r_1 = 0.53\text{ Å}$$

Substituting in equation 25:

$$L = 1.054571817 \times 10^{-34} \quad joule\ second \tag{27}$$

Bohr saw that this number is equal to ħ (I believe that understanding this point was the key to Bohr's theory, which opened all doors for Bohr). After Realizing this point, Bohr was forced to use the results of William Nicholson's atomic model to obtain the spectrum of the hydrogen atom. Although the atomic model of these two people was completely different from each other. In 1911, based on a ringy atomic model, Nicholson obtained the total angular momentum of 5 electrons in a 5-electron atom equal to $25\hbar$ (Nicholson 1911). Then, based on inductive reasoning, he concluded that for a one-electron atom, this value should be equal to $\hbar$ (it should be noted that Nicholson's atomic model is such that, for example, in a 5-electron atom, the electrons are all on one ring revolves around the nucleus, which is completely different from Bohr's atomic model). Bohr used the result of Nicholson's work and wrote Nicholson's formula $L = n^2\hbar$ in his atomic model in the form of:

$$L = n\hbar \tag{28}$$

This choice of Bohr had important results for him and allowed him to obtain the main spectrum of the hydrogen atom. In Nicholson's atomic model, n is the number of electrons, but in Bohr's atomic model, it is the orbital number. Using the equation 28 and 25, Bohr obtained the radius of the orbit n as follows:

$$r_n = \frac{n^2\hbar^2}{mke^2} = n^2 r_1 \tag{29}$$

These orbits are special orbits. These are the orbits that if an electron jumps from any of them to a lower orbit, it emits light with the frequency of one of the main spectrum lines of the hydrogen atom. On the other hand, these orbits are orbits whose perimeter is equal to the correct multiple of the wavelength of the electron that moves on these orbits:

$$L = n\hbar \Rightarrow mvr = n\frac{h}{2\pi} \xRightarrow{\lambda = h/mv} 2\pi r = n\lambda \Rightarrow S_n = n\lambda \tag{30}$$

This was the issue that de Broglie realized (e.g., Eisberg & Resnick 1985, Gasiorowicz 1974). But what does the equation 30 want to say?? Perhaps Fig. 4 can help us understand this. Researches show that the sun has a wavy movement around the center of the galaxy as seen in Fig. 4 (yellow path in Fig. 4). It can be concluded from this issue that maybe the electron, in addition to having an associated wave (green wave in Fig. 2), for some unknown reason, has a wavy movement while traveling around the nucleus. A motion like the motion of sun in Fig. 4. We have shown this electron motion in Fig. 5.

Based on equation 30, we can conclude that, for some unknown reason, the electron tends to settle into an orbit that can perform complete wave motion. Not an incomplete wave motion.

The only orbits in which the electron can perform complete wave motion are the Bohr model orbits. In addition to the circular motion, the electron on these circuits also has an up and down motion. so that the horizontal component of electron velocity in the first orbit is equal to $1/137$. This means that in the first circuit, the shadow of the electron in figure 5 moves on the circle with a velocity of $1/137$. In fact, in formula 24, v is the horizontal component of electron velocity.

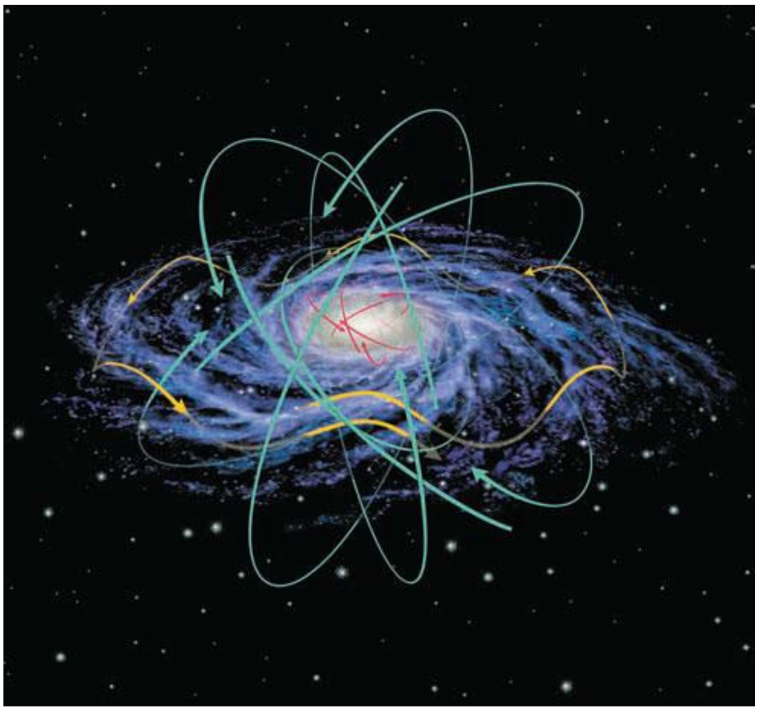

Fig. 4. The yellow line is the path of sun around center of Milky Way.

