$Long\ Proof$
$Taha's\ \ Golden\ Proof$

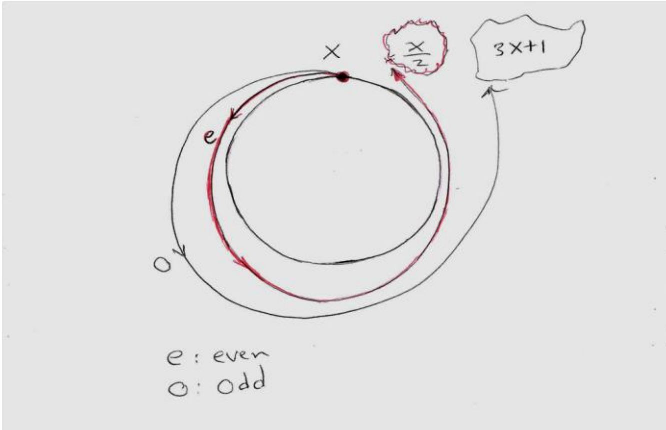
$Collatz\ Sequence\ by\ Taha\ M.Muhammad/USA\ Kurd\ 2022$

$Collatz\ Sequence's\ Rules$: $(Even\ number\ \div 2), \& \ (Odd\ number\ \times 3+1)$
$I\ have\ to\ prove\ \forall n \in N_+\ , lS(n) = \{4,2,1\}.\ (lS(n)\ is\ a\ loop\ of\ n)$

$Proof$:
$Collatz\ Sequence's\ Rules$: $(Even\ number\ \div 2), \& \ (Odd\ number\ \times 3+1)$
$let\ lS(n) = loop\ of\ n\ \&\ \ S(n) = \left\{\frac{n}{2}\ or\ (3n+1), \ldots\right\}.$
$S(1) = \{4,2,1\}\ , S(2) = \{1,4,2\}\ , S(3) = \{10,5,16,8,4,2,1\}$
$\Rightarrow lS(1) = lS(2) = lS(3) = \{4,2,1\}.$
$I\ have\ to\ prove\ \forall n \in N_+\ , lS(n) = \{4,2,1\}$
$let\ \ \ x, y, z, t, k, r, \ldots \in N_+$

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264 Content not peer-reviewed by Cambridge University Press. **License:** All Rights Reserved

$A. let\ lS(n) = \{x\}, \&\ \forall n \in N_+, x \in S(n)$



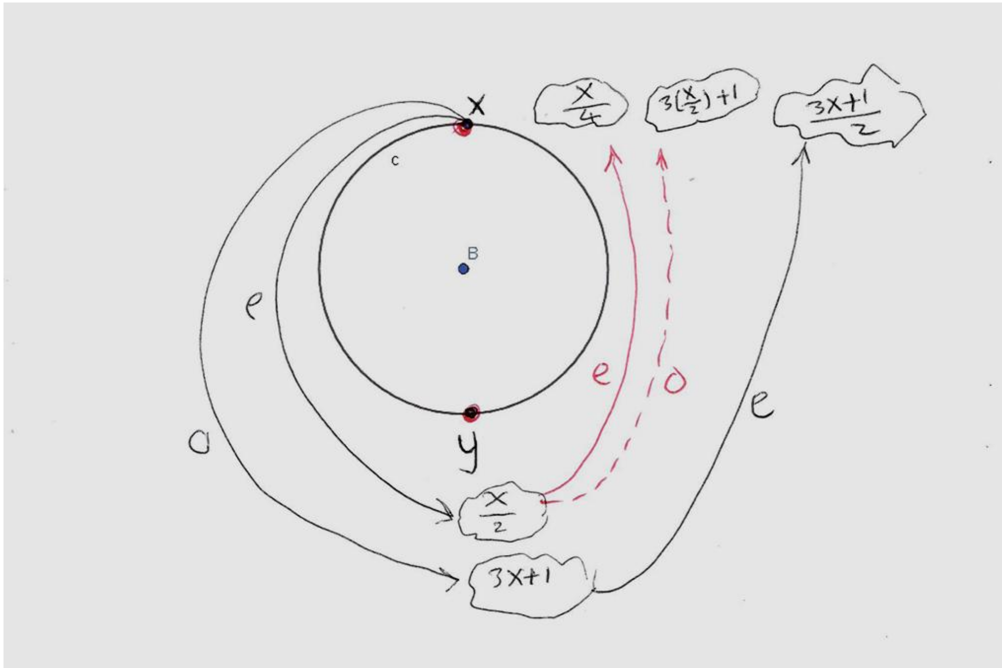
$Taha's\ lS(n) Collatz\ Sketch\ to\ find\ equivalent\ expresion\ (Cloud)\ to\ x$

