The seven-dimensional universe

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Abstract:

This paper elucidates the concept of dark energy and the acceleration of the universe through the quantization of space in hidden dimensions, which provides the foundation for gravitational force. Space-time is conceptualized as a four-dimensional elastic grid within a seven-dimensional universe, wherein matter expands concurrently with the universe. Each cube of the grid is considered a quantum of hidden three-dimensional space of Planck volume containing Planck charge, rendering the universe seven-dimensional. The phenomenon of dark energy is attributed to the electrostatic repulsion between Planck charges in each quantum of the hidden space, and the corresponding energy density is correlated with the Hubble parameter H(z), which accounts for the accelerated expansion of the universe and the increase in the relative cosmological potential energy/rest mass of matter. The expansion of space-time is posited to result not from the creation of new space but from the stretching of existing space-time, analogous to an elastic ruler where proper length and volume remain constant. The relative values of the Planck constant, gravitational constant, permittivity of free space, and Boltzmann constant are demonstrated to vary due to the expansion or contraction of space in cosmological and gravitational frameworks, but are compensated for by the proportional change in the relative rest mass. This theory also establishes a framework for the relativistic Newtonian theory of gravity, the respective MONDian (Modified Newtonian dynamics) gravity, identifies a rationale for the transition of Newtonian gravity to MOND at a_0, and elucidates the dynamics of galaxy clusters without invoking dark matter.

1. Introduction

In this theoretical framework, Hubble expansion is attributed to the stretching of existing space-time rather than the creation of new space. This concept is analogous to the markers on an elastic ruler stretching along with the expansion of the ruler, as opposed to the raisin bread model, where matter does not expand along with space. Consequently, matter (e.g., protons, neutrons, electrons, and atoms) is posited to stretch uniformly along with space-time and not become diluted with the expansion of the universe. As matter also stretches with space, the expansion is non-observable locally, but can be detected through the redshift of light emanating from a distant space of lower stretch. Thus, matter exists in different energy states based on the magnitude of the space-time stretch. Matter continues to transition to higher potential energies as space-time expands, while the energy of the photon remains constant. Therefore, in a lower stretch space-time, a blue photon possesses less energy than a blue photon with the same frequency emitted in a higher stretch space-time. Consequently, when light travels from a lower stretch of space-time to a higher stretch of space-time, it becomes redshifted without violating the law of conservation of energy.

The cosmological potential energy of mass increases with space-time expansion. In contrast, the energy of the photon remains unaltered by space-time expansion and does not gravitate, as it does not resist space-time expansion due to its zero mass. However, it is affected by the gravity of other objects, indicating that the energy of a photon does not curve the space-time around it but follows a curved path around matter. In other words, within this theory, any entity that curves space-time can only travel at velocities less than the speed of light. Consequently, the mass-energy equivalence principle is only partially satisfied, as not all forms of energy curve space-time in this theory. However, this theoretical framework upholds the weak equivalence principle by maintaining the equivalence of passive gravitational mass and inertial mass.

This theory proposes an absolute inertial frame, although it cannot be uniquely identified, and reintroduces the concept of relativistic mass to be considered as part of the inertial or passive gravitational mass. However, in this theory, only the rest mass is regarded as the active gravitational mass, as the associated kinetic energy or momentum, if present, does not generate additional curvature in the space-time grid, but only gravitomagnetic effects. Nevertheless, a change in gravitational or cosmological potential energy is stored as a change in the relative rest mass, specifically due to the expansion or contraction of matter. Consequently, a higher stretch space-time requires more energy than a lower stretch space-time for the same work done, owing to the relative increase in the rest mass/potential energy. However, the numerical value of the locally consumed energy remains consistent for the work done in both cases because the local stationary inertial mass equals the rest mass in both instances, despite the difference in relative rest mass. Thus, in this theory, energy is considered relative rather than absolute but is still conserved with the expansion of the universe or the change in the gravitational potential.

The proper distance between any two points in space is considered to remain constant despite the stretching of space-time, analogous to the markers on an elastic ruler. Therefore, the volume of the universe does not change with space expansion. The accelerated expansion of space-time and matter is attributed to electrostatic repulsion between the Planck charges present in the Planck volumes, thereby providing an explanation for dark energy. Space-time is conceptualized as a four-dimensional elastic grid in a seven-dimensional universe, with each cube of the grid considered a quantum of the hidden three-dimensional space of the Planck volume containing the Planck charge. These cubes could potentially form loose connections with surrounding cubes to create black holes.

