"Efficient Complexity": Evolutionary Perspectives in Natural Learning Systems

Serge Dolgikh[1][0000-0001-5929-8954]
Dept. of Information Technology,
National Aviation University, Kyiv
Lubomira Huzara Ave, 1

Abstract. A fundamental question in the conjunction of information theory, biophysics, bioinformatics and thermodynamics relates to the principles and processes that guide and control the development of natural intelligence in natural environments where information about external stimuli may not be available at prior. A novel approach to the challenge of natural learning is proposed in the framework of constrained optimization where maximums of the information fitness of the internal states of the system with the states of external stimuli under the natural constraints of natural learning are associated with the optimal learning. The progress of natural intelligence can be interpreted in this framework as a strategy of approximation of the solutions of the optimization problem via a traversal or "hopping" over the extrema network of the objective function, the information fitness under the natural constraints that were examined and described. Nontrivial conclusions on the relationships between the complexity, variability and efficiency of the structure, or architecture of learning models made on the basis of the proposed formalism can explain the effectiveness of neural networks as collaborative groups of small intelligent units in biological and artificial intelligence.

Keywords: Natural learning systems; representation learning; conceptual representations; constrained optimization; statistical thermodynamics.

1 Introduction

The emergence and development of natural intelligence happen at the conjunction of several thriving disciplines in modern science: information theory, biophysics, bioinformatics, computational, statistical and topological fields and aspects in mathematics and others. In this analysis, practical constraints imposed by the requirements of the physical existence of the system can be as important as general theoretical principles of the transfer of information and learning.

An essential observation in the theory of natural intelligence is that an intelligent system that is developing in a natural environment cannot be expected to have the information, description or a model, etc. of its environment that are accurate; detailed; and complete at its prior state; therefore, this knowledge has to be acquired through some process of interaction with it.

https://doi.org/10.33774/coe-2024-d0s2d **ORCID:** https://orcid.org/0000-0001-5929-8954 Content not peer-reviewed by Cambridge University Press. **License:** All Rights Reserved

This observation in the analysis of the processes and principles of the emergence and development of natural intelligence resonates with the studies of unsupervised artificial learning systems, a well-researched field in theoretical and applied computer science that explores the possibility and processes of acquisition of information from the interaction of a learning system with its environment that are not dependent on, and do not require significant prior knowledge about it.

In this work we attempted to develop a view on the progress of natural intelligence that combines information-theoretical and physical characteristics of the process of acquisition of information from the interactions with the sensory environment in the framework of the constrained optimization problem. This approach leads to the formulation of a unified general framework of the evolution of natural intelligent systems as a continuous process of fitting their information models to the sensory environments, guided by the optimization principle under the essential physical constraints of natural intelligent systems as a basis for the production of differentiated responses to sensory stimuli the effectiveness of which is verified in empirical trials.

2 Related Work

Information-based approaches in the theory of natural intelligent systems were introduced and developed since the groundbreaking works of Shannon [1] and Shroedinger [2]. An immense body of research has been compiled since in the active and expanding field that will be challenging to review with any fairness in a limited space, so we will focus on the directions and results that are directly related to the subject of our study: the theory of unsupervised learning, where essential information about the sensory environments can be inferred directly from the interactions with the environment (represented by samplings or “data”); and the thermodynamic and evolutionary approaches that attempt to formalize and describe the ability of natural learning systems to attain ever higher levels of attunement or fitness to their environments.

The ability of pioneering unsupervised generative models such as Restricted Boltzmann Machines, Deep Belief Networks [3,4] and related ones to create effective information models of simpler types of data is well-known and researched. Many effective approaches, types and architectures were examined since including neural models [5,6] that proved effective in modeling complex and realistic data such as high-quality images, music and others [7].

In the experimental studies, many important results were reported, such as the spontaneous concept learning experiment that observed the emergence of concept sensitivity on a single neuron level with an unsupervised generative neural model trained with massive sets of realistic images [8]. Distributions in informative representations of visual data were studied with generative neural models [9,10] pointing at the effect of geometric structuring in the generative representations. Geometric and topological structure of conceptual representations of visual data was described in detail in [11]. Studies [12,13] offered both broad and in-depth reviews of methods and approaches in unsupervised learning and dimensionality reduction.

Intriguingly, concurrently with these studies into artificial intelligent systems recent advances in the research in biological intelligence demonstrated the commonality of low-dimensional neural representations in the processing of sensory information by animals and humans [14,15]. These results suggest both interesting and intriguing parallels between the learning processes of artificial and biological systems, perhaps guided by common laws and principles.

Following the foundational works of Shannon, many studies focused on the information processes in natural learning systems based on the framework of thermodynamics.

Studies [16-18] to name only a few, explored the principle of maximum entropy as the formalism of the description of the processes in the natural systems. Information entropy, introduced by Shannon can be seen as both a close and natural measure of a "fitness" of a natural learning system to its physical environment and maximization of it, as a natural existential objective for such systems. However, there can be certain caveats in a direct application of this framework to the theory of natural intelligence; evidently, natural biological and learning systems exist in essentially open thermodynamic environments with strong flows of energy and materials, far from the equilibrium state.

As well, the essential existential constraints of natural systems cannot be ignored [19]. An important aspect of feasibility of any natural learning system, model, architecture, etc. is its ability to exist and operate within the essential physical constraints. We will attempt to address some of these questions in this study.

In [20], the authors approached the problem of learning the essential information in the signal in the framework of constrained optimization. It can yield results on the optimal level of compression of the original signal and the generalization ability of the learning system where the correctness of the encoding is known either via a certain distance function (the rate distortion function) or another variable, such as a known category or class. We will attempt to show that the problem of natural learning can be formulated in the general case in terms of constrained optimization as well.

Another active area of research lies in the plane of incremental variations and adaptations of the information model and structure (architecture) of learning systems: what processes guide their ability to model sensory environments of increasing complexity? Such processes require information models and architectures, for example, neural ones, to be flexible and adaptable that is, possess the ability to change incrementally [21,22], attaining progressively higher information fitness to their sensory environment while remaining with the limits of the essential existential constraints.

In the view of the extensive body of research and the results accumulated over the time in the field, open areas and questions remain. What are the connections between the general principles of learning of natural intelligent systems and the problems and directions in the theory and practice of learning, including data compression, dimensionality reduction and learning without prior knowledge? What principles and processes guide the ability of learning systems to evolve and adapt toward the ability to interpret progressively more complex sensory inputs and build differentiated responses that are both effective and efficient?

In this work, we attempted to approach these questions by first formulating the problem of natural learning as that of the constrained optimum: maximization of the objective function represented by the mutual information/entropy of the external states of the environment and the internal states of the learning system under the essential material constraints of natural learning systems including, critically, memory; computation resources; and other physical resources and energy. This formalism can yield non-trivial insights into the essential functions of natural learning systems including the ability to compress sensory stimuli as the basis for forming intelligent differentiated responses; and conceptualize them for efficient production of responses verified by empirical trials. As well, it can provide a conceptual basis and a formalism for examination of the evolutionary ability of natural intelligent systems [23].

The rest of the paper is organized as follows: in Section 3, we discuss the statement of the constrained optimization problem of natural intelligence, including the formulation of the objective and essential constraints. Section 4 focuses on the formalization of the problem of generative learning and its connection to geometric categorization in informative representations of sensory data. Section 5 contains the formulation and analysis of the framework of evolutionary description of progressing natural intelligence as a traversal of the extrema network of the optimization problem under the objective of maximization of the information fitness and the results obtained in this plane of analysis. Section 6 is devoted to the discussion of proposed approaches, results and their connections to other results and directions of research in the field.

3 Natural Intelligent Systems: Objectives and Constraints

As was commented earlier in the opening section, natural intelligent systems are related to models in unsupervised learning field of computer science by the fact that they cannot rely on massive prior knowledge about their sensory environments.

One of the essential problems in unsupervised learning relates to the ability of learning models to conceptualize sensory information, that is, compact it into a manageable set of common types, states or “concepts”. Correct interpretation of general types of sensory observations, external and internal is a critical foundation for constructing differentiated responses to sensory stimuli that maintain or advance the state of the system relative to its existential objective. Then, it can be concluded that the problem of natural intelligence can be related, directly and closely, to interpretation of sensory observations and their samplings that will be generally referred to as “sensory data” in terms of characteristic types, concepts or states.

3.1 Essential Constraints of Natural Intelligent Systems

Natural intelligent systems are characterized by their ability to produce differentiated responses as a result of their interactions with the environment, while satisfying certain essential constraints governed by the physical reality of their existence. Compression of sensory data is one of these requirements that are fundamental to the existence of a natural intelligent system.