$Cloud$	$Cloud = x$
$\frac{x}{2}$	$\frac{x}{2} = x \Rightarrow x = 0 \notin lS(\mathrm{n})$
$3x + 1$	$3x + 1 = x \Rightarrow x = -\frac{1}{2} \notin lS(\mathrm{n})$

$Taha's\ Chart\ to\ Find\ x\ number\ values$

$\therefore lS(\mathrm{n}) \neq \{x\}, \forall n \in N_+$

$B. let\ lS(n) = \{x, y\}, \&\ \forall n \in N_+, x, y \in S(n)$



$Taha's\ lS(n) Collatz\ Sketch\ to\ find\ equivalent\ expresion\ (Cloud)\ to\ x$

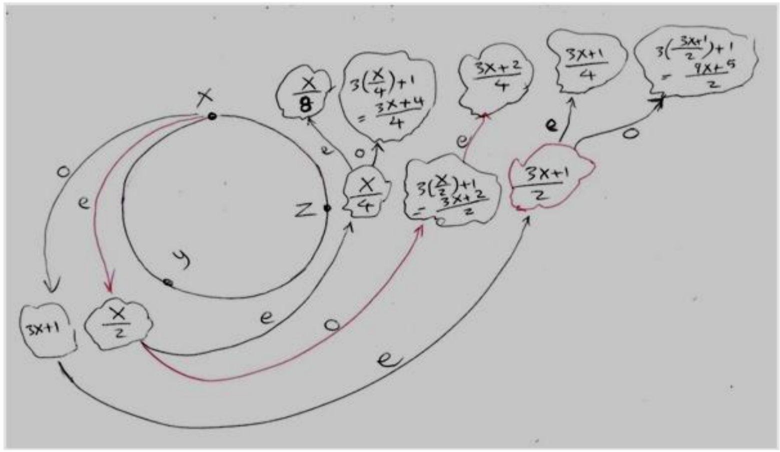
$Cloud$	$Cloud = x$	y
$\frac{x}{4}$	$\frac{x}{4} = x \Rightarrow x = 0 \notin lS(n)$	….
$\frac{3x+2}{2}$	$\frac{3x+2}{2} = x \Rightarrow 3x + 2 = 2x \Rightarrow x = -2 \notin lS(n)$	….
$\frac{3x+1}{2}$	$\frac{3x+1}{2} = x \Rightarrow x = -1 \notin lS(n)$	….

$Taha's\ Chart\ to\ Find\ x, y\ number\ values$

$\therefore lS(n) \neq \{x, y\}, \forall n \in N_+$

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264 Content not peer-reviewed by Cambridge University Press. **License:** All Rights Reserved

$C. \, let \; lS(n) = \{x, y, z\}, \; \forall n \in N_+, x, y, z \in lS(n)$



$Taha's \; lS(n) Collatz \; Sketch \; to \; find \; equivalent \; expresion \; (Cloud) \; to \; x$