As the Hubble parameter H(z) decreases over time, the rate of acceleration of the universe also diminishes due to the contraction force of the space-time elastic grid opposing the electrostatic repulsion between Planck charges. The space-time membrane functions as a dielectric material between the Planck charges. Matter exists as a wave function (Ψ) exclusively in the four-dimensional space-time grid, but not in the hidden three spatial dimensions. The presence of matter in space-time would increase the force required to expand the space-time grid, as matter also expands concurrently with space. The presence of matter increases the permittivity of space-time, thereby reducing the electrostatic repulsion within the grid enveloped by the matter compared to the electrostatic repulsion outside the matter. As the expansion of the space-time grid surrounding the matter will be greater than the expansion of the matter due to the permittivity difference and the net compressing force on the matter from the surrounding Planck charges, space-time naturally becomes curved around any mass. In this theory, this compression force is considered to be the gravitational force. Therefore, in this theory, with the exception of photons, gravitational force is considered a real force rather than a fictitious force of the general theory of relativity.

In this theoretical framework, the electrostatic potential energy between Planck charges in the hidden three-dimensional space is considered equivalent to dark energy, causing accelerated expansion of the universe. The electrostatic dark energy in three-dimensional space does not gravitate because this energy itself is the cause of the gravitational force in the four-dimensional space-time enveloping the three-dimensional Planck charges. The constancy of the speed of light should not limit the apparent expansion velocity of the universe in this model, as it is considered applicable only for objects moving through space but not for the expansion of space-time. From the inception of the universe, the first law of thermodynamics is strictly adhered to in this theory to uphold the law of conservation of energy, which includes energy conservation in dark energy and cosmological redshift. Linear and angular momenta are also conserved in this theoretical framework.

This theory proposes a flat or zero-curvature, isotropic, and homogeneous universe, wherein the rate of acceleration continues to decrease proportionally to the age of the universe, and its velocity only approaches zero after an infinite amount of time. However, the universe continues to accelerate, albeit at a continuously decreasing rate, asymptotically approaching zero. The invocation of critical density ($\Omega_0 = 1$) or cosmic inflation is not necessary to explain the flatness in this model of the universe, as the uniform expansion of the entire universe due to electrostatic repulsion elucidates why the universe is flat rather than closed or open. As there is no increase in the volume of the universe with time, the concept of the Big Bang should be supplanted by that of Big Repulsion. This theory posits that gravity is electrostatic background dependent to explain the curvature of space-time, dark energy, and cosmological redshift. The electrostatic background provides an additional substrate to the quantum fields on top of the gravitational background, which necessitates an absolute inertial frame and universal time, which is the Hubble time or age of the universe in this theory. Consequently, this theory proposes a three-layered universe:

1) Electrostatic or power layer
2) Space-time or gravitational layer
3) Quantum fields layer

The first layer functions as the power or energy source for the universe. The gravitational layer generates gravity, and the quantum field layer enables matter, energy, and the rest of the fundamental forces of nature to operate on top of the space-time or gravitational layer. This theory also eliminates the cosmic event horizon and the cosmic scale factor (a) because the proper length and volume remain constant despite the expansion of space-time. Therefore, it is posited that we should be able to observe light from any part of the universe, given sufficient time, without any distance limits such as a cosmic event horizon. The cosmological constant Λ in the gravitational field equations to account for dark energy is not required in this model of the universe, as dark energy does not gravitate and is confined to the electrostatic background, which is treated independently of gravity.

The relative values of the Planck constant h, gravitational constant G, Boltzmann constant k, and permittivity of free space ε_0 are demonstrated to vary due to the expansion of the space-time grid, thus providing evidence for the existence of an absolute frame of reference. These constants also exhibit variation with gravitational potential. Nevertheless, the absolute inertial frame cannot be uniquely identified in this theory because the alteration in the physical constants is compensated by the change in the rest mass/potential energy. However, this theory necessitates an absolute inertial frame to elucidate dark energy, MONDian gravity, and the change in rest mass/potential energy resulting from space expansion/contraction. This theory upholds the special theory of relativity in local inertial frames, and consequently maintains the local invariance of the speed of light.