Indeed, physical resources available to natural intelligent systems are limited and in the early stages of development, strongly constrained. Memory and computing power (compute) are the two of the critical resources that are required for production of differentiated responses and strongly constrained by the physical factors.

Memorizing previous sensory observations and their association to the internal states of the system is critical for production of differentiated responses: in the absence of such prior information, the correct interpretation and response to any sensory input would have to be relearned again and again. However, realistic natural systems simply cannot “afford” to store raw sensory records due to the practical constraints of the available memory and other essential resources.

Compute in production of differentiated responses is another constrained resource: a realistic intelligent system cannot afford to build an entirely new response for each new sensory input within the imposed constraints of resources and time. Then, it must attempt to group or “conceptualize” sensory stimuli into characteristic types, concepts or states; prioritize; and use pre-built general responses to similar stimuli.

These observations help to formalize the essential constraints of natural intelligence as:

1. The observed sensory samplings (data) D must be compressed or compacted for storage in a representation or “embedding” L of reduced dimensionality: $\dim(L) \ll \dim(D)$, while the information in D that is essential for the production of correct differentiated responses is preserved to an acceptable extent.
2. Sensory data needs to be conceptualized, that is, effectively grouped into classes, types, concepts or states of similarity that can be associated with similar responses.
3. The constraints on the energy and other material resources must be satisfied at all times during the lifetime of the system.

It can be concluded then that for a natural intelligent system, compression of data that describes its sensory environment is a critical practical necessity. Natural intelligence can emerge and develop only within the region of the internal parameters defining the system that satisfies the outlined constraints. Physical and practically feasible intelligent systems must, have no choice but to compress sensory information as it relates directly to the essential constraints of their existence. Then, in its turn, an effective compression and conceptualization of sensory inputs can provide the basis for production of differentiated responses to the sensory stimuli that maintain or advance the system toward its existential objective.

3.2 Objectives of Natural Intelligent Systems

As discussed earlier, intelligence that can be described by the ability to produce differentiated responses to the stimuli from the environment to advance its existential objective. This ability requires some way of storing information about the earlier observations and trials.

Indeed, having no information about prior states and inputs, and short of testing all possible options, there would be no information to produce responses differentiated between the inputs. For example, attempting a feeding response where a predator is nearby can have catastrophic consequences for the learner.

The second important observation that was made is that this data (that is, samplings of the sensory environment) has to be compressed in such a way that it preserves the essential (for the intelligent system) information about the sensory stimuli. If compression results in a significant loss of information, it cannot be used to construct effective responses that will be successful in the empirical trials.

Then, compression of sensory data with preservation of its essential information content can be defined as the objective for natural intelligent systems.

3.3 Natural Learning as a Constrained Optimization Problem

Let us consider sensory inputs in a space D that can be expressed or "observed" in certain observable factors x: the observable distribution $X \in D$, and its compressed representation in a certain space L described by "hidden" or latent factors t. The constraint of compression of sensory data with the preservation of information can be described by the class of encoding mappings $E = \{\, e \,\}\,(D\ \rightarrow L)$, the associated distribution $p_e(x, t), x\ \in D, t\ \in L$, and the mutual information [1] of the distributions X and its latent image, $E(X)$, $I_e(X, E(X))$.

Then, the objective of retaining the essential information about the sensory environment under the constraints of natural learning as discussed earlier, can be formulated as maximization of the mutual information of the sensory data in the input to the learning system and the latent distribution, I_e over the space of encoding transformations $e \in E$, or, equivalently, distributions p_e *under the essential constraints of natural learning.*

In the view of the discussion above, the optimization problem thus defined is clearly constrained, specifically by:

- the critical constraints on the memory and computing power;
- other resource constraints: physical materials and energy, in training and operation.

Then, the problem of natural intelligence can be defined as a case of the classical multi-parameter constrained optimization:

$$Max\big(I_e(e)\big)\,\Big|_{e:\Lambda};\ d(e) \le d_{max};\ c(e)\ \ge c_{min};\ \rho(e)\ \le \rho_{max} \qquad (1)$$

where d, c: the memory and conceptualization constraints, $\rho(e, g)$: the combined resource constraint (physical resources, energy, etc.); Λ: the set of parameters that describe the encoding mappings, $E\,/\,p_e$.

The optimization problem in (1) then describes a case of inequality constrained optimization (Kunh-Tucker conditions, [24]). It is known that solutions of the problem (1) are the extrema, local or global of the Lagrangian functional:

$$L(I_e, \mu_{red}, \eta_{con}\ ) =\ I_e(e) -\ \eta_{con}(d(e) - d_{max}) - \sum_k \mu_k (g_k(e) - c_k) \qquad (2)$$

where η_{con}, μ_k: the Lagrangian multipliers for the constraints of memory, conceptualization and other essential constraints in (1); $g_k(e)$, c_k: the constraints.

Next, one can make some practical observations on the application of the thus formulated problem statement to realistic natural intelligent systems.

Let us assume that the optimization variables (e) in (1) and (2) are described by certain parameters that combined, define the full set of the learning parameters of the system, Λ.

With natural learning systems, the learning parameters are commonly divided in two classes. The physical constraints that are considered nearly immutable are described by the structure or "architecture" of the learning model: $A = \{ a_k \} \sim const$, whereas the training parameters V: $\{ v_j \}$ can be updated in the process of learning to achieve an optimum of the objective. It can be the architecture of an artificial neural network, other artificial learning systems or models or physical biological networks of neurons and synapses. Then, $\Lambda = \{ A, V \}$.

Next, a realistic natural system can be challenged to learn the exact value of the mutual information I_e in (1) as it would require the explicit and complete description of the encoding distribution. Instead, in practical processes of learning, the unknown value of I_e can be effectively approximated by a measure of accuracy or "fitness" $F_e(S)$ on a representative subset of samples S in the original data space that can be calculated with a given configuration of the architecture and training parameters, $\lambda = (a, v)$ and a representative subset of samples in the input space, S. There are different approaches to this approximation as will be discussed further in the section on unsupervised learning.

Then, with a fixed architecture and an effective approximation of the objective functional, the system can seek a solution of the optimization problem (2) via a Bayesian process [25] of updating the training parameters V based on the difference (distance) of the prior (the original set of samples) and the posterior (the prediction produced by the learning system) achieving the minimization of the learning error and a solution, at least local, to the constrained optimization problem. In this interpretation, the general problem of natural intelligence (2) translates into the standard problem of machine learning:

$$T_{opt}: Max\big(F_e(t)\big)\,\big|_{S(D)} \tag{3}$$

where T_{opt}: the optimal configuration of trainable parameters of the model that achieves the maximum of the measure of generative accuracy over the distribution of the original data; $S(D)$: a representative sample (training set) of the original distribution, D. Note that the introduction of an immutable architecture and the approximation of the objective, the information entropy, effectively "hid" the constraints in the unconstrained problem (3) that can be seen as an approximation of (2).

4 Generative Learning and Conceptualization

In this section we will consider a well-established direction in self-supervised learning that can be instrumental in the analysis of the constraints of compression and conceptualization. In this approach, the ability of the system to learn and conceptualize sensory inputs stems from the capacity to restore, or generate the observable distribution from its compressed, "encoded" form. Models of generative self-supervised learning are thus trained to reproduce the input distribution with high accuracy and precision by imposing the incentive to reduce the error or distance between the batches of input data and generations produced by the model in the process of learning.

4.1 No "Natural Limit" in Unsupervised Data Compression

Dimensionality reduction was and is being studied at a great depth with a large number of insightful results obtained to date including those mentioned in the review section. An essential observation that will be made from the outset of this analysis is that in the case of unsupervised dimensionality reduction, that cannot rely on prior knowledge of the essential characteristics of the sensory data, there is no theoretical, information theoretical or other cause or principle that can determine or point to the best or optimal level to which the data needs to be compressed.

The argument first observes that any generative data compression of an original sampling, distribution or data D, that is capable of restoring it from a compressed representation R via a generative transformation G: $R \rightarrow D$ would result in a loss of information about if the dimensionality of the representation is lower than that of the original distribution, excluding special cases as degenerate parameters, limited region of the distribution, etc.

This conclusion follows from the results on the invariance of topological dimension (Brouwer's invariance of domain and the related). Indeed, if the perfect compressed representation existed in the lower dimension, there would exist a homeomorphism between the topological distributions of the data in the original space and its compressed image in the latent space, of two different topological dimensions. That supposition would then contradict the results on the invariance of the topological dimension.

Then, a trivial case of the perfect, zero-loss representation is always available if the dimensionality restriction is lifted with the identity transformation $E(D \rightarrow D): E(x) = x$, where D: the original data (a distribution of data points that represent sensory stimuli in the space of the original observable parameters).

