

Cubic Root by Hand for Real Numbers

I started from decimal point and grouping each 3 digits left and right direction.

$$\therefore (a + b)^3 - a^3 = 3a^2b + 3ab^2 + b^3$$

Find value of $3a^2 \rightarrow 3(a_1(10))^2, 3(a_1(1))^2$, or $3(a_1(0.1))^2, \dots \rightarrow \rightarrow \downarrow$

Find value of $3a^2b + 3ab^2 + b^3 \rightarrow \downarrow$

$\downarrow a_1$

Example:

$$\sqrt[3]{76590.8876724}$$

$$\sqrt[3]{076\ 590.\ 887\ 672\ 400} = d(10) _ . _ _ _ \dots$$

Answer:

b_2

$$\begin{aligned} & b_1 \\ & = 4(10) _ \\ & = 4(10) + \underline{2} _ \text{ Row1} \\ & = \underline{42} _ \\ & (4(10) + 2)^3 \\ & (42 + \underline{0.4})^3 \text{ Row2} \end{aligned}$$

$$\sqrt[3]{076\ 590.\ 887\ 672\ 400} = 42.4$$

$a_1 = 40, b_1 = 2$ $3a_1^2 = 3(40)^2 = \underline{4800} \rightarrow \rightarrow \downarrow$ $3a_1a_1^2b_1 + 3a_1b_1^2 + 3b_1^3 =$ $3(40^2)(2) + 3(2^2) + 2^3 =$ $\underline{9620} \rightarrow \downarrow$	$076\ 590.\ \underline{887\ 672\ 400}$ $- 064$	$\sqrt[3]{076} = 4 \uparrow$
$3a_2^2 = 3(42)^2 = \underline{5292} \rightarrow \rightarrow \downarrow$ $3a_2^2b_2 + 3a_2b_2^2 + b_2^3 =$ $3(42)^2(0.4) + 3(42)(0.4)^2 + (0.4)^3 =$ $\underline{2137.024} \rightarrow \downarrow$	012590 $- 00\underline{9620}$	$012590 \div \underline{4\ 800} = 3, \underline{2} \uparrow$
	$2970.\ \underline{887}$ $- \underline{2137.024}$	$2970.\ \underline{887} \div \underline{5292} = 0.5,$ $\underline{0.4} \uparrow$
	$833.863\underline{672}$	$833.863\underline{672} \div$

Stop or continue repeat the same steps.

Cubic Root by Hand for Real Numbers

Example 1: $\sqrt[3]{12.81} = ?$

$$(a + b)^3 - a^3 = 3a^2b + 3ab^2 + b^3$$

Answer = 2.339

$(a + b)^3 = (2 + 0.3)^3$ i) $3(a)^2 = 3(2)^2 = 12 \rightarrow \rightarrow \downarrow$ ii) $3(a)^2(b) + 3(a)(b)^2 + (b)^3 =$ $3(2)^2(0.3) + 3(2)(0.3)^2 +$ $(0.3)^3 = 4.167 \rightarrow \downarrow$	Start 012.810 000 000 (-) 8	$\sqrt[3]{8} = 2 \uparrow$
$(a + b)^3 = (2.3 + 0.03)^3$ i) $3(a)^2 = 3(2.3)^2 = 15.87 \rightarrow \rightarrow \downarrow$ ii) $3(a)^2(b) + 3(a)(b)^2 + (b)^3 =$ $3(2.3)^2(0.03) + 3(2.3)(0.03)^2 +$ $(0.03)^3 = 0.482337 \rightarrow \downarrow$	4.810 (-) 4.167	$4.810 \div 12 = 0.4$, or 0.3 \uparrow
$(a + b)^3 = (2.33 + 0.009)^3$ i) $3(a)^2 = 3(2.33)^2 = 16.2867 \rightarrow \rightarrow \downarrow$ ii) $3(a)^2(b) + 3(a)(b)^2 + (b)^3 =$ $3(2.33)^2(0.009) +$ $3(2.33)(0.009)^2 +$ $(0.009)^3 = 0.147147219 \rightarrow \downarrow$	0.643000 (-) 0.482337	$0.643 \div 15.87 = 0.04$, or 0.03 \uparrow
	0.160663000 (-) 0.147147219	$0.160663 \div 16.2867 = 0.009 \uparrow$
	0.013515781=R	

$$3a^2b + 3ab^2 + b^3 = (a + b)^3 - a^3$$

Example 2: $\sqrt[3]{5832} = ?$

Answer= **18**

$i) 3a^2 = 3(10)^2 = 300 \rightarrow \rightarrow \downarrow$ $(a + b)^3 = (10 + 8)^3$ $ii) 3a^2b + 3ab^2 + b^3 =$ $3(10)^2(8) + 3(10)(8)^2 + (8)^3 = 4832 \rightarrow \downarrow$	$\overline{005\ 832.000\ 000}$ $(-) \quad 1 \quad \rightarrow$	$\sqrt[3]{1} = 1 \uparrow$
	$\underline{4\ 832}$ $(-) \ 4832$	$4\ 832 \div 300$ $\cong 16, 9, 8 \uparrow$
Answer = 18	000000= R	

