

Collatz Sequence

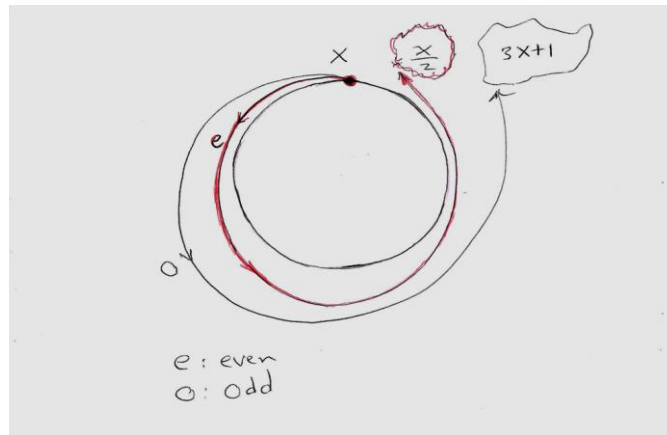
let $x, y, z, t, k, r, \dots \in N_+$

Collatz Sequence: let $LS(n) = \text{loop of } n$, & $S(n) = \left\{ \frac{n}{2} \text{ or } (3n + 1), \dots, ? \right\}$

I have to prove $\forall n \in N_+, LS(n) = \{4, 2, 1\}$

Pattern of Collatz Sequence's Loop, $r \in N_+$, r is number of elements in a loop, $x \in N_+$, $x \in S(n)$, & $x \in LS(n)$

A. let $LS(n) = \{x\}$, & $\forall n \in N_+, x \in S(n)$



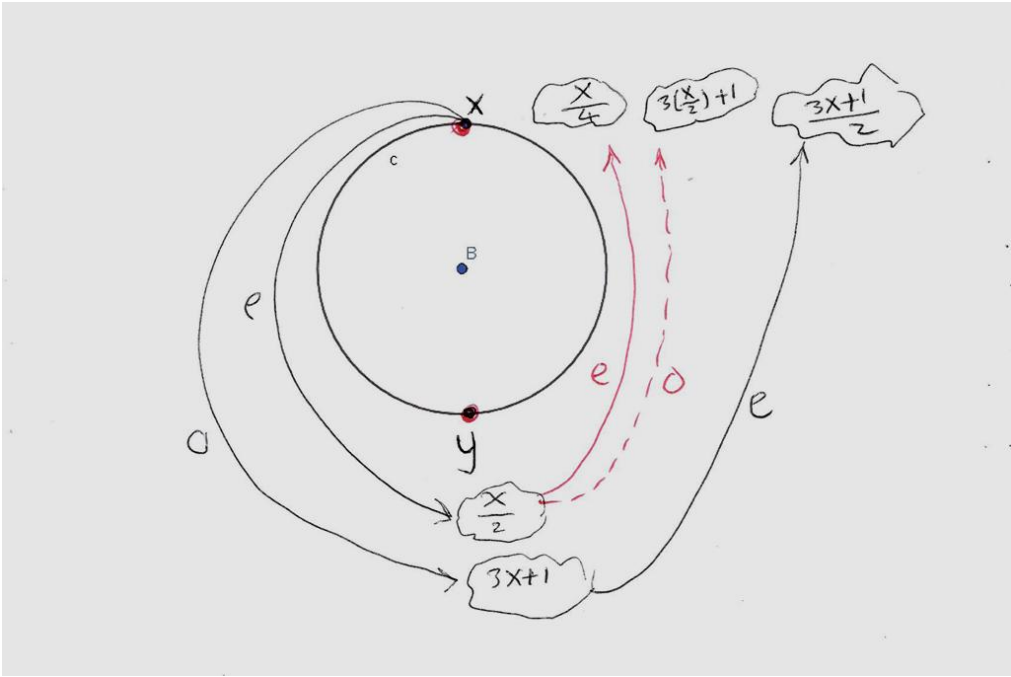
$LS(n)$ Collatz Sketch to find equivalent expresion (Cloud) to x

Cloud	Cloud = x
$\frac{x}{2}$	$\frac{x}{2} = x \Rightarrow x = 0 \notin LS(n)$
$3x + 1$	$3x + 1 = x \Rightarrow x = -\frac{1}{2} \notin LS(n)$