$Cloud$	$Solve \; equation: Cloud = x$	y	z
$\frac{x}{8}$	$\frac{x}{8} = x \Rightarrow x = 0 \notin lS(n)$	---	---
$\frac{3x+4}{4}$	$\frac{3x+4}{4} = x \Rightarrow x = 4 \in lS(n)$	$\frac{x}{2} = 2$	$\frac{x}{4} = 1$
$\frac{3x+2}{4}$	$\frac{3x+2}{4} = x \Rightarrow x = 2 \in lS(n)$	$\frac{x}{2} = 1$	$\frac{3x+2}{2} = 4$
$\frac{3x+1}{4}$	$\frac{3x+1}{4} = x \Rightarrow x = 1 \in lS(n)$	$3x + 1$ =3(1)+1=4	$\frac{3x+1}{2} = 2$
$\frac{9x+5}{2}$	$\frac{9x+5}{2} = x \Rightarrow x = -\frac{5}{7} \notin lS(n)$	---	---

To Find Equivalents Expressions (Clouds) to 1st Element (x) of the Collatz Loop

$S(n) = \{x, y, z, t, g, u, v, \dots, ?\}, \forall n \in N_+$
$lS(n) \neq \{x\}, \forall n \in N_+, \; lS(n) \neq \{x, y\}, \forall n \in N_+$
$but \; lS(n) = \{x, y, z\} = \{4,2,1\}, \forall n \in N_+$
$\therefore lS(n) \neq set \; of \, (0,1,2,4,5,6,7, \dots, \infty) elements, except \; 3, \forall n \in N_+$
$Because \; lS(10) = \{4,2,1\} \neq \{a, b, c\}, a \neq 4 \neq b \neq 2 \neq c \neq 1, \&$
$lS(10) \neq set \; of \, (0,1,2,4,5,6,7, \dots, \infty) elements$
$\therefore S(n) = \left\{\frac{n}{2} \, or \; (3n+1), \dots, 4,2,1\right\}, \forall n \in N_+$

Extra Work for Support to my solution of Collatz Sequence

This instead of the Taha's Sketch & Chart

$\mathbf{1}\boldsymbol{st}$: $\boldsymbol{find\ y\ from\ x}$

$i)\ x(even) \rightarrow\ y = \frac{x}{2}(even\ or\ odd),\ or$

$x(odd) \rightarrow\ y = 3x + 1(even)$

$\mathbf{2}\boldsymbol{nd}$: $\boldsymbol{find\ z\ from\ y}$

$i)\ y = \frac{x}{2} = even \rightarrow z = \frac{x}{4}\ (even\ or\ odd),$

$or\ y = \frac{x}{2} = odd \rightarrow z = 3\left(\frac{x}{2}\right) + 1 = \frac{3x+2}{2}(even)$

$ii)\ y = 3x + 1 = even \rightarrow z = \frac{3x+1}{2}(even\ or\ odd)$

$\mathbf{3}\boldsymbol{rd}$: $\boldsymbol{find\ x\ from\ z}$

$i) z = \frac{x}{4} = even \rightarrow x = \frac{x}{8} \Rightarrow x = 0 \notin N_+$

$or\ z = \frac{x}{4} = odd \rightarrow x = 3\left(\frac{x}{4}\right) + 1 = \frac{3x+4}{4} \Rightarrow x = 4 \in N_+ \Rightarrow y = 2, \&\ z = 1$

$ii)\ z = \frac{3x+2}{2} = even \rightarrow x = \frac{3x+2}{4} \Rightarrow x = 2 \in N_+\ \Rightarrow y = 1, \&\ z = \frac{3x+2}{2} = 4$

$iii)\ z = \frac{3x+1}{2} = even \rightarrow x = \frac{3x+1}{4} \Rightarrow x = 1\ \in N_+ \Rightarrow y = 4, \&\ z = \frac{3x+1}{2} = 2$

