1st Way Solution of Collatz Sequence

$Abstract$:

$1 - let\ \ n, x, y, z, t, u, v, k, r, \ldots \in N_+$

$let\ x \in S(n), \&\ x \in lS(n) = loop\ of\ Collatz\ Sequence\ (n),$

$i)\ I\ proved\ lS(n) = \emptyset, \{x\}, \{x, y\}, \{x, y, z, t\} = \cdots = \{x, y, z, t, \ldots, v, \ldots\}\ is\ false$

$ii)\ I\ proved\ lS(n) = \{x, y, z\} = \{1,2,4\} = \{1,2,4,1,2,4\} = \cdots = \{1,2,4,1,2,4, \ldots, 1,2,4\}\ is\ true$

$2 - I\ created\ special\ sketch\ to\ find\ algebra\ expresions\ equal\ to\ x.$

$3 - I\ created\ special\ Table\ to\ find\ algebra\ expresions\ of\ elements\ in\ the\ lS(n).$

$4 -\ \ I\ created\ pattren\ to\ find\ value\ of\ x \in\ lS(n).$

$5 -\ \ Observation: let\ r = number\ of\ elememts\ in\ lS(n), therefore:$

$i)\ if(\ r = 3k\ ) \Rightarrow x = 1\ \Rightarrow lS(n) = \{4,2,1\}, \forall n\ when\ r \in \{3,6,9,12, \ldots\}$

$ii)\ if\ (r = 3k + h; h = 1, or\ h = 2\ ) \Rightarrow x \notin N_+ \Rightarrow lS(x) = \emptyset, , \forall n\ when\ r \notin \{3,6,9,12, \ldots\}$
$\because N_+ = \{r = 3k\} \cup \{r = 3k + h\}$

$lS(n)_{\mathrm{n}\in N_+} = lS(n)_{\mathrm{r}\in\{r=3k\}} \cup lS(n)_{\mathrm{r}\in\{r=3k+h\}}$

$\therefore lS(n)_{\mathrm{n}\in N_+} = \{1,2,4\} \cup \emptyset = \{1,2,4\}, \forall n \in N_+$

$Proof$:

$let\ \ \ x, y, z, t, k, r, \ldots \in N_+$

$Collatz\ Sequence: let\ lS(n) = loop\ of\ n, \&\ \ S(n) = \left\{\frac{n}{2} or\ (3n + 1), \ldots, ?\right\}$

$I\ have\ to\ prove\ \forall n \in N_+\ , lS(n) = \{4,2,1\}$

Pattern of Collatz Sequence's Loop, $r \in N_+, r\ is\ number\ of\ elements\ in\ a\ loop, x \in N_+,$

$x \in S(n), \&\ x \in\ lS(n)$

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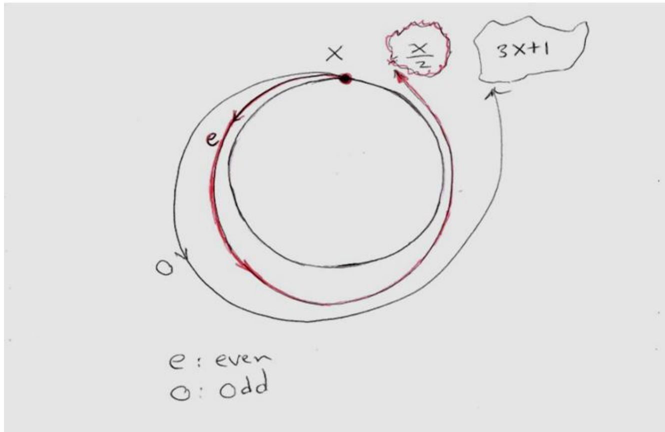
$A. \, let \; lS(n) = \{x\}, when \; x \in S(n) = \{x, \ldots, ?\}$



$lS(n) Collatz \; Sketch \; to \; find \; equivalent \; expresion \; (Cloud) \; to \; x$

$Cloud$	$Cloud = x$
$\frac{x}{2}$	$\frac{x}{2} = x \Rightarrow x = 0 \notin lS(\mathrm{n})$
$3x + 1$	$3x + 1 = x \Rightarrow x = -\frac{1}{2} \notin lS(\mathrm{n})$