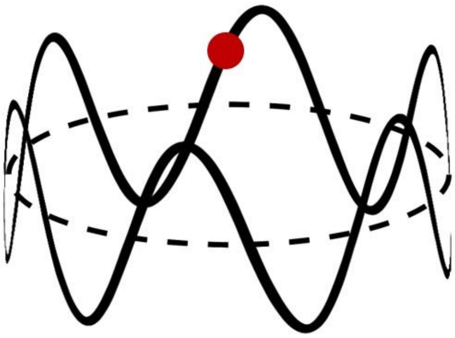

Fig. 5. Vertical oscillation of bound electron of Fig. 2 around Hydrogen nucleus. In this figure the associated wave packet with electron (green wave packet in Fig. 2) is not shown.

7. Quantum Achievements

It seems that in this paper we have been able to put wave mechanics in the right path and achieve the correct atomic model. This article puts an end to many confusions and mistakes. The quantum achievements of the article are as follows:

1- Non-spreading of wave packet: In this article, in section 3, we showed that the superposition of a particular group of matter waves leads to a wave packet (equation 20) which does not spread over time. This achievement saves wave mechanics from a major problem. I think no one is as pleased with this achievement as Schrodinger. Schrodinger was always annoyed by the spread of the wave packet. The Schrödinger's wave packets had an important problem: Lorentz warned Schrodinger that his wave packet would spread over time (e.g. Gasiorowicz 1974, Cohen 1977, McEvoy & Oscar 2013). This point of Lorentz showed that the Schrödinger's wave packet should be destroyed after a while. In this article, in section 3 we showed that the superposition of a special group of plane waves reach us to a quantum wave packet that does not spread over time. And this is an important finding for wave mechanics.

2- Rejecting the use of Max Born's probability concept and use of potential term in wave equation: We explained about removing the potential term from the Schrödinger equation in section 2. It is clear that the elimination of the potential from the Schrodinger equation means the invalidity of the Schrödinger-Born atomic model (namely the invalidity of Fig. 1c and Fig. 6). In addition, our article means getting rid of Max Born's probability concept. In Fig. 2, electron and proton are at the center of their associated wave packets, and so the issue of probability is meaningless. Prior to my article, $\psi(x,t)$ was a meaningless mathematical thing and only $|\psi(x,t)|^2$ meant something to us (e.g., Gasiorowicz 1974). But this article gave meaning to $\psi(x,t)$ and from now on $|\psi(x,t)|^2$ can only be considered as the intensity of the wave packet at x and t, not probability.

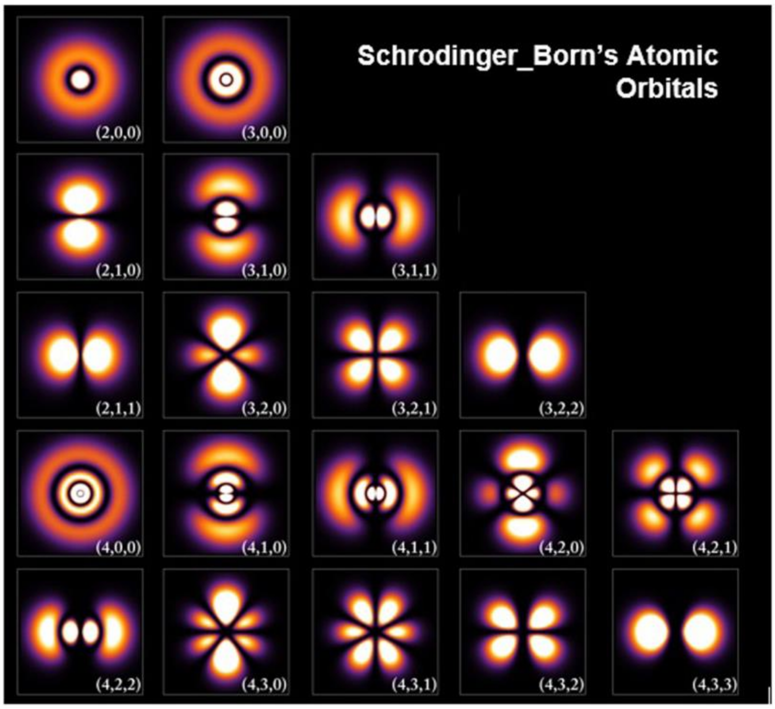

Fig. 6. Schrodinger-Born atomic model. The drawing of these orbitals is obtained by placing the Coulomb potential of hydrogen atom in the Schrodinger equation. Its calculations can be found in any quantum mechanics academic book.

