Product of Geometric Series on Prime Numbers is equal to Sum of Natural Numbers

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Abstract: This paper presents a novel idea to compute the product of infinite geometric series on all prime numbers, which is equal to a series of natural numbers, that is, the sum of natural numbers up to infinite terms.

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1. Introduction

In the earlier days, geometric series [2, 3] with positive exponents served as a vital role in the development of differential and integral calculus and as an introduction to Taylor series and Fourier series. Geometric series have significant applications in physics, engineering, biology, economics, finance, management, queueing theory, computer science and medicine [1].

2. Product of Geometric Series equal to Sum of Natural numbers

Theorem 2.1: The product of geometric series on primes is equal to the sum of natural numbers.

$$\prod_{i=1}^{\infty}\left(\sum_{j=0}^{\infty} p_i^j\right) = \sum_{n=1}^{\infty} n \text{, where } p_i \text{ is the } i^{th} \text{ prime number.}$$

Proof. Let us prove the above theorem as follows:

$$\prod_{i=1}^{\infty}\left(\sum_{j=0}^{\infty} p_i^j\right) = \sum_{j=0}^{\infty} p_1^j \times \sum_{j=0}^{\infty} p_2^j \times \sum_{j=0}^{\infty} p_3^j \times \sum_{j=0}^{\infty} p_4^j \times \sum_{j=0}^{\infty} p_5^j \times \sum_{j=0}^{\infty} p_6^j \times \cdots \tag{1}$$

$$\prod_{i=1}^{\infty}\left(\sum_{j=0}^{\infty} p_i^j\right) = (1 + p_1 + p_1^2 + p_1^3 + \cdots)(1 + p_2 + p_2^2 + p_2^3 + \cdots)\cdots \tag{2}$$

By simplifying the expression (2), we get

$$\prod_{i=1}^{\infty}\left(\sum_{j=0}^{\infty} p_i^j\right) = 1 + \sum_{1\le i}^{\infty} p_i + \sum_{1\le i\le j}^{\infty} p_i p_j + \sum_{1\le i\le j\le k}^{\infty} p_i p_j p_k + \cdots \tag{3}$$

By rearranging the expansion (3), we obtain

$$\prod_{i=1}^{\infty}\left(\sum_{j=0}^{\infty} p_i^j\right) = 1 + p_1 + p_2 + p_1 p_1 + p_3 + p_1 p_2 + p_4 + p_1 p_1 p_1 + p_2 p_2 + \cdots \tag{4}$$

where $p_1 = 2,\ p_2 = 3,\ p_3 = 5,\ p_4 = 7,\ p_5 = 11,\ p_6 = 13,\ p_7 = 17,\ p_8 = 19, \cdots$

By substituting the numerical values of p_i in the equation (4), we conclude that

$$\prod_{i=1}^{\infty}\left(\sum_{j=0}^{\infty} p_i^j\right) = 1+2+3+4+5+6+7+8+9+\cdots = \sum_{n=1}^{\infty} n. \qquad (5)$$

Hence, proved.

3. Conclusion

In the article, the author of this article has proved that the product of infinite geometric series on all prime number is equal to the sum of natural numbers up to infinite terms. This idea can enable the scientific researchers for further involvement in research and development.

References

[1] Annamalai, C. (2010) "Application of Exponential Decay and Geometric Series in Effective Medicine", *Advances in Bioscience and Biotechnology*, Vol. 1(1), pp 51-54. DOI: 10.4236/abb.2010.11008.

[2] Annamalai C (2017) "Computational modelling for the formation of geometric series using Annamalai computing method", Jñānābha, Vol. 47(2), pp 327-330.

[3] Annamalai C (2018) "Annamalai's Computing Model for Algorithmic Geometric Series and its Mathematical Structures", *Journal of Mathematics and Computer Science*, Vol.3(1), pp 1-6. DOI: 10.11648/j.mcs.20180301.11.