Chart to Find x number values

$\therefore LS(n) \neq \{x\}, \forall n \in N_+$

B. let $lS(n) = \{x, y\}, \& \forall n \in N_+, x, y \in S(n)$



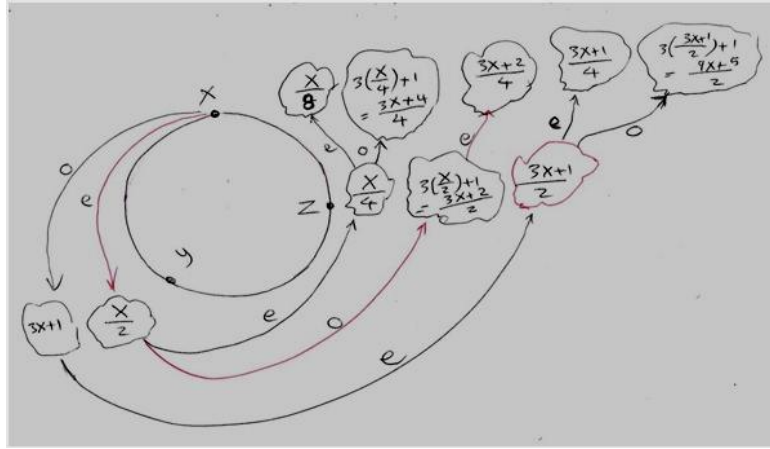
lS(n)Collatz Sketch to find equivalent expresion (Cloud) to x

Cloud	Cloud = x	y
$\frac{x}{4}$	$\frac{x}{4} = x \Rightarrow x = 0 \notin lS(n)$	
$\frac{3x + 2}{2}$	$\frac{3x+2}{2} = x \Rightarrow 3x + 2 = 2x \Rightarrow x = -2 \notin lS(n)$	
$\frac{3x + 1}{2}$	$\frac{3x+1}{2} = x \Rightarrow x = -1 \notin lS(n)$	

Chart to Find x,y number values

$\therefore lS(n) \neq \{x, y\}, \forall n \in N_+$

C. let $LS(n) = \{x, y, z\}, \forall n \in N_+, x, y, z \in LS(n)$



$LS(n)$ Collatz Sketch to find equivalent expression (Cloud) to x

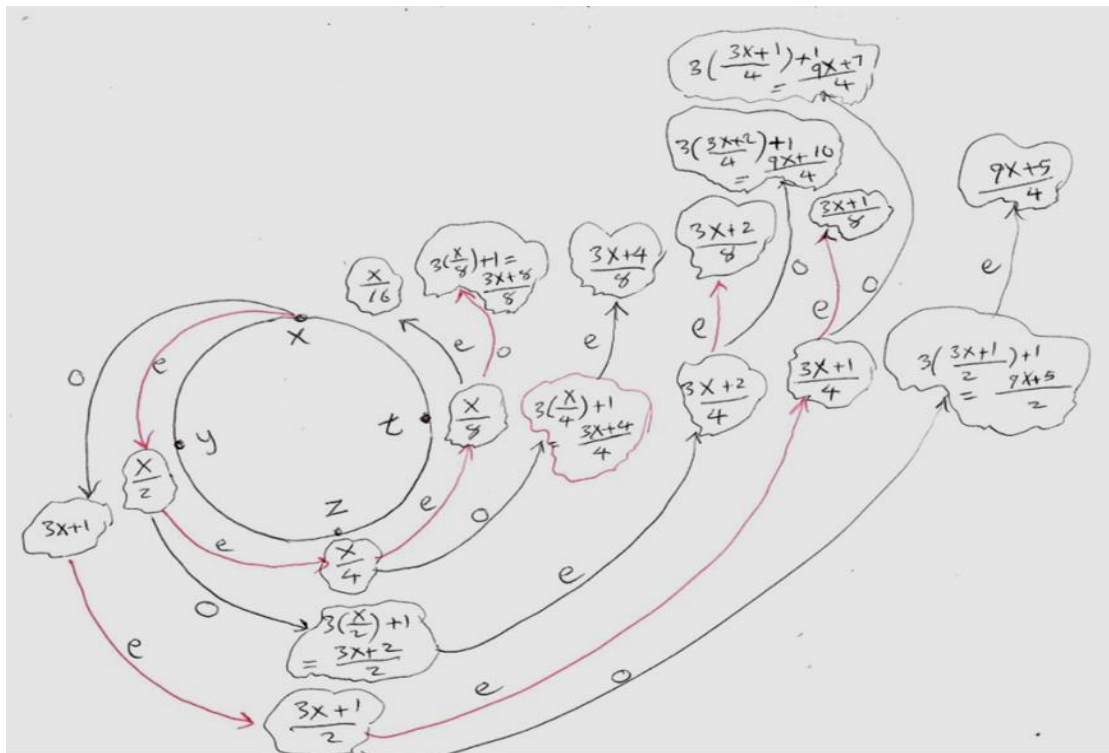
Cloud	Solve equation: Cloud = x	y	z
$\frac{x}{8}$	$\frac{x}{8} = x \Rightarrow x = 0 \notin LS(n)$	---	---
$\frac{3x+4}{4}$	$\frac{3x+4}{4} = x \Rightarrow x = 4 \in LS(n)$	$\frac{x}{2} = 2$	$\frac{x}{4} = 1$
$\frac{3x+2}{4}$	$\frac{3x+2}{4} = x \Rightarrow x = 2 \in LS(n)$	$\frac{x}{2} = 1$	$\frac{3x+2}{2} = 4$
$\frac{3x+1}{4}$	$\frac{3x+1}{4} = x \Rightarrow x = 1 \in LS(n)$	$3x+1$ $= 3(1)+1=4$	$\frac{3x+1}{2} = 2$
$\frac{9x+5}{2}$	$\frac{9x+5}{2} = x \Rightarrow x = -\frac{5}{7} \notin LS(n)$	---	---