Example 3: $\sqrt[3]{12\ 812\ 904} = ? = \text{Answer} = \mathbf{234}$

i) $3a^2$

ii) $3a^2b + 3ab^2 + b^3 = (a + b)^3 - a^3$

$(a + b)^3 = (2(10) + 3)^3$ $i) 3(a)^2 = 3(2(10))^2 = 3(20)^2 =$ $1200 \rightarrow \rightarrow \downarrow$ $ii) 3(20)^2(3) + 3(20)(3)^2 + (3)^3 =$ $4167 \rightarrow \downarrow$	$\overline{3\ 012\ 812\ 904.000\ 000}$ $(-) \quad 8 \rightarrow$	$\sqrt[3]{8} = 2 \uparrow$
$(a + b)^3 = (23(10) + 4)^3 =$ $(230 + 4)^3$ $i) 3(a)^2 = 3(230)^2 = 158700 \rightarrow \rightarrow \downarrow$ $ii) 3(230)^2(4) + 3(230)(4)^2 +$ $(4)^3 = 645904 \rightarrow \downarrow$	$\underline{4\ 812}$ $(-) \ 4167$	$4812 \div 1200 \cong 4, \text{ or}$ $b = 3 \uparrow$
	$\mathbf{645904}$ $(-) \ \mathbf{645904}$	$\mathbf{645904} \div \mathbf{158700} \cong$ $b = 4 \uparrow$
	000000= R	

Ex 4: $\sqrt[3]{625} = 8.54$

$i) 3a^2 = 3(8)^2 = 192 \rightarrow \rightarrow \downarrow$ $ii) 3a^2b + 3ab^2 + b^3 =$ $3(8)^2(0.5) + 3(8)(0.5)^2 + (0.5)^3 =$ $102.125 \rightarrow \downarrow$	625.000 $8^3 = 512$	$\sqrt[3]{512} = 8 \uparrow$
$i) 3a^2 = 3(8.5)^2 = 216.75 \rightarrow \rightarrow \downarrow$ $ii) 3a^2b + 3ab^2 + b^3 =$ $3(8.5)^2(0.04) +$ $3(8.5)(0.04)^2 + (0.04)^3 =$ $102.125 \rightarrow \downarrow$	113.000 102.125	$113 \div 192 = 0.58854166666$ $= 0.5 \uparrow$
	10.875000 8.710864	$10.875 \div 216.75 = 0.04 \uparrow$

Example 5: $\sqrt[3]{1288637.538} = 108.820385329$

$3(10)^2 (0) + 3(10) (0)^2 + (0)^3 = 0$ $3(10)^2 = 300$	$\sqrt[3]{001288637.538000000}$ (-) 1	$\sqrt[3]{1} = 1$
$3(100)^2 (8) + 3(100) (8)^2 + (8)^3 = 259712$ $3(100)^2 = 30000$	0288 (-) 0	$288 \div 300 \cong 0.96 \cong 1$, or 0
$3(108.0)^2 (0.8) + 3(108.0) (0.8)^2 + (0.8)^3 = 28201.472$ $3(108.0)^2 = 3499.2$	288637 (-) 259712	$288637 \div 30000 \cong 9.621$, or 8
$3(108.8)^2 (0.02) + 3(108.8) (0.02)^2 + (0.02)^3 = 710.376968$ $3(108.8)^2 = 35512.32$	028925.538 (-) 28201.472	$28925.538 \div 3499.2 \cong 8.266 \cong 8$, 7,..., 1, 0.9, 0.8
$3(108.80)^2 (0.000) + 3(108.80) (0.000)^2 + (0.000)^3 = 0$ $3(108.8)^2 = 35512.32$	724.066000 (-) 710.376968	$724.066 \div 35512.32 \cong 0.02038914945$
$3(108.82)^2 (0.0003) + 3(108.82) (0.0003)^2 + (0.0003)^3 = 10.6576425414$ $3(108.82)^2 = 35525.3772$	13.689032 (-) 0.	$13.689032 \div 35512.32 \cong 0.00038547275$
$3(108.8203)^2 (0.00008) + 3(108.8203) (0.00008)^2 + (0.00008)^3 = 2.84204793545$ $3(108.8203)^2 = 35525.5730763$	13.689032000 10.6576425414 (-)	$13.689032 \div 35525.3772 \cong 0.00038533108$
$3(108.82038)^2 (0.000005) + 3(108.82038) (0.000005)^2 + (0.000005)^3 = 0.17762813471$ $3(108.82038)^2 = 35525.3772$	3.0313894586 (-) 2.84204793545	$3.0313894586 \div 35525.5730763 \cong 0.00008533025$
$3(108.820385)^2 (0.0000003) + 3(108.820385) (0.0000003)^2 + (0.0000003)^3 = 0.0106576886$ $3(108.820385)^2 = 35525.6285746$	0.18934152315 (-) 0.17762813471	$0.18934152315 \div 35525.3772 \cong 0.00000532975$
$3(108.8203853)^2 (0.00000002) + 3(108.8203853) (0.00000002)^2 + (0.00000002)^3 = 0.00071051257$ $3(108.8203853)^2 = 35525.6287705$	0.01171338844 (-) 0.0106576886	$0.01171338844 \div 35525.6285746 \cong 0.000000329716571$
$3(108.82038532)^2 (0.000000009) + 3(108.8203853) (0.000000009)^2 + (0.000000009)^3 = 0.00031973065$ $3(108.82038532)^2 = 35525.6287836$	0.00105569984 (-) 0.00071051257	$0.00105569984 \div 35525.6287705 \cong 0.0000000297165702$
108.820385329	0.00034518727 (-) 0.00031973065	$0.00034518727 \div 35525.6287836 \cong 0.00000000971657031$