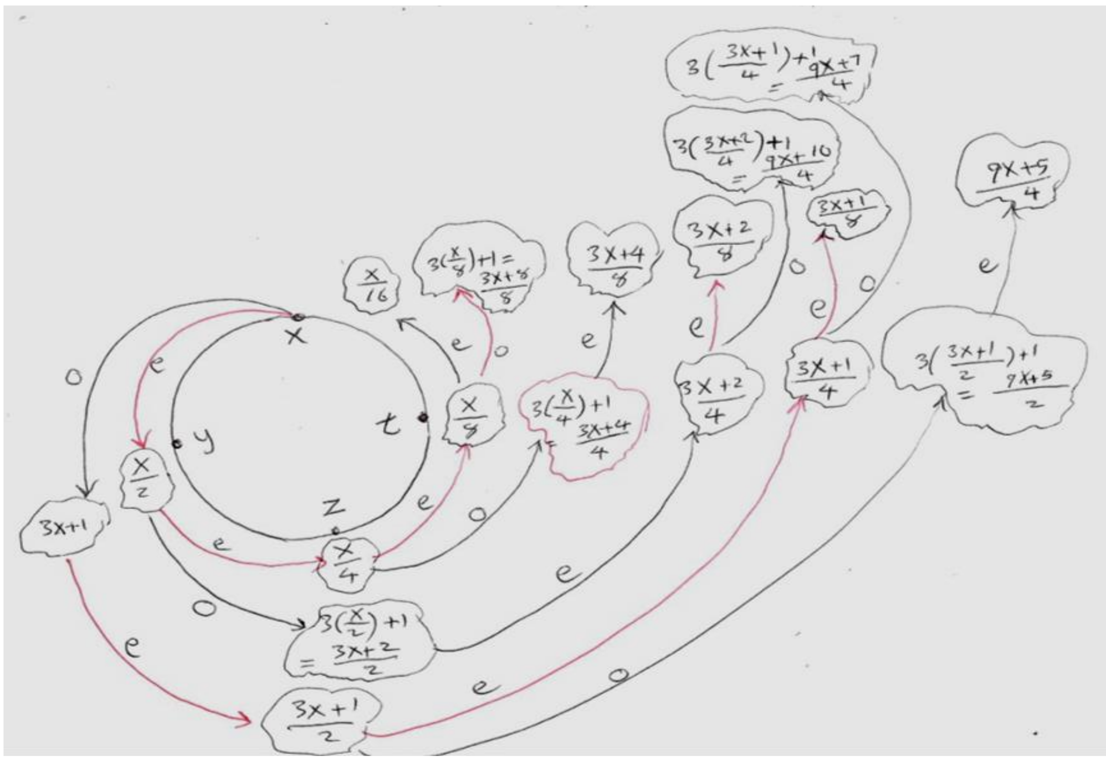
$or\ z = \frac{3x+1}{2} = odd\ \rightarrow x = 3\left(\frac{3x+1}{2}\right) + 1 = \frac{9x+5}{2} \Rightarrow x = -\frac{5}{7} \notin N_+$

$\therefore S(n)\ = \left\{\frac{n}{2}\, or\ (3n+1), \dots, 4,2,1\right\} \Rightarrow lS(n) = \{4,2,1\}, \forall n \in N_+$

$\therefore S(n)\ has\ only\ one\ loop\ with\ 3\ elements\ 4,2,1.$

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264 Content not peer-reviewed by Cambridge University Press. **License:** All Rights Reserved

$D. let\ lS(n) = \{x, y, z, t\}, \&\ \forall n \in N_+, x, y, z, t \in S(n)$



$Taha's\ lS(n) Collatz\ Sketch\ to\ find\ equivalent\ expresion\ (Cloud)\ to\ x$

$\because x = \frac{x}{16}, x = \frac{3x+8}{8}, x = \frac{3x+4}{8}, x = \frac{3x+2}{8}, x = \frac{3x+1}{8}, x = \frac{9x+10}{4}, x = \frac{9x+7}{4}, \&\ x = \frac{9x+5}{2}$
$\Rightarrow x \notin N_+ \Rightarrow lS(n) \neq \{x, y, z, t\}, \forall n \in N_+$
$Then\ I\ have\ to\ find\ a\ pattren\ and\ formula\ to\ get\ elements\ of\ any\ other\ lS(n)$