To Find Equivalents Expressions (Clouds) to 1st Element (x) of the Collatz Loop

$$\therefore LS(n) = \{x, y, z\} = \{4, 2, 1\}, \forall n \in N_+$$

$$\therefore S(n) = \left\{ \frac{n}{2} \text{ or } (3n+1), \dots, 4, 2, 1 \right\}, \forall n \in N_+$$

D. let $LS(n) = \{x, y, z, t\}$, & $\forall n \in N_+, x, y, z, t \in S(n)$



$LS(n)$ Collatz Sketch to find equivalent expression (Cloud) to x

$$\because x = \frac{x}{16}, x = \frac{3x+8}{8}, x = \frac{3x+4}{8}, x = \frac{3x+2}{8}, x = \frac{3x+1}{8}, x = \frac{9x+10}{4}, x = \frac{9x+7}{4}, \& x = \frac{9x+5}{2}$$

$$\Rightarrow x \notin N_+ \Rightarrow LS(n) \neq \{x, y, z, t\}, \forall n \in N_+$$

Then I have to find a pattern and formula To get elements of $LS(n)$

Pattern of Collatz Sequence's Loop, $r \in N_+$, r is number of elements in a loop, $x \in N_+$, $x \in S(n)$, & $x \in IS(n)$

$r = \#IS(n)$ $x = e \text{ or odd}$ 
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$r = 1$	 $\frac{x}{2}$ $= e \text{ or odd}$ 	 $3x + 1$ $= e$ 
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$r = 2$	 $\frac{x}{4}$ $= e \text{ or odd}$ 	 $\frac{3x + 2}{2}$ $= e$ 	 $\frac{3x + 1}{2}$ $= e \text{ or odd}$ 
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$r = 3$	 $\frac{x}{8}$ $= e \text{ or odd}$ 	 $\frac{3x + 4}{4}$ $= e$ 	 $\frac{3x + 2}{4}$ $= e \text{ or odd}$ 	 $\frac{3x + 1}{4}$ $= e \text{ or odd}$ 	 $\frac{9x + 5}{2}$ $= e$ 
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$r = 4$	 $\frac{x}{16}$ $= e \text{ or odd}$ 	 $\frac{3x + 8}{8}$ $= e$ 	 $\frac{3x + 4}{8}$ $= e \text{ or odd}$ 	 $\frac{3x + 2}{8}$ $= e \text{ or odd}$ 	 $\frac{3x + 1}{8}$ $= e \text{ or odd}$ 	 $\frac{9x + 10}{4}$ $= e$ 
 $\frac{9x + 7}{4}$ $= e$ 	 $\frac{9x + 5}{4}$ $= e \text{ or odd}$ 					

$r = 5$	$\frac{x}{32}$ =e or odd *	$\frac{3x + 16}{16}$ =e or odd \$	$\frac{3x + 8}{16}$ =e or odd !	$\frac{3x + 4}{16}$ =e or odd @	$\frac{3x + 2}{16}$ =e or odd #	$\frac{3x + 1}{16}$ =e or odd %
$\frac{9x + 20}{8}$ =e @@	$\frac{9x + 14}{8}$ =e &	$\frac{9x + 11}{8}$ =e ((	$\frac{9x + 10}{8}$ =e or odd {{	$\frac{9x + 7}{8}$ =e or odd [[	$\frac{9x + 5}{8}$ =e or odd **	$\frac{27x + 19}{4}$ =e ***

$r = 6$	$\frac{x}{64}$	$\frac{3x + 32}{32}$	$\frac{3x + 16}{32}$	$\frac{3x + 8}{32}$	$\frac{3x + 4}{32}$	$\frac{3x + 2}{32}$	$\frac{9x + 19}{16}$
$\frac{3x + 1}{32}$	$\frac{9x + 64}{16}$	$\frac{27x + 38}{8}$	$\frac{9x + 28}{16}$	$\frac{9x + 22}{16}$	$\frac{3x + 1}{32}$	$\frac{9x + 64}{16}$	$\frac{9x + 40}{16}$
$\frac{@@}{9x + 20}$ 16	$\frac{9x + 14}{16}$	$\frac{9x + 11}{16}$	$\frac{9x + 10}{16}$	$\frac{9x + 7}{16}$	$\frac{27x + 29}{16}$	$\frac{9x + 5}{16}$	
$\frac{27x + 23}{8}$	$\frac{27x + 19}{8}$						