Black-hole singularities do not exist in this model of the universe, as the mass is transformed into informational entropy and stored on the surface of the black hole at the event horizon, and the space-time terminates at the event horizon. It cannot be extended beyond the event horizon, as in the general theory of relativity, since the proper length, time, and tangible mass become zero at the event horizon. Furthermore, charged black holes do not exist in this model of the universe as the permittivity of free space becomes infinite at the event horizon. As a result, stationary black holes can only possess two properties: informational (entropic) mass and angular momentum, thus only partially fulfilling the no-hair theorem. This theory also establishes a framework for the relativistic Newtonian theory of gravity, which is local, Lorentz-covariant, and gauge-invariant, with potential for further generalization. The relativistic Newtonian gravity proposed in this theory produces similar or identical results to the general theory of relativity for the following phenomena:

1) Schwarzschild radius
2) Innermost stable circular orbit (ISCO)
3) Innermost bound circular orbit (IBCO)
4) Photon sphere
5) Gravitational lensing
6) Perihelion precession of Mercury
7) Shapiro time delay

This theory generates relativistic Poisson equations and GEM (Gravitoelectromagnetism) equations or Maxwell-like equations for gravity that are applicable in strong gravitational fields and at relativistic velocities. These equations can elucidate the frame-dragging effect, orbital precession, geodetic effect, and gravitational waves. Furthermore, this theory produces the equivalent of the Schwarzschild and FLRW (Friedmann–Lemaître–Robertson–Walker) metrics utilizing the Minkowski space-time.

The universe expansion model proposed in this theory presents a plausible explanation for the transition of Newtonian gravity to MOND at a_0 ($\sim 1.2 \times 10^{-10}$ m/s^2), thereby presenting a case for the absolute reference frame. However, this theory posits that transitioning to a deep-MOND regime is feasible only in the radial space surrounding black holes, and not in the vicinity of ordinary matter. Additionally, this theory constructs a framework for MONDian gravity derived from the relativistic Newtonian theory of gravity and elucidates the dynamics of galaxy clusters without dark matter by employing an updated virial theorem. Moreover, this novel model could serve as a precursor to theories explicating baryogenesis and primordial nucleosynthesis, predicated on the interaction between the initial extremely high expansion energy of space-time and quantum vacuum fluctuations in the creation of matter.

2. Relativistic acceleration of space-time

Let us consider two points, A and B, on the space-time fabric. We can calculate the apparent outward acceleration of B when observed from A based on the redshift of the light coming from B.

D = Proper distance between A and B
λ = D (Let us consider the wavelength of light to be equal to D)

Thus, by the time light travels from B to A, its wavelength expands by D(1+z), based on the cosmological redshift (z) phenomenon. Therefore, B apparently should have moved from its original location by D(1+z)–D, which is equal to Dz.

The apparent velocity v of B due to the redshift is given by $v = \frac{\text{Distance}}{\text{Time}} = \frac{Dz}{t}$, where t is the time taken for the light to travel from B to A. Because the proper distance between A and B remains constant, the real velocity of B is zero. Thus, the apparent acceleration (a) is given by $a = \frac{Dz}{t^2}$. As space stretches like an elastic ruler, the proper distance between A and B should always remain the same, irrespective of the apparent acceleration. Therefore, light should take the same amount of time t to travel the apparent distance D(1+z), which is actually D owing to the constancy of the speed of light. Therefore, the time t is given by $t = \frac{D}{c}$.

$$v = \frac{Dz}{\left(\frac{D}{c}\right)} = zc \quad \text{and} \quad a = \frac{Dz}{\left(\frac{D}{c}\right)^2} = \frac{zc^2}{D}$$

$$v = zc \tag{1}$$

As v = zc (1) has already been established in conjunction with the Hubble law, the above derivation proves that space stretches like an elastic ruler, where the length and volume remain constant, as opposed to the raisin bread model, where length increases and matter is diluted with the expansion of space-time. Therefore, this theory provides a theoretical basis for v = zc, whereas the raisin-bread model does not provide any theoretical reasoning. Based on the equations v = H_0D and v = zc from Hubble law, $\frac{z}{D} = \frac{H_0}{c}$, where v is the apparent receding velocity, H_0 is the Hubble constant, D is the distance, z is the cosmological redshift, and c is the speed of the light. Therefore, the apparent acceleration $a = \frac{H_0 c^2}{c} = cH_0 =$ **7.55×10^{-10} m/s²**, which is the current acceleration of the universe for H_0 = 77.7 (km/s)/Mpc based on H_0 values within the range mentioned in references [1], [2], [3]. Therefore, the local acceleration of the universe is given below, where H(z) is the Hubble parameter.

$$a(z) = cH(z) \tag{2}$$

As the universe is stretching with a constant volume, there must be a length contraction and time dilation in the past, which are given below, where l(z) and t(z) are the proper length and time, respectively, at the cosmological redshift z, and l_0 and t_0 are the coordinate length and time, respectively, as measured from the current cosmological reference frame.