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264 Content not peer-reviewed by Cambridge University Press. **License:** All Rights Reserved

Taha's Pattern of Collatz Sequence's Loop

$r \in N_+, r$ *is number of elements in a loop*

$x \in N_+, \&\ x \in lS(n)$

$\#lS(n)$ $r = 0$	x =e or odd $\rightarrow$

$r = 1$	$\rightarrow$ $\frac{x}{2}$ =e or odd $\rightarrow$	$\rightarrow$ $3x + 1$ =e $\rightarrow$

$r = 2$	$\rightarrow$ $\frac{x}{4}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{3x + 2}{2}$ =e $\rightarrow$	$\rightarrow$ $\frac{3x + 1}{2}$ =e or odd $\rightarrow$

$r = 3$	$\rightarrow$ $\frac{x}{8}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{3x + 4}{4}$ =e $\rightarrow$	$\rightarrow$ $\frac{3x + 2}{4}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{3x + 1}{4}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{9x + 5}{2}$ =e $\rightarrow$

$r = 4$	$\rightarrow$ $\frac{x}{16}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{3x + 8}{8}$ =e $\rightarrow$	$\rightarrow$ $\frac{3x + 4}{8}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{3x + 2}{8}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{3x + 1}{8}$ =e or odd $\rightarrow$	$\rightarrow$ $\frac{9x + 10}{4}$ =e $\rightarrow$
$\rightarrow$ $\frac{9x + 7}{4}$ =e $\rightarrow$	$\rightarrow$ $\frac{9x + 5}{4}$ =e or odd $\rightarrow$					

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264 Content not peer-reviewed by Cambridge University Press.

$r=5$	→ $\frac{x}{32}$ =e or odd *	→ $\frac{3x+16}{16}$ =e or odd $	→ $\frac{3x+8}{16}$ =e or odd !	→ $\frac{3x+4}{16}$ =e or odd @	→ $\frac{3x+2}{16}$ =e or odd #	→ $\frac{3x+1}{16}$ =e or odd %
→ $\frac{9x+20}{8}$ =e @@	→ $\frac{9x+14}{8}$ =e &	→ $\frac{9x+11}{8}$ =e ((	→ $\frac{9x+10}{8}$ =e or odd {{	→ $\frac{9x+7}{8}$ =e or odd [[	→ $\frac{9x+5}{8}$ =e or odd **	→ $\frac{27x+19}{4}$ =e ***

$r=6$	* $\frac{x}{64}$	* $\frac{3x+32}{32}$	$ $\frac{3x+16}{32}$	! $\frac{3x+8}{32}$	@ $\frac{3x+4}{32}$	# $\frac{3x+2}{32}$	% $\frac{9x+19}{16}$
% $\frac{3x+1}{32}$	$ $\frac{9x+64}{16}$	{{ $\frac{27x+38}{8}$	@ $\frac{9x+28}{16}$	# $\frac{9x+22}{16}$	% $\frac{3x+1}{32}$	$ $\frac{9x+64}{16}$	! $\frac{9x+40}{16}$
@@ $\frac{9x+20}{16}$		& $\frac{9x+14}{16}$	(($\frac{9x+11}{16}$	{{ $\frac{9x+10}{16}$	[[$\frac{9x+7}{16}$	[[$\frac{27x+29}{16}$	** $\frac{9x+5}{16}$
** $\frac{27x+23}{8}$		*** $\frac{27x+19}{8}$					

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264 Content not peer-reviewed by Cambridge University Press. **License:** All Rights Reserved