$Chart \; to \; Find \; x \; number \; values$

$\therefore lS(\mathrm{n}) \neq \{x\}, \forall n \in N_+$

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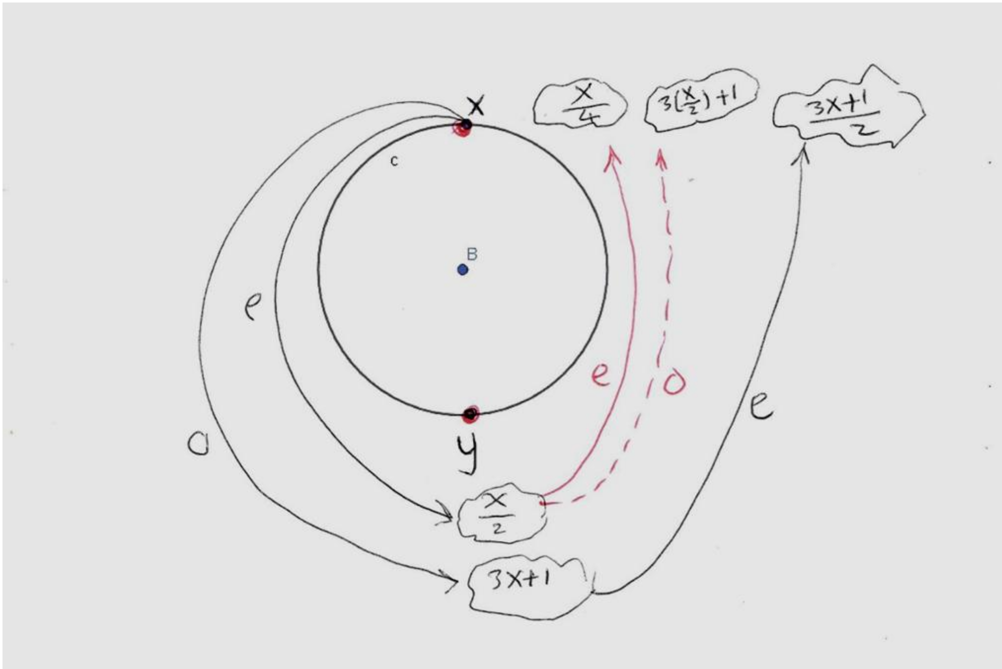
$B. let\ lS(n) = \{x, y\}, \&\ \forall n \in N_+, x, y \in S(n) = \{x, y, \dots, ?\}$



$lS(n) Collatz\ Sketch\ to\ find\ equivalent\ expresion\ (Cloud)\ to\ x$

$Cloud$	$Cloud = x$	y
$\frac{x}{4}$	$\frac{x}{4} = x \Rightarrow x = 0 \notin lS(n)$	….
$\frac{3x+2}{2}$	$\frac{3x+2}{2} = x \Rightarrow 3x + 2 = 2x \Rightarrow x = -2 \notin lS(n)$	….
$\frac{3x+1}{2}$	$\frac{3x+1}{2} = x \Rightarrow x = -1 \notin lS(n)$	….

$Chart\ to\ Find\ x, y\ number\ values$

$\therefore lS(n) \neq \{x, y\}, \forall n \in N_+$

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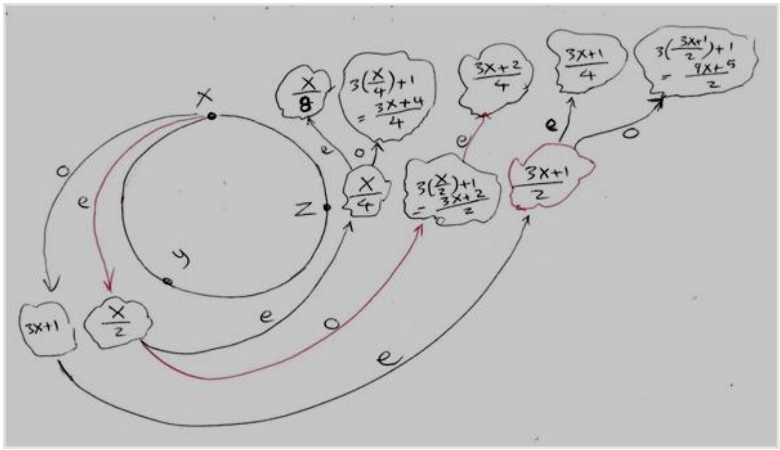
$C. let\ lS(n) = \{x, y, z\},\ \forall n \in N_+, x, y, z \in lS(n)$