Table for Collatz Sequence Pattern

$r = 1 = 3(0) + 1$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{1-0}} \Rightarrow x = 1, \text{ but } y = 4 \notin lS(n) \Rightarrow lS(n) \neq \{x\}, \forall n \in N_+$
$r = 2 = 3(0) + 2$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{2-0}} \Rightarrow x = \frac{1}{3} \notin N_+ \Rightarrow x, y \notin lS(n) \Rightarrow lS(n) \neq \{x, y\}, \forall n \in N_+$
$r = 3 = 3(1)$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{3-1}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{4, 2, 1\}, \forall n \in N_+$
$r = 4 = 3(1) + 1$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{4-1}} \Rightarrow x \notin N_+, \forall n \in N_+$
$r = 5 = 3(1) + 2$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{5-1}} \Rightarrow x \notin N_+, \forall n \in N_+$
$r = 6 = 3(2)$ $k = 2$	$x = \frac{3^2 x + 3(2^0) + 2^2}{2^{6-2}} \Rightarrow x = 1, y = 4, z = 2. lS(n) = \{x, y, z\}, \forall n \in N_+$
$r = 9 = 3(3)$ $k = 3$	$x = \frac{3^3 x + 3^2(2^0) + 3^1(2^2) + 2^4}{2^{9-3}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{1, 4, 2\},$ $\forall n \in N_+$
$r = 12 = 3(4)$ $k = 4$	$x = \frac{3^4 x + 3^3(2^0) + 3^2(2^2) + 3(2^4) + 2^6}{2^{12-4}} \Rightarrow x = 1, y = 4, z = 2, \forall n$ $\in N_+$
$r = 15 = 3(5)$ $k = 5$	$x = \frac{3^5 x + 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8}{2^{15-5}} \Rightarrow x = 1, y = 4,$ $Z=2 \forall n \in N_+$
$r = 3(k),$ $\text{let } x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \dots + 3^{k-k+1}(2^{2(k-2)}) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow x = 1$ <i>Note:</i> $3^{k-1}(2^0), 2^2, 2^4, \dots, 2^{2(k-2)}, 2^{2(k-1)}, i, e 2^m, \text{ and } m \in N = \{0, 2, 4, 6, 8, \dots\}$	
$\text{if } r + 3 = 3(k + 1),$ $\text{is } x = \frac{3^{k+1} x + 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \dots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}}{2^{r+3-(k+1)}} \Rightarrow x = 1 ?$ <i>Proof:</i> $(2^{r+3-(k+1)} - 3^{k+1})x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \dots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}$ $(2^{3k+3-(k+1)} - 3^{k+1})x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \dots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}$ $(2^{2k+2} - 3^{k+1})x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \dots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}$ $\text{let } k = 4 \Rightarrow$ $(2^{10} - 3^5)x = 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8$ $(781)x = 781 \Rightarrow x = 1 \Rightarrow y = 4, z = 2$ $\therefore lS(n) = \{4, 2, 1\}$	

if $r = 3(k) + h, h < 3, k, h \in N_+$

$$\text{let } x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \dots + 3^{k-k+1}(2^{2(k-2)}) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow x \notin N_+$$

if $r + 1 = 3(k) + 2$

$$\text{is } x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \dots + 3^{k-k+1}(2^{2(k-2)}) + 2^{2(k-1)}}{2^{r+1-k}} \Rightarrow x \in N_+?$$

Proof: $r + 1 = 3(k) + 2$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \dots + 3^{k-k+1}(2^{2(k-2)}) + 2^{2(k-1)}}{2^{3k+2-k}}$$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \dots + 3^{k-k+1}(2^{2(k-2)}) + 2^{2(k-1)}}{2^{2k+2}}$$

$$(2^{2k+2} - 3^k)x = 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \dots + 3^{k-k+1}(2^{2(k-2)}) + 2^{2(k-1)}$$

let $k = 4$

$$(2^{10} - 3^4)x = 3^3(2^0) + 3^2(2^2) + 3^1(2^4) + 2^6$$

$$943x = 175 \Rightarrow x \notin N_+$$

$\therefore$ if $r = 3(k) \Rightarrow x \in N_+ \Rightarrow lS(x) = \{4, 2, 1\}, \forall n \in N_+ \Rightarrow$ There is one loop $\forall n \in N_+$

[If $r = 3(k) + h \Rightarrow$ all $x \notin N_+, h \in N_+, h < 3] \Rightarrow$ No loop for Collatz Sequence

$\therefore lS(n) = \{4, 2, 1\}$ the only loop with three elements, $\forall n \in N_+.$