Thus, between the identity transformation and a compressed representation R in a latent space L, $R(D \rightarrow L)$, $\dim(L) < \dim(D)$, any non-trivial reduction of dimensionality would result in a loss of some information about D, defined as the non-existence of a mapping that allows to restore D from L precisely:

$$G(L) \simeq D \Rightarrow \mathrm{Dim}(L) \geq D \tag{4}$$

where G: the generative transformation, and excluding aforementioned exceptions.

From this result, as the optimal level, dimension or criteria of the compression of data cannot be formulated in general terms based only on information theoretical principles, there follows an essential for natural learning systems observation that *the constraints and incentives of dimensionality reduction in their information models have to be dictated by the practical conditions of their existence and interaction with the environment*. These conditions, as discussed earlier, can be expressed formally as the essential constraints of natural learning.

4.2 Constrained Optimization in Generative Learning

As in Section 3.3 we consider sensory input space D and its compressed representation in the latent space L described by latent factors t, encoding transformations $E(D \rightarrow L)$, the associated distribution $p_e(x,t), x \in D, t \in L$, and the mutual information of the distributions X and its latent image, $E(X)$, $I_e(X, E(X))$.

In the generative approach, the intelligent system must also have the means to restore the compressed representations to the input space to compare the stored information with new inputs, and to verify the correctness of the encoding function. This function can be described by the class of generative transformations: $G(L \rightarrow D)$, the distribution $p_g(t,x), x \in D, t \in L$, and the mutual information $I_g(t, G(t))$.

Then, following the approach outlined earlier in Section 3.3 and the concept of generative learning outlined above, the fitness objective of generative learning can be defined as maximization the mutual information of the sensory data in the input to the learning system and distribution generated distribution: $I_r = I(X, X'=G(t))$ over the space of encoding and generative transformations described by the tuple (e, g), $e \in E$, $g \in G$) again, under the essential constraints of natural learning:

$$Max\big(I_r(e,g)\big)\,\big|_{e,g:\Lambda};\ d(e,g) \leq d_{max};\ c(e,g)\ \geq c_{min};\ \rho(e,g)\ \leq \rho_{max}$$

As in (1), d, c: the memory and conceptualization constraints, $\rho(e,g)$: the combined resource constraint (physical resources, energy, etc.); Λ: the set of parameters that describe the encoding/generative tuple (e, g) with the Lagrangian of generative learning:

$$L(I_r, \mu_{red}, \eta_{con}) = I_r(e,g) - \sum_k \mu_k(g_k(e,g) - c_k) \tag{5}$$

where μ_k: the Lagrangian multipliers for the constraints, g_k(e, g), c_k: the constraints of dimensionality reduction, conceptualization and other material constraints.

Similarly to what has been outlined in Section 3.3 the optimization variables (e, g) of generative learning in (5) are defined by the architectural/structural and training parameters: $\Lambda = \{ A, V \}$.

An effective approximation of the mutual information in (5) can be modeled by a measure of generative accuracy $F_g(S, G(E(S))$ on a representative subset of samples S in the original data space, i.e., the distance between the original sampling and the result generated by the model in the metric of the input data space ("the generation trick"); it can be calculated readily with a given configuration of the architecture and training parameters, $\lambda = (a, v)$ and a representative subset of samples in the input space, S.

Then, as in (3), Section 3.3 a solution of the optimization problem (5) can be sought via a Bayesian process of updating the training parameters V based on the difference (distance) of the prior (the original set of samples) and the posterior (the generation produced by the learning model):

$$T_{opt}: Max\left(F_g(t)\right)\Big|_{S(D)}$$

where T_{opt}: the optimal configuration of trainable parameters of the model that achieves the maximum of the measure of generative accuracy over the distribution of the original data; $S(D)$.

The statement above describes the standard formulation of the objective of self-supervised learning [12,13]. Note, again, how the introduction of the immutable architecture and the generation trick effectively “hide” the essential constraints in the unconstrained problem of unsupervised learning (5).

4.3 Generative Compression and Geometric Conceptualization

In this section we will attempt to provide some arguments that the constraint of conceptualization that is related to the ability of the system to factorize or classify stimuli into common types in some cases in some cases can be effectively realized by that of a strong dimensionality reduction. We will consider the following lemma of geometric conceptualization:

Under the conditions of:

1. Learning success: the learning system is capable of achieving generative accuracy above certain minimum: $A(D, G(D)) > A_{min} = \alpha$
2. Generalization: the accuracy is preserved across all and any representative set of samples in the original data space: $\forall\, S(D){:}\, A\big(S, G(S)\big) > \alpha$.
3. A strong dimensionality reduction to a low dimension d_L that allows simplified topological classification of the data distribution manifolds.
4. Conceptualization or factorization of data in the observable space: the original distribution is comprised of characteristic types of similarity: $D = \{C_1, .. C_k\}$, C_k: the classes or types of similarity in the observable space. Note that neither the types nor their observable or latent distributions are known to the model at prior.
5. Constant structure (architecture) of the model in the stages of training and trials.

generative models with well-separated (as defined below) regions of the latent distributions of the concepts, i.e., the types of similarity are prevalent in the ensemble of learning models of the same architecture trained on the same representative input sample.

Outline of the proof

We will consider the simplest case of the composition of data with minimal number of types: two, $D = \{A, B\}$ and a generative intelligent model M that satisfies the conditions of the lemma.

Next, we consider a representative sample $S \subseteq D$ and the latent region X of the strong intersection (intermixing) of the latent distributions of the types A, B in the latent image of S. The condition 3 (dimensionality reduction) ensures that the latent distributions $L_A = E(A)$, $L_B = L(B)$ are well defined topologically (for example, as one or finite number of compact manifolds of dimension) and therefore, so is their intersection. The condition of strong intersection means that it contains significant populations of both classes, in a general manner, that is, for all representative sets of D. Formally, $\forall\, g \subseteq$

$X\ P_A(g) \sim \beta\ P_B(g) > 0, P_A, P_B = Card(L_A\ \cap g), Card(L_B\ \cap g), \beta\colon const.$ Any subregion of $L_A \cap L_B$ that does not satisfy the condition of strong intersection is removed from X.

For the simplicity of the abridged version of the proof presented here, it will be assumed that the populations of the types in the observable data, and in the condition of significant intersection region X are approximately equal: $\beta \sim 1$. The proof can be extended to the general case straightforwardly and will be provided in another study.

Now, assuming $X\ \neq\ \emptyset$ let us take a latent point $y\ \in X$ with the image in the observable space $G(y)$ is of type A: $G(y)\ \in A$, where G: generative transformation of M. Then, applying the condition of strong intersection to an arbitrarily small neighborhood of y, ε(y) one can conclude that it has to contain approximately equal populations of latent positions of classes A, B.

It is known that models of finite complexity, including deep neural networks [26] commonly have finite resolution, described by the Lipschitz constant, l: $\|G(y), G(x)\| \leq l\ \|y, x\|$. Then, for sufficiently small ε it follows that $G(x) \in$ A $\forall\, x\ \in\ \varepsilon(y)$. Then, based on the conclusion of equal populations of classes above, it would follow that the model produces an erroneous generation in approximately ½ of the population of ε(y). Finally, as y represents an arbitrary position in X, this conclusion can be extended to the entire region. Where w_g is the generative error of the model,

$$w_g(S)\ \geq w_g(X) \sim \frac{1}{2}\ P(X) = \frac{c}{2}\ \frac{m(X)}{m(E(S))}$$

where P: the population of the latent region X with an observable set S, m: a measure of volume in the latent space; c: a constant.

Then, for the overall generative error of the model on S one obtains:

$$\mathrm{w_g}(S) \geq \frac{c}{2}\ \mu_X$$

where μ_X: the relative volume of X in the latent space, $\frac{m(X)}{m(E(S))}$.

Now, the conditions 1 (accuracy) and 2 (generalization) of the lemma require that $w_g(S) \sim$ const and $w_g(S) \leq 1 - A_{min}$. Then,

$$\mu_X\ \leq\ \beta\ (1 -\ A_{min}),\ \beta\colon \mathrm{const} \qquad (6)$$

connecting the generalized accuracy with the relative measure of the latent region of the strong intermixing of the general types in the observable sample.

This argument can be extended straightforwardly to the arbitrary number of general types. It follows then that under the conditions of the lemma, successful generative models with latent representations of sufficiently low dimension must prefer conceptualized latent distributions of the general types (concepts), with minimal intersection of their latent regions of distribution.

Note that all of the conditions were essential in the proof. Another condition that was assumed implicitly is that the essential characteristics of the original distribution D does not change during the training or any of the trail stages.