$lS(n) Collatz\ Sketch\ to\ find\ equivalent\ expresion\ (Cloud)\ to\ x$

$Cloud$	$Solve\ equation: Cloud = x$	y	z
$\frac{x}{8}$	$\frac{x}{8} = x \Rightarrow x = 0 \notin lS(n)$	---	---
$\frac{3x+4}{4}$	$\frac{3x+4}{4} = x \Rightarrow x = 4 \in lS(n)$	$\frac{x}{2} = 2$	$\frac{x}{4}$ =1
$\frac{3x+2}{4}$	$\frac{3x+2}{4} = x \Rightarrow x = 2 \in lS(n)$	$\frac{x}{2} = 1$	$\frac{3x+2}{2} = 4$
$\frac{3x+1}{4}$	$\frac{3x+1}{4} = x \Rightarrow x = 1 \in lS(n)$	$3x + 1$ =3(1)+1=4	$\frac{3x+1}{2}$ =2
$\frac{9x+5}{2}$	$\frac{9x+5}{2} = x \Rightarrow x = -\frac{5}{7} \notin lS(n)$	---	---

To Find Equivalents Expressions (Clouds) to 1st Element (x) of the Collatz Loop

$\therefore lS(n) = \{x, y, z\} = \{4,2,1\}, \forall n \in N_+$

$\therefore S(n) = \left\{\frac{n}{2} or\ (3n+1), \dots, 4,2,1\right\}, \forall n \in N_+$

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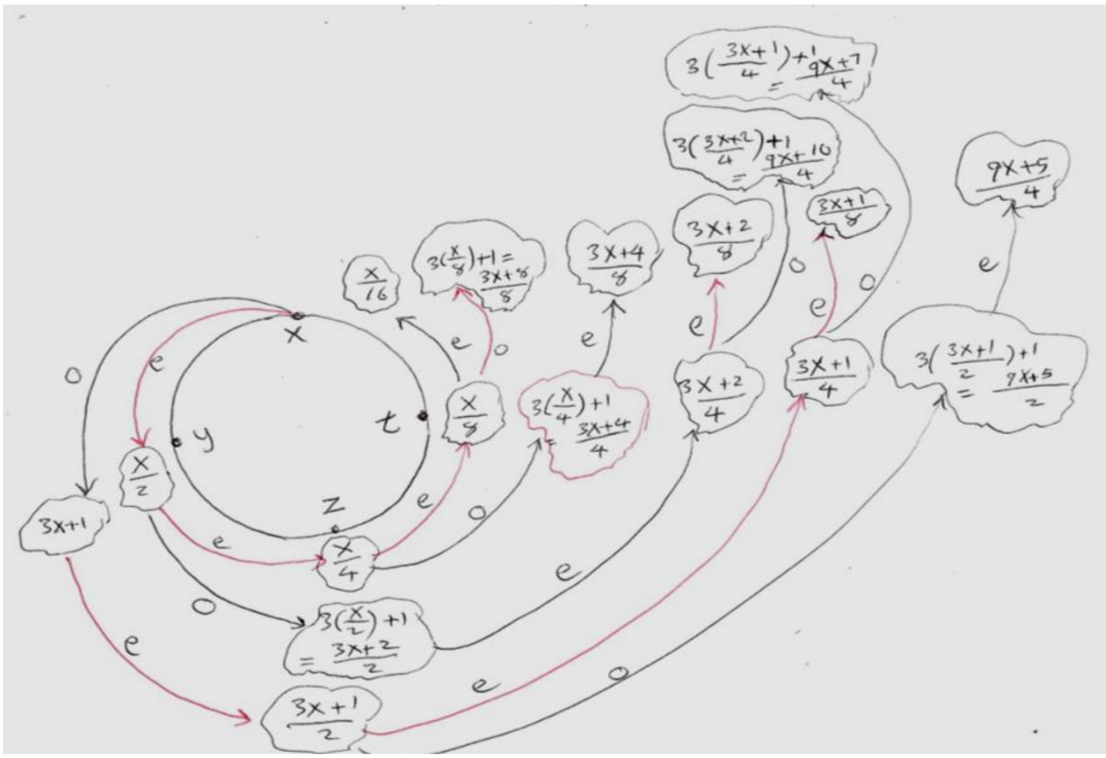
$D. let\ lS(n) = \{x, y, z, t\}, \&\ \forall n \in N_+, x, y, z, t \in S(n)$