3- The end of the wave-particle duality controversy for matter: Our article ends the particle-wave duality for matter. Based on our article, every matter quantum entity is a wave with a particle at its center (Fig. 2). For example, in diffraction experiments of electron, what appears on the screen is the effects of interference of the green wave packet of electron in Fig. 2 with the wave packet of another electron. And in the face-to-face collisions of, for example proton-proton, the cause of elastic interaction is because of elastic interaction of the center particle of the wave packet of proton in Fig. 2 with the center particle of another proton, not interaction of the wave packets. Before our article, the reason for the dual behavior of a quantum entity in different phenomena was unknown to us. But from today, the nature of a quantum entity is clear for us and so we know the reason for its dual behavior.

4- The reason for the existence of Bohr's atomic orbits: After the presentation of Bohr's theory, the question that was raised for scientists was that in an excited atom, what is the reason of quantization of Bohr's orbits? Or how does an electron that goes to a higher orbit know where the higher orbit is? "It's as if the electrons in your model know where to stop", Ernest Rutherford told Bohr. Our theory answers this problem. This is the wave packet of the atom's nucleus (namely blue wave packet in Fig. 2) that determines the stopping place of the electrons (location of the Bohr's orbits). The wave packet of nucleus does not allow the electron to stop in the space between the nodes.

5- The unity of wave mechanics and Bohr's theory: As you observe in Fig. 2, the new atomic model is a unifying model that includes both the wave packet of wave mechanics and the Bohr atomic model inside.

Conclusion

In this article, in section 2, we showed the problems of Schrodinger wave mechanics and showed that the Schrodinger wave equation and some assumptions of De Broglie and Schrodinger should be modified. In section 7 of the article, we explained the achievements of these modifies and changes in detail. Davisson–Germer experiment showed that the wave nature is a part from the reality of subatomic entities (like electrons, protons), and here we showed that wave nature is a part from the reality of celestial entities (like sun) too. Neither of these two conclusions is strange. Rather, they are truths that we should become accustomed to.

In this paper, we had achieved a new model for the hydrogen atom. In addition to predicting the main lines of the hydrogen spectrum (that is, the Ballmer, Lyman series, etc.), the new atomic model also predicts many other lines of the spectrum.

We know very little about the truth or spirit of quantum physics. Richard Feynman and Brian Greene better explain this. In 1965, Richard Feynman wrote: ‘’ There was a time when the newspapers said that only twelve men understood the theory of relativity. I do not believe there ever was such a time. There might have been a time when only one man did because he was the only guy who caught on, before he wrote his paper. But after people read the paper a lot of people understood the theory of relativity in one way or other, certainly more than twelve. On the other hand I think I can safely say that nobody understands quantum mechanics’’(Greene 2000). Or Brian Greene writes in his book (Greene 2000): " . . . By 1928 or so, many of the mathematical formulas and rules of quantum mechanics had been put in place . . .. But in a real sense those who use quantum mechanics find themselves following rules and formulas laid down by the "founding fathers" of the theory calculational procedures that are straightforward to carry out **without really understanding why the procedures work or what they really mean**. **Unlike relativity, few if any people ever grasp quantum mechanics at a "soulful" level**. . . . . Does it mean that on a microscopic level the universe operates in ways so obscure and unfamiliar that the human mind, evolved over eons to cope with phenomena on familiar everyday scales, is unable to fully grasp "what really goes on"? Or, might it be that through historical accident physicists have constructed an extremely awkward formulation of quantum mechanics that, although quantitatively successful, obfuscates the true nature of reality? No one knows. Maybe some time in the future some person will see clear to a new formulation that will fully reveal the "whys" and the "whats" of quantum mechanics" (Greene 2000). Based on these sentences, I think this article has been able to reveal the spirit and truth of Quantum mechanics to a great extent, and it seems that a big step has been taken.

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