With practical implementations of intelligent systems, the assumption of infinitesimal continuity of the latent space on which the outlined proof was based can be substantiated by an observation based on the condition of generalization of the lemma. Even if, with a finite representative set *S*, an arbitrarily small neighborhood may not be expected to contain the latent positions of physical samples in *S*, it can still contain the those of samples in other such sets, S_1, … S_k. Then, the argument in the proof that all such samples in a sufficiently small neighborhood would be classified to the same type still holds.

Geometric conceptualization in informative low-dimensional representations was observed in a number of empirical results in artificial and natural neural systems.

Empirical Results in Generative Geometric Conceptualization

The results on geometric conceptualization of generative representations are supported by a number of published experimental results obtained with different types of data and generative architectures. The results [9,10] demonstrated and described "disentangled" representations of visual data with several datasets of images. Emergence of concept-correlated structures in unsupervised generative learning with very large sets of images was reported in [8]. Geometric and topological characteristics of generative representations were studied in [11] with several datasets of images describing a well-defined continuous structure of concept regions.

These results provide direct experimental support for the conclusions on geometric conceptualization of generative representations outlined in the preceding sections. Recent results in experimental neuroscience point at the ubiquitous character of low-dimensional representations of sensory stimuli in biological organisms, including visual, olfactory and audio signals in animals and humans [14,15]. It can be conjectured that this observed effect can be related to the higher effectiveness of low-dimensional generative representations in identifying characteristics types, or concepts in the sensory data.

Additional Benefits of Deep Data Compression

It may be worth noting certain additional "coincidental" benefits of deep compression of sensory data for natural learning systems that can be important or even critical for the success of conceptualization:

1. Compressed distributions of lower dimensionality can have simpler topological structure. A number of known results in topology point to this observation: Smith conjecture breaks in $d > 3$; Milner conjecture breaks in $d > 5$; classification of compact manifolds is solved in $d < 4$; algorithmic PL homeomorphism problem for compact manifolds solved in $d < 4$; and other results (*d*: the dimension of topological space). All of these results can have a direct connection to topological characteristics of distributions in the low-dimensional representations of sensory data and the ability of a learning system to determine the concept structure in them.
2. Significantly reduced dimensionality of internal representations can be essential for some computational methods in the task of conceptualization such as clustering, that

can require progressively more time and computing power with higher dimensionality of the data or even become intractable [27].

3. If strong reduction of dimensionality is successful (i.e. avoids significant loss of essential information in the original distribution) it can point at the original data being strongly redundant. There can be downsides in attempting conceptualization directly with strongly redundant data such as a possibility of overrepresentation of some factors.

All of these factors can be linked directly and significantly to the effectiveness of conceptualization of sensory observations: as already commented, a critical need and constraint for any realistic natural intelligent system.

4.4 Practical Corollaries of the Geometric Conceptualization

For an illustration of the importance of the dimensionality constraint and geometric categorization for practical intelligent systems let us consider an example of two intelligent systems. One (A) trained to achieve lower accuracy (and therefore, higher information loss) for example, 70% but more effective conceptualization, possibly due to producing an effective low-dimensional representation: 80%; the other system (B) achieves near perfect accuracy, but less effective conceptualization: 95% and 70%, respectively.

Then, assuming that correct responses are associated with the identified general types of sensory inputs with perfect accuracy, A would produce 80% of correct responses, whereas the second one (B), 70%. Then the model A will be selected in empirical trials due to higher effectiveness of its responses.

This simple example underlines that the objective for practical intelligent systems is not an unconditional and unconstrained maximization of the correlation between the sensory information and its internal representation, commonly described by the standard problem of unsupervised learning; but rather, an effective conceptualization of it with preservation of essential information and within the subspace of the parameters of the system that satisfies the essential constraints (2). Strong dimensionality reduction thus plays a key role in satisfying two of the essential constraints of natural intelligence: that of limits on the critical resources, the memory and the compute; and effective conceptualization. The result on geometric categorization is important in this perspective as it allows to effectively replace the conceptualization constraint with that of the dimensionality of the internal model of the sensory data that can be defined and described explicitly in the architectural parameters of learning models.

Therefore, an essential corollary of the result on geometric conceptualization of generative representations is that the conceptualization constraint that can be challenging to express explicitly in the architectural parameters of the system can be effectively replaced, in some cases at least, with the constraint on the effective dimensionality of the internal representation space. Then, both of the constraints of the memory and conceptualization in (2) and (5) can be described by a strong constraint on the dimensionality of the internal representation:

$$d(\lambda) \leq d_{max};\ c(\lambda)\ \geq c_{min} => \ d(\lambda) \leq d_{con},$$

d_{con}: the constraint of effective compression and conceptualization.

This observation allows to obtain the explicit form of the Lagrangian (3) based on the results on inequality-constrained problems:

$$L(F, \mu, \eta) = F_e(\lambda) - \mu\,(d(\lambda) \le d_{con}) - \eta\,(\rho(\lambda) \le \rho_{max}) \tag{7}$$

where the objective function $F_e(\lambda)$ and the constraint operate in the space of architectures Λ, with the Kuhn – Tucker conditions on the solutions (and similarly, for the case of generative learning, (5)).

It can be noted in conclusion that generative learning and conceptualization considered in this section present one possible direction and strategy for a learning system to comply with the essential constraints of natural learning, including critically, data compression and conceptualization. However, there are no reasons to expect it to be either the general one or the only possible. Other possibilities, for example, effective one-way dimensionality reduction methods and strategies whose effectiveness, including accuracy and precision, can be established empirically in trials. From here onwards it will be assumed that the learning system is able to achieve the compression and conceptualization objective under the constraints via some strategy and the approach outlined in (1)-(3), (7) will be used from this point on.

5 Evolving Natural Intelligence

5.1 Information Fitness and Information Model

Let us return to the optimization problem (2) with the objective function I_e, the mutual information of the observable (sensory) and the encoding distributions.

An intelligent system that has attained an objective maximum under the constraints as described earlier can associate input stimuli x in the observable space to their encoded positions t in the space of internal factors that describe the state of the system satisfying the constraints of the problem (2). Then, with sufficient empirical trials, the following probability distributions can be estimated empirically:

- $P_s(x)$: the empirical probability distribution of the sensory inputs x,
- $P_i(t)$: the empirical probability distribution of the internal variables, $t = E(x)$,
- $P_m(t, x)$: the empirical joint probability distribution of the sensory inputs and their internal images or representations.

With these empirical variables, the mutual information between the internal variables of the system and the sensory stimuli $x \in X$ can be calculated by the Shannon formula [1]:

$$F = \sum_x \sum_t P_m(t, x) \log \frac{P_m(t,x)}{p_s(x)\, p_i(t)} \tag{8}$$

The mutual information thus defined will be referred to as the “information fitness” of an intelligent system. It can be seen immediately that it is directly related to the empirical success of the responses produced by the system.

Indeed, assuming for now that the system can produce perfectly effective responses based on the identified internal state of the stimuli (as the model of production of the responses will be discussed elsewhere), the empirical effectiveness of the response will be determined by the correctness of the association between the sensory stimuli and the internal states of the system that is used to construct the response, and this is exactly what the information fitness characteristic is a measure of F.

The joint probability distribution $P_m(t, x)$ in (8) will be referred to as the information model of the intelligent system:

$$M = P_m(t, x) = P_{[T,X]} \tag{9}$$

As noted above, it describes the empirical correctness of the interpretation of the observables x by the internal variables of the system t, verified empirically by the effectiveness of the responses produced by the system. An discussed earlier, the information model can be described or realized by a parametric function of the observable factors, $\mu(\Lambda, X)$:

$$M: \mu_{v,a}(X) \rightarrow E(X)$$

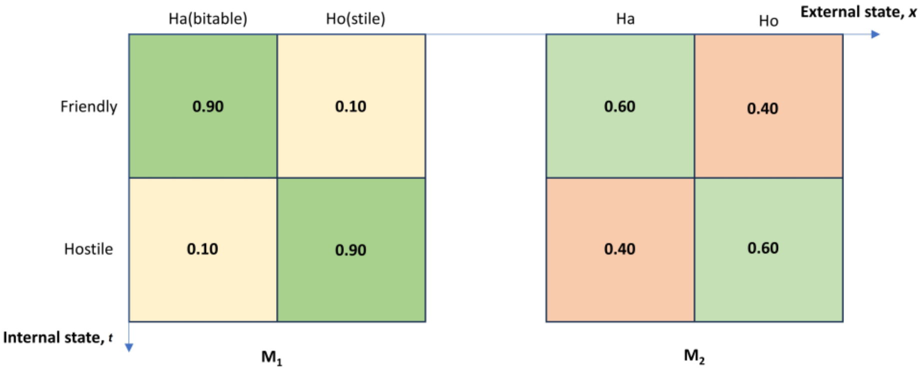
Next, based on the result of the lemma of geometric conceptualization in Section 4.2, we will assume that the distributions X, $E(X)$ in the observable and latent spaces are conceptualized: $X = \bigcup_C C_k,\ \ E(X) = \bigcup_j K_j$, C_k: general types of similarity of sensory inputs; K_j: internal concepts or “states” of the system associated with distinct regions in the latent space, according to the lemma of conceptualization. Then, as easy to see, the definitions (8) and (9) can be rewritten in terms of characteristic classes C and internal states K that will be referred to as the external states of the environment and the internal states of the system, respectively. To eschew cluttering of symbols, the variables x, t will denote both external factors and states, and internal (latent) factors and states, respectively. Where the distinction is essential and not obvious from the context an explicit note will be made.