Taha's Golden Table for Collatz Sequence Pattern

$r = 1 = 3(0) + 1$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{1-0}} \Rightarrow x = 1, but\ y = 4 \notin lS(n) \Rightarrow lS(n) \neq \{x\},\ \forall n \in N_+$
$r = 2 = 3(0) + 2$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{2-0}} \Rightarrow x = \frac{1}{3} \notin N_+ \Rightarrow x, y \notin lS(n) \Rightarrow lS(n) \neq \{x, y\},\ \forall n \in N_+$
$r = 3 = 3(1)$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{3-1}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{4,2,1\},\ \forall n \in N_+$
$r = 4 = 3(1) + 1$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{4-1}} \Rightarrow\ x \notin N_+, \forall n \in N_+$
$r = 5 = 3(1) + 2$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{5-1}} \Rightarrow\ x \notin N_+, \forall n \in N_+$
$r = 6 = 3(2)$ $k = 2$	$x = \frac{3^2 x + 3(2^0) + 2^2}{2^{6-2}} \Rightarrow x = 1, y = 4, z = 2. lS(n) = \{x, y, z\},\ \forall n \in N_+$
$r = 9 = 3(3)$ $k = 3$	$x = \frac{3^3 x + 3^2(2^0) + 3^1(2^2) + 2^4}{2^{9-3}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{1,4,2\},$ $\forall n \in N_+$
$r = 12 = 3(4)$ $k = 4$	$x = \frac{3^4 x + 3^3(2^0) + 3^2(2^2) + 3(2^4) + 2^6}{2^{12-4}} \Rightarrow x = 1, y = 4, z = 2, \forall n$ $\in N_+$
$r = 15 = 3(5)$ $k = 5$	$x = \frac{3^5 x + 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8}{2^{15-5}} \Rightarrow x = 1, y = 4,$ Z=2 $\forall n \in N_+$

$r = 3(k)$,

$$let\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow x = 1$$

$Note\!:\ 3^{k-1}(\ 2^0), 2^2, 2^4, \ldots, 2^{2(k-2)}, 2^{2(k-1)}, i, e\ 2^m, and\ m \in N = \{0,2,4,6,8, \ldots\}$

$if\ r + 3 = 3(k + 1)$,

$$is\ x = \frac{3^{k+1} x + 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}}{2^{r+3-(k+1)}} \Rightarrow x = 1\ \ ?$$

$Proof\!: \left(2^{r+3-(k+1)} - 3^{k+1}\right)x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}$

$\left(2^{3k+3-(k+1)} - 3^{k+1}\right)x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}$

$(2^{2k+2} - 3^{k+1})x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}$

$let\ k = 4\ \Rightarrow$

$(2^{10} - 3^5)x = 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8$

$(781)x = 781 \Rightarrow x = 1 \Rightarrow\ y = 4, z = 2$

$\therefore lS(n) = \{4,2,1\}$

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264 Content not peer-reviewed by Cambridge University Press. **License:** All Rights Reserved

$if\ \ r = 3(k) + h, h < 3, k, h \in N_+$

$$let\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow x \notin N_+$$

$if\ \ r + 1 = 3(k) + 2$

$$is\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r+1-k}} \Rightarrow x \in N_+?$$

$Proof{:}\ r + 1 = 3(k) + 2$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{3k+1-k}}$$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{2k+1}}$$

