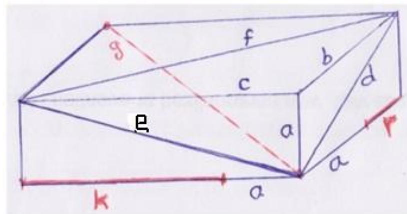
Euler Perfect Box

$a, b, c, d, e, f, g, r, k \in N_+$. *Does Euler Perfect Box exist?*

Proof:

$A)\ a < b < c$



$g^2 = a^2 + b^2 + c^2$, $let\ b = a + r, c = a + k,\ a \neq r \neq k$

$\because a < b < c \Rightarrow\ a < a + r < a + k \Rightarrow r < k, but\ a < r, or\ a > r$

$\because g^2 = a^2 + b^2 + c^2$

$\therefore g^2 = a^2 + (a + r)^2 + (a + k)^2$

$g^2 = a^2 + a^2 + 2ar + r^2 + a^2 + 2ak + k^2$

$g^2 = 3a^2 + 2ar + r^2 + 2ak + k^2$

$g^2 = 3a^2 + 2ar + r^2 + 2ak + k^2 + 2ak + a^2 - 2ak - a^2$

$g^2 = (4a^2 + 4ak + k^2) + (2ar + r^2 - 2ak - a^2)$

$\therefore g^2 = (2a + k)^2 + (2ar + r^2 - 2ak - a^2) \ldots$ (eq1)

$if\ (2ar + r^2 - 2ak - a^2) = 0 \Rightarrow 2ar + r^2 = 2ak + a^2 \Rightarrow$

$a = r = k\ \ldots Contradiction\ to\ a \neq r \neq k\ \Rightarrow$

$2ar + r^2 - 2ak - a^2 \neq 0 \Rightarrow g^2\ is\ a\ complete\ square\ or\ g^2\ is\ not\ a\ complete\ square$

$\because (2a + k)^2 + (-3a^2 - 2ak) = (a + k)^2\ is\ a\ complete\ square\ \ldots eq2, \&$

$\because\ g^2 = (2a + k)^2 + (2ar + r^2 - 2ak - a^2) \ldots eq1$

$\therefore is\ (-3a^2 - 2ak) = (2ar + r^2 - 2ak - a^2)? \ldots in\ eq1\ \&\ eq2] \ldots eq3]:$

$i)\ \ a < r, r < k: let\ a = 3, r = 4, k = 5, and\ substitute\ in\ eq3$

$is - 3(3^2) - 2(3)(5) =\ 2(3)4 + 4^2 - 2(3)5 - 3^2?$

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$is - 57 = 1?$

$\therefore -57 \neq 1 \Rightarrow (-3a^2 - 2ak) \neq (2ar + r^2 - 2ak - a^2) \Rightarrow$

$in\ eq1: g^2\ is\ not\ a\ complete\ square\ \Rightarrow g \notin N_+$

$ii)\ a > r, r < k: let\ a = 6, r = 4, k = 5, and\ substitute\ in\ eq3$

$is - 3(6^2) - 2(6)(5) = 2(6)4 + 4^2 - 2(6)5 - 6^2?$

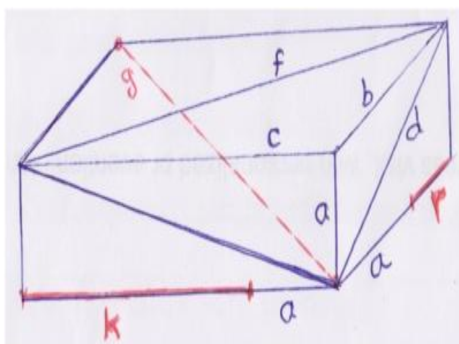
$is - 168 = -32?$

$\therefore -168 \neq -32 \Rightarrow (-3a^2 - 2ak) \neq (2ar + r^2 - 2ak - a^2) \Rightarrow$

$in\ eq1: g^2\ is\ not\ a\ complete\ square\ \Rightarrow g \notin N_+$

$\therefore$ *The Euler Perfect Box does not exist*

$B)\ a < b < c, let\ b = a + r,\ a = r$



$\because d^2 = a^2 + b^2$

$d^2 = a^2 + (a + r)^2$

$d^2 = a^2 + (a + a)^2$

$d^2 = a^2 + (2a)^2$

$d^2 = a^2 + 4a^2$

$d^2 = 5a^2 \Longrightarrow \mathrm{d} = a\sqrt{5} \notin \mathrm{N}_+$

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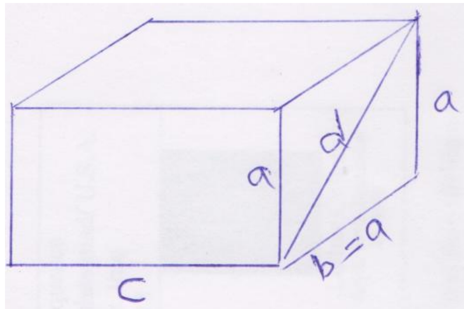
$C)\ if\ a = b < c$



$\because d^2 = a^2 + b^2$

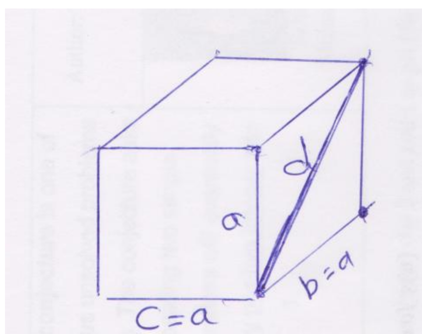
$\therefore d^2 = a^2 + a^2$

$d^2 = 2a^2$

$\therefore\ \ d = \sqrt{2}\,a \implies d \notin \mathrm{N}_+$

The Euler Perfect Box does not exist

$D)\ if\ a = b = c$



$\because d^2 = a^2 + b^2$

$\therefore d^2 = a^2 + a^2$

$d^2 = 2a^2$

$\therefore\ \ d = a\sqrt{2} \implies d \notin \mathrm{N}_+$

The Euler Perfect Box does not exist

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