For an illustration of the definitions given above, let us consider information models M_1 and M_2 of a simplest type that interpret a single observable v mapping it to a single internal variable, “viability” of the system t with two possible states. Such a model can be realized with a single intelligent unit, such as a diode or a neuron.

$$if\ x\ \in\ r_a: t = True\ (\text{"friendly"});\ else\ t =\ False\ (\text{"hostile"})$$

r_a being the viable or hospitable range of the observable x.

We consider a model M_1 of this type that maps the observable to the internal state mostly correctly, producing correct interpretation with the probabilities 0.9/0.1 in each of the ranges of the observable x; whereas the other model, M_2 fails to learn or has yet to, producing near-random responses. The information models M_1 and M_2 are described by the probability matrices (t, x), Fig.1.



M₁	Ha(bitable)	Ho(stile)
Friendly	0.90	0.10
Hostile	0.10	0.90

M₂	Ha	Ho
	0.60	0.40
	0.40	0.60

Fig. 1. A two-state intelligent system and information model.

Then, taking as an example the empirical distribution of the observables p_s = (0.7, 0.3) one can calculate the values of the fitness function for the models as: $F(M_1) = 2.28$; $F(M_2) = 1.26$. A model with the best fitness possible in this case, represented by a diagonal probability matrix has, under these settings, the maximum possible fitness value, $F(M_p)$ = 2.99. Indeed, it can be seen that the models that produce responses in a better correlation of the internal and external states have higher value of the information fitness.

The correctness of the mapping of the external states of the environment to the internal ones by the intelligent system is verified in empirical trials. Indeed, in the example above, let us presume that the systems can produce the simplest differentiated response *R*: "if the internal state is "friendly": remain in place; else, move in a randomly chosen direction for a random distance; repeat".

Then, in an environment that has sufficient prevalence of habitable regions the simple intelligent system described above, with a correct or "fit" information model described above can survive and even thrive. On the other hand, as easy to see, incorrect mapping of the external and internal states can be detrimental to its survival.

The characteristics of information fitness *F* and information model *M* of an intelligent system thus define its ability to produce effective responses to sensory stimuli in a given empirical environment incorporating all essential constraints discussed earlier. Then, the development of natural intelligence can be seen as a continuous process of "fitting" to the environment of the system, characterized by progressive increase in the value of the information fitness objective.

5.2 Evolving Natural Intelligence

In the framework of the constrained optimization and its connections to the architecture and training of learning models in natural environments outlined in this work, one question remains: how, through which processes could natural intelligent systems acquire the ability to process, interpret and utilize sensory data of increasing complexity in production of more complex and effective responses, solving the optimization problem in (2) in progressively more complex environments?

It can be noted that an emerging natural intelligent system may not and commonly, does not have detailed information about its sensory environment and for that reason, would have to acquire it through some process of interaction with the it. It may not be able to implement complex algorithms to seek the solution to the optimization problem for the same reason, or due to the essential constraints.

The two areas where nature is not constrained is the time and the number of trials. Rather than employing complex and often theoretically intractable approaches to multidimensional non-linear constrained optimization, the nature can choose the evolutionary path of incremental variations and adaptations with selection according to certain essential criteria.

As an illustration, let us first consider a practical example of simplest intelligent systems that can be feasible in natural environments. Possibly, the simplest intelligent system, with a single intelligent unit was used in the example in the preceding section. It is capable of mapping at least two intervals of a single sensory observable to a binary internal state. As was shown, even a simple system of this kind is capable of surviving and even thriving in certain simple sensory environments.

By adding one more intelligent unit, with a two-neuron information model, an intelligent system would acquire the ability to model significantly more complex single channel input distributions including linear functions; and/or approximate two-channel ones as logical functions and others [28].

An incremental adaptation of a different type: convolutional vectorization [29], a function or component that can translate visual signals to scale-invariant numerical vectors and vice versa can be considered a processing improvement that preserves higher information content of the sensory inputs; by combining it with a small number of intelligent units (from as low as 2-3, [11]), an intelligent system can acquire the ability to recognize some simple geometric shapes such as circles, triangles, and others . Clearly, an ability to conceptualize and differentiate even simple visual forms can be essential for survival of simple intelligent species.

The sequence of incremental adaptations in the architecture of artificial neural models was shown to produce a noticeable improvement in the ability to recognize characteristic types (concepts) in the visual data, up to complex visual forms such as handwritten digits [23]. The sequence of incremental architectural adaptations capable of conceptualizing sensory data from the simplest, binary variable mapping to complex visual data comprising up to 10^2 distinct conceptual states is illustrated in Fig.2.

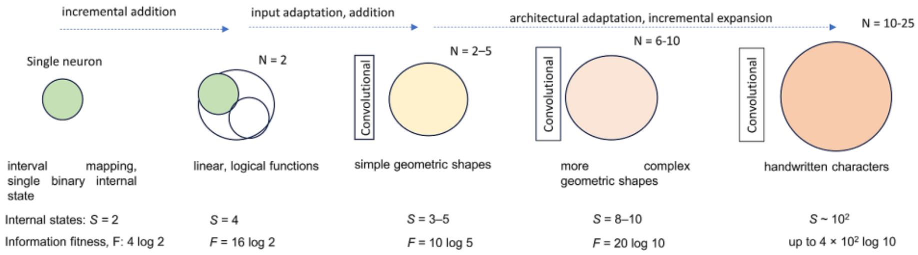


Fig. 2. Chain of incremental architectural adaptations characterized by improvement in information fitness; *N*: the number of intelligent units.

As shown below (16), on the assumption of the perfect mapping of the observable states to the internal ones, a chain of adaptations would have produced a significant improvement in the information fitness of the information models in Fig.2: from F ~ 2.77 for the simplest single intelligent unit system to F ~ 9.2 10^2 as a result of the sequence of architectural adaptations in this example.

On the basis of the comments made earlier on the initial state and the constraints of a developing intelligent system and the example of architectural adaptations above, it can be conjectured that a natural intelligent system can seek solutions to the optimization problem (2) via an evolutionary process of incremental adaptations in the space of parameters that define its structure (architecture) with selection by the objective and within the essential constraints of the problem.

5.3 Evolutionary Formalism of Natural Intelligence

Generally, an evolution can be defined by incremental variations with selection by the fitness to the objective until an optimum under the essential constraints is achieved. Then, it can be formally described a quad tuple of components $E_v = (A, V, R, K)$ [30] where:

- A: the space of possible states of the system which are formally describable, with explicit descriptions represented by certain variables { a };
- $V(a)$: a subspace of incremental variations of the states that are allowed within the constraints of the system;
- $R(a)$: a certain factor of selection that can obtained or calculated from the current state of the system and its interactions with the environment;
- the selection criteria (and the process) on the basis of the selection factor, $K(a)$.

An evolving population $P(t_0)$ = { p_k }, $p_k \in A$ at the initial point t_0 of the evolution can be described by a certain distribution of possible states of the system satisfying the constraints. Then, at the next evolutionary point t_1, the evolved population can be described as: $P(t_1)$ = { $p_k + v_k$}, $v_k \in V(p_k)$ with the range of the selection factor $R(t_1) = R(p_k + v_k)$.

$$R(t_1) = R(p_k + v_k) = R(p_k) + v_k \nabla R^k, \nabla R^k = \left.\frac{\partial R}{\partial v_k}\right|_{p_k}$$

In the evolutionary models both observed and studied, it is presumed the selection process K selects the individuals in the population with “better” characteristics of the selection factor, R. In the examples of natural systems considered earlier, individuals with more effective empirical responses can survive longer, expand to a larger area and so on.