$lS(n) Collatz\ Sketch\ to\ find\ equivalent\ expresion\ (Cloud)\ to\ x$

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$Cloud$	$Solve\ equation: Cloud = x$	y	z	t
$\frac{x}{16}$	$\frac{x}{16} = x \Rightarrow x = 0 \notin lS(n)$	---	---	---
$\frac{3x+8}{8}$	$\frac{3x+8}{8} = x \notin lS(n)$	---	---	---
$\frac{3x+4}{8}$	$\frac{3x+2}{4} = x \notin lS(n)$	---	---	---
$\frac{3x+2}{8}$	$\frac{3x+1}{4} = x \notin lS(n)$	---	---	---
$\frac{3x+1}{8}$	$\frac{9x+5}{2} = x \notin lS(n)$	---	---	---
$\frac{9x+10}{4}$	$\frac{9x+10}{4} = x \notin lS(n)$	---	---	---
$\frac{9x+7}{4}$	$\frac{9x+7}{4} = x \notin lS(n)$	---	---	---
$\frac{9x+5}{2}$	$\frac{9x+5}{2} = x \notin lS(n)$	---	---	---

$\Rightarrow lS(n) \neq \{x, y, z, t\}, \forall n \in N_+$

$Then\ I\ have\ to\ find\ a\ pattren\ and\ formula\ To\ get\ elements\ of\ lS(n)$

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Pattern of Collatz Sequence's Loop, $r \in N_+, r$ *is number of elements in a loop*, $x \in N_+$, $x \in S(n), \& \ x \in lS(n)$

$r = \#lS(n)$ x = e or odd   →

$r = 1$	→ $\frac{x}{2}$ = e or odd →	→ $3x + 1$ = e →

$r = 2$	→ $\frac{x}{4}$ =e or odd →	→ $\frac{3x+2}{2}$ =e →	→ $\frac{3x+1}{2}$ =e or odd →

$r = 3$	→ $\frac{x}{8}$ =e or odd →	→ $\frac{3x+4}{4}$ =e →	→ $\frac{3x+2}{4}$ =e or odd →	→ $\frac{3x+1}{4}$ =e or odd →	→ $\frac{9x+5}{2}$ =e →

$r = 4$	→ $\frac{x}{16}$ =e or odd →	→ $\frac{3x+8}{8}$ =e →	→ $\frac{3x+4}{8}$ =e or odd →	→ $\frac{3x+2}{8}$ =e or odd →	→ $\frac{3x+1}{8}$ =e or odd →	→ $\frac{9x+10}{4}$ =e →

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→ $\frac{9x+7}{4}$ =e →	→ $\frac{9x+5}{4}$ =e or odd →

$r = 5$	→ $\frac{x}{32}$ =e or odd *	→ $\frac{3x+16}{16}$ =e or odd $\$$	→ $\frac{3x+8}{16}$ =e or odd !	→ $\frac{3x+4}{16}$ =e or odd @	→ $\frac{3x+2}{16}$ =e or odd #	→ $\frac{3x+1}{16}$ =e or odd %
→ $\frac{9x+20}{8}$ =e @@	→ $\frac{9x+14}{8}$ =e &	→ $\frac{9x+11}{8}$ =e ((	→ $\frac{9x+10}{8}$ =e or odd {{	→ $\frac{9x+7}{8}$ =e or odd [[	→ $\frac{9x+5}{8}$ =e or odd **	→ $\frac{27x+19}{4}$ =e ***

$r = 6$	* $\frac{x}{64}$	* $\frac{3x+32}{32}$	$\$$ $\frac{3x+16}{32}$	! $\frac{3x+8}{32}$	@ $\frac{3x+4}{32}$	# $\frac{3x+2}{32}$	% $\frac{9x+19}{16}$
% $\frac{3x+1}{32}$	$\$$ $\frac{9x+64}{16}$	{{ $\frac{27x+38}{8}$	@ $\frac{9x+28}{16}$	# $\frac{9x+22}{16}$	% $\frac{3x+1}{32}$	$\$$ $\frac{9x+64}{16}$	! $\frac{9x+40}{16}$
@@ $\frac{9x+20}{16}$		& $\frac{9x+14}{16}$	(($\frac{9x+11}{16}$	{{ $\frac{9x+10}{16}$	[[$\frac{9x+7}{16}$	[[$\frac{27x+29}{16}$	** $\frac{9x+5}{16}$
** $\frac{27x+23}{8}$		*** $\frac{27x+19}{8}$					

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Table for Collatz Sequence Pattern