$$(2^{2k+1} - 3^k)x = 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}$$

$let\ k = 4$

$$(2^9 - 3^4)x = 3^3(2^0) + 3^2(2^2) + 3^1(2^4) + 2^6$$

$$431x = 175 \Rightarrow x \notin N_+$$

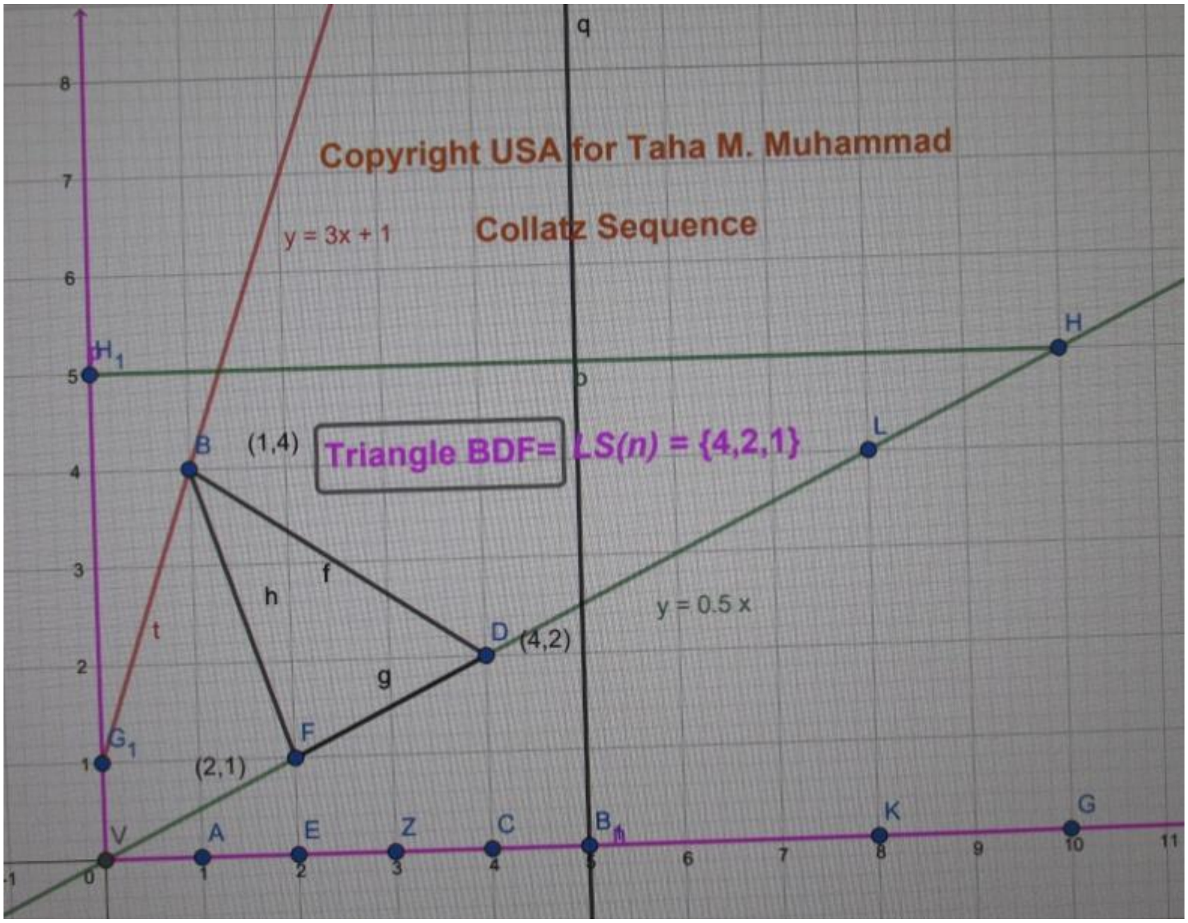
$\therefore if\ r = 3(k) \Rightarrow x \in N_+ \Rightarrow lS(x) = \{4,2,1\}, \forall n \in N_+ \Rightarrow There\ is\ one\ loop\ \forall n \in N_+$

$[If\ r = 3(k) + h \Rightarrow all\ x \notin N_+ ,\ h \in N_+, h < 3] \Rightarrow No\ loop\ for\ Collatz\ Sequence$

$\therefore lS(n) = \{4,2,1\}\ the\ only\ loop\ with\ three\ elements, \forall n \in N_+.$

https://doi.org/10.33774/coe-2024-c4f7p **ORCID:** https://orcid.org/0000-0002-8954-9264

Taha's Graph to Find Collatz Sequence's & Loop's Elements



Collatz Sequence Graph

https://doi.org/10.33774/coe-2024-c4f7p ORCID: https://orcid.org/0000-0002-8954-9264