In application to the problem of the natural intelligence, the evolution can be described by the following components: $A = \Lambda$, the architectural parameters of the information model; $V = \Delta(\lambda)$, the subspace of possible incremental variations of the architecture under the constraints of natural learning; the information fitness F of the information system defined earlier; and the empirical selection mechanism, K that selects the individuals in the population that achieve higher information fitness. Then the evolution process of natural intelligence can be defined as:

$$E_{nat} = (\Lambda,\ \Delta,\ F, K) \qquad (10)$$

Next, one can describe the necessary conditions of a successful evolutionary strategy as:

1. The solutions of the optimization problem, both global and local, exist.
2. At least in some points or regions in the system architecture space the subspace of incremental variations is not empty: $\exists\ \lambda: \Delta(\lambda)\ \neq\ \emptyset$.
3. Transitions between the local extrema λ, λ' of the objective function (2) are possible for some regions in the system architecture space: $\exists\ \lambda, \lambda': p_{tr}(\lambda, \lambda') > 0$.

where $p_{tr}(\lambda, \lambda')$: the probability of transition between the architecture states λ, λ' that satisfy the constraints of the problem.

Under these conditions, the evolutionary process just described can effectively traverse the system parameter space finding solutions with overall improvement in the fitness factor. The sequence of architectural adaptations discussed earlier in Section 5.2 demonstrated an example of such an evolutionary path or trajectory in the neural architecture space.

5.4 Evolutionary Dynamics of Natural Intelligence

In the framework defined in the preceding section let us consider the probability distribution $p(F)$ of a certain information model described by architectural parameters λ by the information fitness it achieves (the F-distribution). It can be an actual distribution in the population of intelligent systems of similar architecture, $p(F) = \frac{n(F)}{N}$, $n(F)$ being the sub-population with the fitness of F in the total population N, or the probability of an individual model to attain certain range of fitness, $p(F) = \frac{\partial P(F)}{\partial F}$ following training. Then, in the framework of the variational evolution defined by (10), the change in the fitness distribution as a result of an architectural variation $\delta\lambda$ at the next evolutionary point t_{k+1} can be written as:

$$\delta p(F, \delta\lambda, t_{k+1}) = \Xi(F, \lambda, \delta\lambda)$$

where $\Xi(F, \lambda, \delta\lambda)$ is the adaptation functional that defines the relation between the architecture and the fitness factor; it is therefore, specific to the problem.

Next, let us consider the subspace of all possible incremental variations at a given architecture point λ: $\Delta_f(\lambda)$. Then, the sum of the F-distributions of the possible architectural variations at λ is:

$$\delta p(F, \lambda, t_{k+1}) = \sum_{\Delta_f} \Xi(F, \lambda, \delta\lambda)\, q(\lambda, \delta\lambda), \qquad (11)$$

$q(\lambda, \delta\lambda)$: the probability of the incremental variation $\delta\lambda$ at λ.

The architecture described by λ will be defined as "adaptable", if $\exists\, F \geq p(F)\sim :$ $p(F) = 0;\ \ \delta p(F, \lambda, t_{k+1}) > 0$. Then, $\exists\, \delta\lambda$: $\delta p(F, \delta\lambda, t_{k+1}) > 0$; the variation $\delta\lambda$ is then defined as an architectural adaptation.

The role of the selection function *K* in (10) is to select or "prune" the evolved distribution (11) according to some existential objective or the imperative. In the framework described here it can be the improvement in the information fitness of the evolved distribution that can be defined in different ways in a specific problem. Then, only the variations that satisfy the criteria imposed by *K* in the subset of variations $\Delta_a(\lambda)$ are selected in the resulting distribution. As a result,

$$\delta p(F, \lambda, t_{k+1}\,) = \sum\nolimits_{\Delta_f} \Xi(F, \lambda, \delta\lambda)\, q(\lambda, \delta\lambda)\, K(\delta\lambda) = \sum\nolimits_{\Delta_a} \Xi(F, \lambda, \delta\lambda)\, q(\lambda, \delta\lambda) \quad (12)$$

The last equation of the evolutionary dynamics presumes that the selection process eventually propagates the evolved distribution to the general one:

$$p(F, \lambda',\, t_{k+1}\,) = \delta p(F, \lambda, t_{k+1}\,);\ \Delta_a \neq \emptyset \quad (13)$$

where $\lambda' \in\ \Delta_a$: the new "standard" architecture that was selected in the process of architectural adaptation. As a result of transition (13), the distribution *p*(*F*) shifted toward higher values of the information fitness by locating a new local maximum of the optimization problem (2) in the architecture space, λ' while within the constraints of the problem.

An equivalent formulation of the dynamical equations (11) – (13) in terms of the populations of individuals can be given straightforwardly.

5.5 "Efficient Complexity": Variability Balance in Evolving Natural Intelligence

As stated earlier in Section 5.1 the necessary condition of successful transition to a higher fitness can be written as:

$$\exists\, \lambda: \Delta_a(\lambda) \neq \emptyset, |\, \Delta_a(\lambda)\, | > 0 \quad (14)$$

that implies: $\left| \Delta_f(\lambda) \right| > 0$, as $\Delta_a(\lambda) \subseteq \Delta_f(\lambda)$

Whereas a closer analysis of the explicit form of $\Xi(\delta\lambda)$ in application to intelligent systems will be considered in a dedicated study, it can be noted that a success of evolution described in (11)–(13) is strongly determined by the volume or "richness" of the subspace of incremental variations, Δ_f. Indeed, a small, "narrow" variational space may suppress the variance in the distribution $\delta p(F, \lambda)$ reducing the potential fitness gain from architectural adaptations.

As was noted previously, in the context of the optimization problem (2), architectural adaptations of an evolving natural intelligent system correspond to the local minima of the Lagrangian of the information fitness optimization problem (5). Then, the essential condition of the evolution (10), 1–3 for a natural system can be interpreted as nontrivial constraints imposed on the architecture by the ability to migrate between the local extrema of the objective function.

Indeed, assuming that there exists a characteristic scale, L_{ex} that represents the characteristic minimal distance in the architecture space between the adjacent extrema of the objective function (2); this factor is essentially, dictated by the characteristics of the environment and can be thought of as a characteristic scale of the extrema network of the problem. Let H_{ex} be the cumulative energy cost (including all material constraints) of the transition from the current architecture λ to the adjacent minima of the objective function λ'. Then, the necessary conditions of evolution (10), 1–3 require, simultaneously, the space of incremental variations $\Delta_f(\lambda)$ to be sufficiently "large" to contain the new minima positions of the objective function; while at the same time, be "efficient" with respect to the energy required for the incremental change of the architecture, satisfying both the constraints of the problem (the weaker constraint) and the feasibility of the transition to the new state (the stronger constraint), $p_{tr}(H_{ex}) > p_{min}$.

While an explicit formulation of this condition requires a detailed analysis of the transition probability p_{tr} and will be attempted in another work due to a limitation of the format, it can be interpreted generally as "*efficient complexity*": maximizing the variability of the architecture while minimizing the energy cost of architectural variations, or maximizing the energy gradient of architectural adaptations:

$$G_H(\lambda) = \frac{\partial \lambda}{\partial H} \rightarrow max \tag{15}$$

A natural way to realize such a solution in naturally feasible systems would be to construct them from small intelligent units with a minimal energy cost so that incremental architectural variations can result in non-trivial improvements in learning (as illustrated in Section 5.2). It can be observed then that the ubiquity of neural architectures in natural intelligence may be due to more than just an effective type of intelligent model/strategy but *a natural solution to the constraints of evolvability*. The condition (15) then assures the ability of the learning system to traverse ("hop over") the extrema network of the optimization problem along a trajectory of solutions leading to higher information fitness.

It can be noted in the conclusion, that a challenge for a natural system can be in identifying the successful variations i.e., adaptations in the subspace of possible ones, $\Delta_a(\lambda) \subseteq \Delta_f(\lambda)$. It is not a trivial task as it follows from the definitions of the information fitness and model (7), (9) that empirical trials are required to determine the effectiveness of specific architectural variations. For this reason, advanced intelligent systems can invent complex mechanisms to simulate the empirical effectiveness of the intended actions and select the ones with a higher chance of empirical success.

5.6 A Taxonomy of Incremental Adaptations

In the preceding section we introduced the framework of description of the evolution of natural intelligence via incremental variations of the architecture of the information models in general theoretical terms. Here, we will consider such variations in the closer detail.

First, one can introduce a geometric description of the architecture space where the architectures λ that satisfy the essential constraints in (8) correspond to the points on a certain manifold in the space of parameters that define the information model, $\mu_{v,a}$ (9). Then, different types or classes of incremental variations in the architecture space that can produce an improvement in the fitness can be described. In the discussion that follows we will make a distinction between *variations*, that are incremental changes in the architecture that are allowed by the constraints of the problem, and *adaptations*: the variations that have been verified empirically to produce superior result of the optimization, i.e., the information fitness; thus, the subspace of adaptations is a subset of that of architectural variations.

