Taha's 2nd Way of Collatz Sequence Solution

$Proof:$

$let\ Collatz\ Sequence\ of\ (n) = S(n), loop\ of\ Collatz\ Sequence\ (n) = lS(n), \&$
$Collatz\ Fact = CF.$

$n \in N_+, n\ (Even) : \frac{n}{2}, or\ n\ (Odd): 3n + 1$

$\because S(n) = \left\{[\left(\frac{n}{2}\right)\ or\ (3n + 1)], \ldots, ?\right\} \ldots CF1 \Rightarrow$

$S(n) \supseteq S\left(\left(\frac{n}{2}\right)\ or\ (3n + 1)\right) \ldots CF2\ \Rightarrow$

$lS(n) = lS\left(\left(\frac{n}{2}\right)\ or\ (3n + 1)\right) \ldots CF3.$

$Example:$

$\because S(5) = \{16,8,4,2,1\}, \ldots CF1 \Rightarrow$

$\therefore S(5) \supseteq S(16)\ because\ S(16) = \{8,4,2,1\} \ldots CF2 \Rightarrow$

$lS(5) = lS(16) = lS(8) = lS(4) = lS(2) = lS(1) \ldots CF3.$

$\because S(1) = \{4,2,1\} \Rightarrow lS(1) = \{4,2,1\},$

$S(2) = \{1, 4,2\} \Rightarrow lS(2) = \{4,2,1\},$

$S(3) = \{10, 5,16,8,4,2,1\} \Rightarrow lS(2) = \{4,2,1\},$

$S(4) = \{2,1,4\} \Rightarrow lS(4) = \{4,2,1\},$

$S(5) = \{16,8,4,2,1\} \Rightarrow lS(5) = \{4,2,1\}, and$

$S(6) = \{3,10,5,16,8,4,2,1\}\ \Rightarrow lS(6) = \{4,2,1\}.$

$Then\ let\ lS(r) = \{4,2,1\}, r \in N_+.$

$\therefore\ lS(n) = \{4,2,1\}, for\ n \in set\ C = \{1,2,3,4,5,6,7,8,9, \ldots, ?, \ldots, r\}.$

$Then: is\ lS(r + 1) = \{4,2,1\}?$

$Part\ a)\ If\ (r + 1) \in N_{even}\ \Rightarrow S(r + 1) = \left\{\left(\frac{r+1}{2}\right), \ldots, ?\right\} \ldots CF1 \Rightarrow$

$S(r + 1) \supseteq\ S\ (\frac{r+1}{2}) \ldots CF2\ \Rightarrow$

$lS(r + 1) = lS\ (\frac{r+1}{2})\ \ldots CF3$

$\because lS\ \left(\frac{r + 1}{2}\right) = \{4,2,1\}\ because \frac{r + 1}{2} \leq r \ldots by\ set\ C \Rightarrow$

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$\therefore\ lS(r+1) = \{4,2,1\} \ldots (substitution)$

$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{even}.$

$Part\ b)\ If\ n \in N_{odd} \Rightarrow S(n) = \{3n+1, \ldots, ?\} \ldots CF1 \Rightarrow$

$S(n) \supseteq S(3n+1) \ldots CF2 \Rightarrow$

$\therefore lS(n) = lS(3n+1) \ldots\ CF3.$

$\because lS(3n+1) = \{4,2,1\}\ because\ (3n+1) \in N_{even} \ldots part\ a).$

$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{odd} \ldots (substitution)$

$Therefor\ by\ Part\ a\ \&\ Part\ b \Rightarrow lS(n) = \{4,2,1\}, \forall\ n \in N_{+}$

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