$r = 1 = 3(0) + 1$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{1-0}} \Rightarrow x = 1, but\ y = 4 \notin lS(n) \Rightarrow lS(n) \neq \{x\},\ \forall n \in N_+$
$r = 2 = 3(0) + 2$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{2-0}} \Rightarrow x = \frac{1}{3} \notin N_+ \Rightarrow x, y \notin lS(n) \Rightarrow lS(n) \neq \{\ x, y\},\ \forall n \in N_+$
$r = 3 = 3(1)$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{3-1}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{4{,}2{,}1\},\ \forall n \in N_+$
$r = 4 = 3(1) + 1$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{4-1}} \Rightarrow\ x \notin N_+, \forall n \in N_+$
$r = 5 = 3(1) + 2$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{5-1}} \Rightarrow\ x \notin N_+, \forall n \in N_+$
$r = 6 = 3(2)$ $k = 2$	$x = \frac{3^2 x + 3(2^0) + 2^2}{2^{6-2}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow\ lS(n) = \{x, y, z\},\ \forall n \in N_+$
$r = 9 = 3(3)$ $k = 3$	$x = \frac{3^3 x + 3^2(2^0) + 3^1(2^2) + 2^4}{2^{9-3}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{1{,}4{,}2\},$ $\forall n \in N_+$
$r = 12 = 3(4)$ $k = 4$	$x = \frac{3^4 x + 3^3(2^0) + 3^2(2^2) + 3(2^4) + 2^6}{2^{12-4}} \Rightarrow x = 1, y = 4, z = 2, \forall n$ $\in N_+$
$r = 15 = 3(5)$ $k = 5$	$x = \frac{3^5 x + 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8}{2^{15-5}} \Rightarrow x = 1, y = 4,$ Z=2 $\forall n \in N_+$

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$r = 3(k)$

$$let\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow x = 1$$

$Note\colon\ 3^{k-1}(\ 2^0), 2^2, 2^4, \dots, 2^{2(k-2)}, 2^{2(k-1)}, i, e\ 2^m, and\ m \in N = \{0,2,4,6,8,\dots\}$

$if\ r + 3 = 3(k+1),$

$$is\ x = \frac{3^{k+1}x + 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}}{2^{r+3-(k+1)}} = 1\ \ ?$$

$Proof\colon \left(2^{r+3-(k+1)} - 3^{k+1}\right)x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}$

$\left(2^{3k+3-(k+1)} - 3^{k+1}\right)x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k} \Rightarrow$

$(2^{2k+2} - 3^{k+1})x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k} \dots eq1$

$is\ (2^{2k+2} - 3^{k+1}) = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k} \Rightarrow x = 1?$

$let\ k = 4\ \Rightarrow\ (2^{10} - 3^5) = 781, \&\ 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8 = 781$

$\Rightarrow eq1\colon (781)x = 781 \Rightarrow x = 1 \Rightarrow\ y = 4, z = 2 \Rightarrow lS(n) = \{4,2,1\}, \forall n\ when\ r \in \{\ r = 3(k)\}$

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$if \;\; r = 3(k) + h, h < 3, k, h \in N_+$

$$let \; x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow x \notin N_+$$

$i)if \;\; r + 1 = 3(k) + 1$, *when h=1*

$$is \; x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r+1-k}} \Rightarrow x \in N_+?$$

Proof: $r + 1 = 3(k) + 2$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{3k+1-k}}$$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{2k+1}}$$

$$(2^{2k+1} - 3^k)x = 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)} \ldots eq2$$

$let \; k = 4$

$\Rightarrow eq2{:}\; (2^9 - 3^4)x = 3^3(2^0) + 3^2(2^2) + 3^1(2^4) + 2^6 \Rightarrow 431x = 175 \Rightarrow x \notin N_{+,}$

$\forall n \; when \; r \in \{r = 3(k) + 1\}$

$ii)if \;\; r + 1 = 3(k) + 2$, *when h=2*

$$is \; x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r+1-k}} \Rightarrow x \in N_+?$$

Proof: $r + 1 = 3(k) + 2$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{3k+2-k}}$$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{2k+2}}$$

$$(2^{2k+2} - 3^k)x = 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)} \ldots eq3$$

$let \; k = 4$

$\therefore eq3{:}\; (2^{10} - 3^4)x = 3^3(2^0) + 3^2(2^2) + 3^1(2^4) + 2^6 \Rightarrow 943x = 175 \Rightarrow x \notin N_{+,}$

$\forall n \; when \; r \in \{r = 3(k) + 2\}$

$\because N_+ = \{r = 3k\} \cup \{r = 3k + h\}$, *When h=1, or h=2*

$\therefore lS(n)_{n \in N_+} = lS(n)_{r \in \{r=3k\}} \cup lS(n)_{r \in \{r=3k+h\}}$

$\therefore lS(n)_{n \in N_+} = \{1,2,4\} \cup \emptyset = \{1,2,4\}, \forall n \in N_+$

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