Arguably, the simplest type of architectural adaptation is training ("T"-type), which affects only the mutable parameters of the information model; in this process they are adjusted to maximize the objective function (2). In this process, the meta-description of the model, that is, the types of the training and architectural parameters of the model remains unchanged, and only the values of the trainable parameters participate in incremental variations to achieve the optimum of the objective function.

Experiments and empirical experience show that training with practical implementations of posterior-based methods with fixed architecture, including neural networks, natural and artificial, can produce distributions of individual information models in a certain range of accuracy (that is directly related to the information fitness as defined in this work) whereas some methods produce fully deterministic outcome, while training with similar sets of observables. Then, there is an upper limit of the value of the information fitness that can be achieved through this type of adaptation.

To overcome this ceiling, another type of variation: the architectural ones can be employed. It allows modification of the architectural parameters that are considered immutable in the T-type of adaptations, in an incremental way. In the example considered earlier, a sequence of incremental changes in the architecture of generative neural models was instrumental in successful learning of the conceptual structure in the visual data of progressively higher complexity. In the geometric interpretation introduced above, these adaptations can happen in the subspace of architectural variations that is orthogonal to the T-type.

Architectural Adaptations

Architectural adaptations themselves can be of different types. In one case, the sets of internal and observable variables are unchanged, but the model can attain a better fitness between the observable and internal states via an improvement in the architecture. In the example in Section 4.4, such an improvement can produce an increase of the fitness factor with no change in the sets and ranges of the internals and observables. This type of adaptation can be designated "A_f", "fit-only". More examples of this type of architectural adaptations can be seen in the classification competitions aiming at achieving the best accuracy on the same set of data representing a sampling of a realistic visual environment, though not all of them can be classified as incremental.

A different type of architectural variation: A_v was illustrated in Fig.2 where incremental architectural adaptations in the embedding layer of a generative neural network model resulted in the ability to learn a greater variety of concepts, i.e., essentially, distinct external states in the visual data that modeled simple sensory environments. Such adaptations can lead to an extension of the set of the identifiable observable states and the internal states of the model, i.e., a more detailed and precise information model of the sensory environment.

Indeed, it is easy to see that an addition of any number of non-detectable factors and/or states that have no effect on the internal state of the system does not change the value of the mutual information / information fitness function. At the same time, an addition of new observable state(s) that are correlated with new internal states of the system can increase the value of the fitness factor. In the simplest case, where N distinct states of the observables are perfectly matched to the same number of the internal states, the information fitness definition (4) yields (with the information model $P_m(t, x)$ a unitary diagonal matrix, $p_s(x) \sim p_i(t) \sim 1/N$):

$$F(N) = \sum_i^N \log \frac{1}{1/N^2} = 2N \log N \qquad (16)$$

i.e., monotonous growth of the information fitness with the cardinality of the set of the "fitted" sensory states. This result can be interpreted in the common terms as: the more essential characteristics or features an intelligent observer can detect in its environment, the closer it can fit to it.

A similar outcome, that is, an expansion of the set of the identifiable observable states via addition of new observable factors (A_o) such as new sensory channels. In the nature, this type of adaptation can happen for example, via an accidental copying of an existing sensory channel with an incremental modification. The result can be an acquisition of an additional, parallel sensory channel (such as color vision) that can produce a more detailed description of the sensory environment and an opportunity to identify more distinct states (concepts) in it. Then, the process of fitting them to the internal states via adaptations A_v and T can again achieve superior fitness via more precise information modeling of the environment. This observation aligns with the potential of the "multiple view" approach discussed in [13].

Finally, a different yet type of incremental adaptation, A_c can be described that is related to the relaxation of the essential constraints on the learning system. Suppose a system has developed more efficient architecture of the information model that can be operated within a lower requirement of energy or memory. Then each solution of the optimization problem with the new architecture would satisfy the original constraints, but the opposite is not necessarily true. This can be interpreted as an effective expansion of the solution space that can produce new local maximums of the objective function that were not achievable with the previous iteration of the architecture.

Different types of incremental variations in the parameter space of a natural learning model are illustrated in Fig. 3.

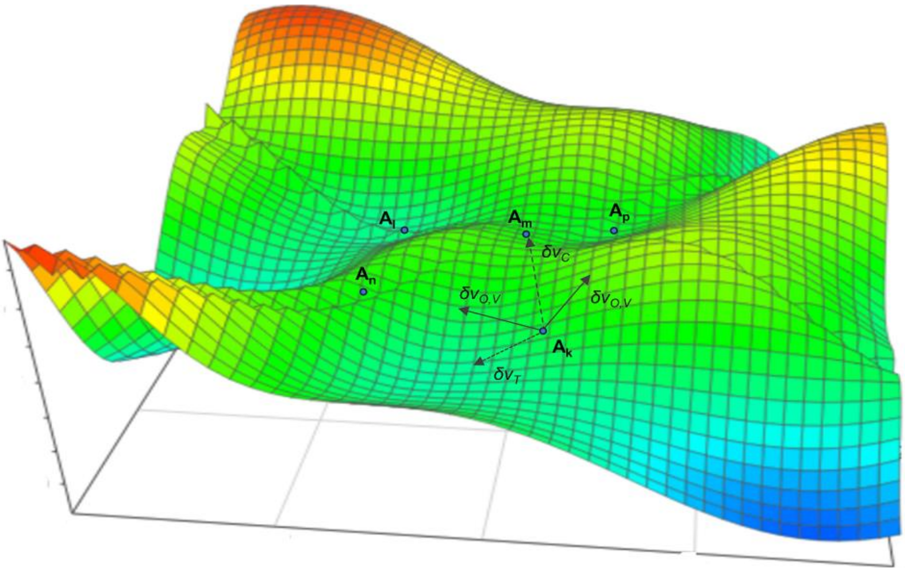


Fig. 3. Incremental variations in the architecture parameter space and the extrema network.

Thus, via the process of incremental variation with selection by the fitness described in this section, a sequence of architectural adaptations can attain progressively higher values of the information fitness to the sensory environment.

5.7 Collective Intelligence and Information Fitness

The concept of information fitness can be applied to groups or ensembles of natural intelligent systems. For example, it can be instrumental in explaining the effectiveness of the communication strategy widely used by social natural species. Indeed, let us consider a simple hypothetical population of information models P of size N of a similar architecture with the fitness distribution $p(F)$ in the range $R_f = (F_{min}, F_{max})$, $F_{min} \leq F(x) \leq F_{max}$, x: an individual model in the population. Suppose $F_a = mean(F(P))$, $F_x = max(F(P))$, the mean and the maximum value of the information fitness in the population.

Then, the total information fitness of the population at some point t can then be written as:

$$F_t(P) = \sum_P F(x) = N\, F_a$$

Next, suppose the population has developed an effective communications strategy that can transfer the information from the best fit information models to the others in the population via transmission of information or "instruction":

$$F(x, t) = C(x, x_{mx}) \sim F_x,$$

where x, x_{mx}: a common and the best sample in the population; C: communication, or instruction function.

Then, the new value of the total population fitness F_{nt} at the next point can be found as:

$$F_{t+1}(P) \sim N\, F_x(t) = F_t + N\,(F_x - F_a) = F_t + N\, G_{com} \quad (17)$$

where $G_{com} \sim F_x - F_a$: the average individual fitness gain in the population due to communication/instruction.

Then, the improvement in the total fitness of the population due to communication can amplified by up to the size of the population. There are numerous examples of this behavior exhibited by natural biological systems.

An intriguing question then is how such an effective strategy could have emerged in the natural evolution of intelligence? Due to limitation of the scope, this discussion will be deferred to another study.

6 Discussion

The connections between the principles of constrained optimization, classical approaches in the theory of natural learning systems and evolutionary processes based on traversal of the extrema landscape of the optimization problem as a strategy for finding the solution, or rather a progression of solutions increasingly approximating the fitness or “adaptation” to the environment examined in this work can be instrumental in a number of ways.

From the theoretical perspectives, the results presented here offer a general conceptual and formal mathematical framework for the description of evolving natural intelligent systems. While some interesting initial results have been presented, such as the substantiation of the effectiveness of neural architectures in natural intelligent by the principle of efficient complexity and the classification of incremental adaptations, other qualitative and quantitative results can be expected from following this formalism.

The framework developed in this work can describe different approaches in natural learning such as generative learning discussed in Section 4. The result on geometric conceptualization of generative representations can offer insights into the ability of early intelligent systems to develop effective differentiated behaviors based on grouping similar stimuli into general types or concepts that require similar response. The ability to conceptualize sensory stimuli can be critical for natural intelligent systems in meeting the essential physical constraints in the search for optimal architectural solutions.

As mentioned earlier, information-based approaches in the theory of natural intelligence have been developed over a long period since the classical works of Shannon, Shroedinger and others. To discuss or even mention all related studies and directions in one section may not be feasible. For this reason, we will limit the discussion to several studies directly related to the subject and scope of this work.

