

2nd Way Solution of Collatz Sequence

let Collatz Sequence of $(n) = S(n)$, loop of Collatz Sequence $(n) = lS(n)$, & Collatz Fact = CF.

$n, a, b, c, w \in N_+, n \text{ (Even)} : \frac{n}{2}, \text{ or } n \text{ (Odd)}: 3n + 1 \dots \text{(Collatz Sequence rules)}$

Proof:

if $S(n) = \{a, b, c, \dots, w\} \Rightarrow lS(n) = lS(a) = lS(b) = lS(c) = \dots = lS(w) \dots \text{Fact1}$

$\therefore S(1) = \{4, 2, 1\} \Rightarrow lS(1) = \{4, 2, 1\}.$

$S(2) = \{1, 4, 2\} \Rightarrow lS(2) = \{4, 2, 1\}.$

$S(3) = \{10, 5, 16, 8, 4, 2, 1\} \Rightarrow lS(3) = \{4, 2, 1\}.$

$S(4) = \{2, 1, 4\} \Rightarrow lS(4) = \{4, 2, 1\}.$

$S(5) = \{16, 8, 4, 2, 1\} \Rightarrow lS(5) = \{4, 2, 1\}.$

Then let $S(r) = \left\{ \frac{r}{2} \text{ or } (3r + 1), \dots, 4, 2, 1 \right\} \Rightarrow lS(r) = \{4, 2, 1\}, r \in N_+.$

$\therefore lS(n) = \{4, 2, 1\}, \forall n \in \text{Set } Z = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \dots, r\}.$

is $lS(r + 1) = \{4, 2, 1\}$?

Part a)

If $(r + 1) \in N_{\text{even}} \Rightarrow S(r + 1) = \left\{ \left(\frac{r+1}{2} \right), \dots, ? \right\} \Rightarrow lS(r + 1) = lS\left(\frac{r+1}{2}\right) \dots \text{Fact1}$

$\therefore \frac{r+1}{2} \leq r \Rightarrow \frac{r+1}{2} \in \text{Set } Z = \left\{ 1, 2, 3, 4, 5, 6, 7, 8, 9, \dots, \frac{r+1}{2}, \dots, r \right\} \Rightarrow$

$\therefore lS\left(\frac{r+1}{2}\right) = \{4, 2, 1\} \dots \text{Fact1}$

$\therefore lS(r + 1) = \{4, 2, 1\} \dots \text{(substitution)}$

$\therefore lS(n) = \{4, 2, 1\}, \forall n \in N_{\text{even}}.$

Part b)

If $n \in N_{\text{odd}} \Rightarrow S(n) = \{3n + 1, \dots, ?\} \Rightarrow lS(n) = lS(3n + 1) \dots \text{Fact1}$

$\therefore (3n + 1) \in N_{\text{even}}$

$\therefore lS(3n + 1) = \{4, 2, 1\} \dots \text{by Part a}$

$\therefore lS(n) = \{4, 2, 1\} \forall n \in N_{\text{odd}} \dots \text{(substitution)}$

$\therefore lS(n) = \{4, 2, 1\}, \forall n \in N_+ \dots \text{by Part a \& Part b.}$