The “information bottleneck” method based on constrained optimization was developed in [20] with the constraint interpreted as the rate distortion function or factor, i.e., in a fundamentally different perspective from the interpretation used in this work. As well, the framework and results of the study apply mostly to the case of learning with classes or categories known at prior (i.e., supervised learning). As was commented, this is rarely the case with natural intelligent systems that cannot rely on prior information and have to “extract” concept structure from their sensory interactions with the environment.

In [13], the problem of compression of data in unsupervised learning processes and approaches from the information-theoretical and practical perspectives were examined in depth. An observation directly related to the scope of this work is the result on the absence of a natural level or limit of compression in unsupervised learning, in contrast to the supervised case. Then, the practical level of compression is dictated by the essential constraints of natural learning, primarily, those on the memory and computing power that are related directly to the ability of the systems to produce effective differentiated responses via conceptualizing sensory stimuli into manageable frameworks of general types, concepts or external states that can be associated with similar responses.

Studies [17,18] among others, used a related approach by defining the objective function in terms of the information entropy. However, they appeared to focus on the maximization of the objective function (entropy), while leaving aside the physical, material constraints of the problem. As we attempted to show in this work, physical constraints, including the critical ones of memory, compute and energy/resources can be of the utmost importance for emerging and developing natural intelligent systems.

Clearly, an unconstrained problem can have essentially different solutions from the strongly constrained one; in fact, even the global minimum of the unconstrained objective function may not satisfy certain practical material constraints. As noted in the introduction section, natural biological systems are by their nature, open allowing strong flow of both energy and materials; in that setting, the interactions and constraints of energy and materials on the natural systems cannot be ignored. The importance of material, specifically energy constraints of biological intelligent systems is supported by experimental results such as the “critical power law” in neuroscience, the tradeoff between the accuracy of coding and the energy it requires [31].

For these reasons we believe that the constrained optimization formalism proposed in this work would offer a more accurate description of the states and evolutionary trajectories of progressing natural intelligent systems. Further to its advantage, it is based on the established principles of statistics and information science and does not introduce any new essential assumptions or postulates.

From the more practical point of view, the analysis of necessary conditions of the evolution of natural intelligence, including the principle of “efficient complexity” that is, minimization of the energy cost of architectural adaptations (actually a corollary of the proposed evolutionary formalism) can provide a basis for further quantitative studies into the models and architectures of evolving intelligence.

The concept and framework of analysis based on the information fitness can be instrumental in examination of collective intelligence and intelligent collective behaviors as discussed in Section 5.7; an immediate observation being that behaviors that can effectively propagate or share more effective information models can trigger a sharp increase in the information fitness in a collective of learners. In a related perspective, recent results point at a possible connection between the conceptualization abilities of generative learning systems and a development of collective intelligent behaviors such as the ability to share interpretations of sensory stimuli via symbolic communications [32].

All in all, there seems to be a wide range of direction of research in both theory and applied models of evolving intelligence within the formalism proposed in this work.

Disclosures

This research has not received any specific funding.

The authors declare no conflicts of interest.

References

1. Shannon, C.E.: A mathematical theory of communication. Bell System Technical Journal 27(3), 379–423 (1948).
2. Shroedinger, E.: What is life? The Physical aspect of the living cell. Cambridge University Press, 1944.
3. Fisher, A., Igel, C.: Training restricted Boltzmann machines: an introduction. Pattern Recognition, 27 25–39 (2014).
4. Hinton, G., Osindero, S., Teh Y.W.: A fast-learning algorithm for deep belief nets. Neural Computation 18(7), 1527–1554 (2006).
5. Welling M., Kingma DP.: An introduction to variational autoencoders. Foundations and Trends in Machine Learning, 12(4), 307–392, 2019.
6. Liu, W., Wang Z., Liu X. et al.: A survey of deep neural network architectures and their applications. Neurocomputing 234 11–26 (2017).
7. Roberts A., Engel J., Raffel C. et al.: A hierarchical latent vector model for learning long-term structure in music. In: 35th International Conference on Machine Learning, Proceedings of Machine Learning Research 80 4364-4373 (2018).
8. Le, Q.V., Ransato, M. A., Monga, R. et al.: Building high level features using large scale unsupervised learning. In: 29th International Conference on International Conference on Machine Learning ICML'12, 507–514 (2012).
9. Higgins, I., Matthey, L., Glorot, X., Pal, A. et al.: Early visual concept learning with unsupervised deep learning. arXiv 1606.05579 (2016).
10. Xiong S., Tang, Y., Wang G.: Explore visual concept formation for image classification. Proceedings in Machine Learning Research 139 11470–11479 (2021).
11. Dolgikh, S.: Topology of conceptual representations in unsupervised generative models. In: 26th International Conference on Information Society and University Studies, Kaunas, Lithuania CEUR-WS.org 2915, 150–157 (2021).
12. Bengio, Y., Courville, A., Vincent, P.: Representation Learning: a review and new perspectives. IEEE Transactions on Pattern Analysis and Machine Intelligence 35 1798–1828 (2012).
13. Shwartz, ZR, Le Cun, Y.: To compress or not to compress - self-supervised learning and information theory: a review. Entropy 26 252 (2024).
14. Yoshida, T., Ohki, K.: Natural images are reliably represented by sparse and variable populations of neurons in visual cortex. Nature Communications 11, 872 (2020).

15. Bao, X., Gjorgiea, E., Shanahan, L.K. et al.: Grid-like neural representations support olfactory navigation of a two-dimensional odor space. Neuron 102 (5), 1066–1075 (2019).
16. Wicken, JS: A thermodynamic theory of evolution. Journal of Theoretical Biology, (87)1, 9–23 (1980).
17. Frank, SA: The common patterns of nature. Journal of Evolutionary Biology, 22(8) 1563–1585 (2009).
18. Vanchurin, V., Wolf, YI, Koonin, EV, Katsnelson, MI: Thermodynamics of evolution and the origin of life. Proceedings of the National Academy of Science, 119(6) e2120042119 (2022).
19. Ortega P.A., Braun D.A.: Thermodynamics as a theory of decision-making with information processing costs. Proceedings of the Royal Society A, 469 (2153) (2012).
20. Tishby, N., Pereira, F.C., Bialek, W. The information bottleneck method. In: 37th annual Allerton Conference on Communication, Control, and Computing, 368–377 (1999).
21. Milstein, A.D., Yiding L., Bittner, K.C. et al: Bidirectional synaptic plasticity rapidly modifies hippocampal representations. eLife 10, e73046, 770–778 (2021).
22. Wagarachchi, M., Karunananda, A.: Optimization of artificial neural network architecture using neuroplasticity. International Journal of Artificial Intelligence 15 (1) 112–125 (2017).
23. Dolgikh, S.: From Data to Model: evolutionary learning with generative neural systems. In: Proceedings of the 14th International Conference on Soft Computing and Pattern Recognition (SoCPaR 2022). Lecture Notes in Networks and Systems, 648. Springer, Cham 729–739 (2022).
24. Kuhn, H. W., Tucker, A. W.: Nonlinear programming. In: Proceedings of 2nd Berkeley symposium. Berkeley, University of California Press 481–492 (1951).
25. Tipping, ME: Bayesian inference: an introduction to principles and practice in machine learning. In O. Bousquet, U. von Luxburg, and G. Ratsch (Eds.), Advanced Lectures on Machine Learning, 41–62. Springer (2004).
26. Scaman, K., Virmaux, A.: Lipschitz regularity of deep neural networks: analysis and efficient estimation. In: Proceedings of the 32nd International Conference on Neural Information Processing System (NIPS) 3839–3848 (2018).
27. Kriegel, HP, Kroger, P., Zimek, A.: Clustering high-dimensional data: A survey on subspace clustering, pattern-based clustering, and correlation clustering. ACT Transactions on Knowledge Discovery from Data (TKDD) 3(1) 1–58 (2009).
28. Crossley, M., Staras, K., Kemenes, G.: A two-neuron system for adaptive goal-directed decision-making in Lymnaea. Nature Communications 7 11793 (2016).
29. Le, Q.V.: A tutorial on deep learning: autoencoders, convolutional neural networks and recurrent neural networks. Stanford University, 2015.
30. Mitchell, M.: An Introduction to Genetic Algorithms. Cambridge, MIT Press p. 41 (2014).
31. Tatsukava, T., Teramae, J-n.: Energy-information trade-off makes the cortical critical power law the optimal coding. arXiv 2407.16215 (2024).
32. Dolgikh, S.: Generative conceptual representations and semantic communications. International Journal of Computer Information Systems and Industrial Management Applications, 14 239–248 (2022).