Cosmological length contraction $l_0 = \frac{l(z)}{(1+z)}$, cosmological time dilation $t_0 = t(z)(1+z)$ **(3)**

According to this theory, the local speed of light c is invariant in all reference frames. However, the relative speed of light c(z) at cosmological redshift z, as measured from the current cosmological reference frame, is given below:

$$c(z) = \frac{l_0}{t_0} = \frac{l(z)}{t(z)(1+z)^2} = \frac{c}{(1+z)^2} \tag{4}$$

This apparently reduced speed of light c(z) leads to a phenomenon called the Shapiro time delay (refer to Section 7.8) in the gravitational potential with gravitational redshift z. The cosmological time dilation proposed in this theory is inherent to the fabric of space-time due to the cosmological potential, similar to the gravitational time dilation caused by the gravitational potential, and the corresponding redshift is the cosmological redshift due to the increase in the cosmological potential caused by the stretching of space-time.

3. Relativistic Hubble law and the cosmological redshift

Based on the relativistic acceleration of space-time (2), we can calculate the relativistic velocity v of B from A (Big Repulsion) on the space-time fabric based on the cosmological redshift z.

$$\frac{dv}{dt} = a \quad ; \quad dv = adt \quad ; \quad dv = cH(z)dt \text{ using (2)}$$

$$\int_0^v dv = \int_{t_p}^t cH(z)dt \quad ; \quad \int_0^v dv = c\int_{t_p}^{T_0} \frac{1}{T(z)} dT \quad ; \quad v = c\ln\left(\frac{1}{H_0 t_p}\right) \quad ; \quad cz = c\ln\left(\frac{1}{H_0 t_p}\right) \text{ using (1)}$$

Planck time t_p is the minimum age of the universe at its beginning owing to the quantization of space and time to the Planck units. T_0 is the age of the universe, which is the Hubble time in this universe model. As ln(0) is undefined, the singularity or zero time of the Big Repulsion is also undefined. Therefore, the maximum possible cosmological redshift can be expressed as follows:

$$z = \ln\left(\frac{1}{H_0 t_p}\right) \tag{5}$$

which is **140.15** for H_0 = 77.7 (km/s)/Mpc, and the maximum apparent relativistic recession velocity that is possible is ~140 times the speed of light.

For $H_0 < H(z)$, $z = \ln\left(\frac{H(z)}{H_0}\right) = \ln\left(\left(H_0\left(\frac{1}{H_0} - \frac{D}{c}\right)\right)^{-1}\right) = \ln\left(\left(1 - \frac{H_0 D}{c}\right)^{-1}\right) = \ln\left(\frac{c}{c - H_0 D}\right)$ **(6)**

where H(z) is the Hubble parameter at the cosmological redshift z or at distance D, and $\frac{1}{H(z)} \geq t_p$.

The local acceleration a(p) at the beginning of the universe (Big Repulsion) can be expressed as follows:

$$a(p) = cH(z) = \frac{c}{t_p} = \mathbf{5.56 \times 10^{51}\ m/s^2} \text{ using (2), (5), and (6)}$$

The local acceleration a(z) of the universe at a distance D from the present can be expressed as follows:

$$a(z) = cH(z) = cH_0 e^z = \frac{cH_0}{\left(1 - \frac{H_0 D}{c}\right)} \text{ using (2) and (6)}$$

$$a(z) = cH_0 \left(1 - \frac{H_0 D}{c}\right)^{-1} \qquad \textbf{(7)}$$

The relativistic Hubble law based on (1) and (6) can be expressed as follows:

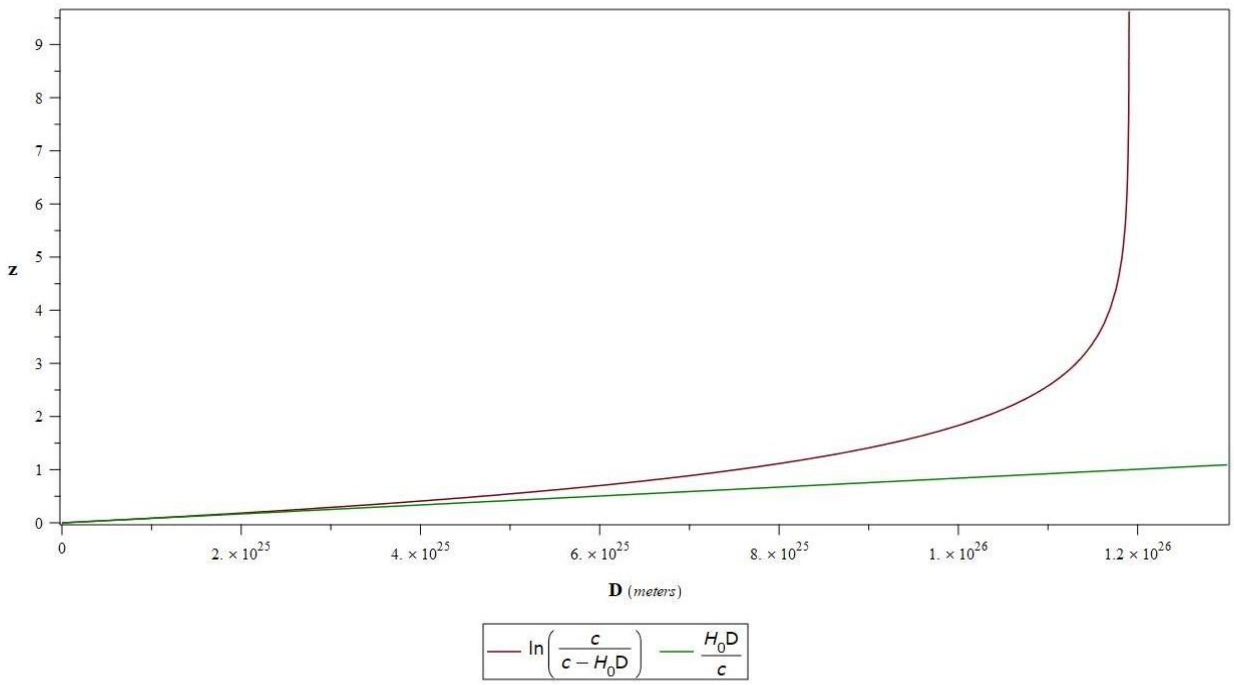
$$v = c \ln\left(\frac{c}{c - H_0 D}\right) \qquad \textbf{(8)}$$

As shown in Table 1, the redshift z values matched the regular Hubble formula and new relativistic formula for low values of D. The new formula restricts the maximum cosmological redshift to 140.15, owing to the universe model that is considered. The z values differ between the two formulas as the distance D increases or as the time traveled by the light approaches the age of the universe, as shown in Table 1. Therefore, the Hubble law, $v = H_0 D$, is only accurate up to moderate distances, as it is the limiting case of the relativistic Hubble law. We can also see that the new z-values in Table 1 are in line with the accelerating model of the universe.

Table 1 (Cosmological redshift values)

D (meters)	$z = \frac{H_0 D}{c}$	$z = \ln\left(\frac{c}{c - H_0 D}\right)$
1×10^{10}	8.40×10^{-17}	8.40×10^{-17}
2×10^{15}	1.68×10^{-11}	1.68×10^{-11}
3×10^{20}	2.52×10^{-6}	2.52×10^{-6}
7.53×10^{25}	0.63	1
1×10^{26}	0.84	1.83
1.18×10^{26}	0.99	4.73
1.19×10^{26}	1	140.15 (Maximum/Big Repulsion)

Redshift(z) Vs Distance(D)



Graph 1

The age of the universe is 12.58 billion years, which is the Hubble time. As the proper distance and volume remain constant, the maximum observable universe is only 12.58 × 2 billion light years across, which is 25.16 billion light-years for $H_0 = 77.7$ (km/s)/Mpc.

Important formulas: based on (6)

Time since Big Repulsion to the emission of light (T_B)	Time since the emission of light (T_z)	Distance vs Redshift
$T_B = \dfrac{1}{H_0 e^z}$	$T_z = \dfrac{1}{H_0}\left(1-\dfrac{1}{e^z}\right)$	$D = \dfrac{c}{H_0}\left(1-\dfrac{1}{e^z}\right)$

Owing to the cosmological time dilation and the lower relative energy in the past, each by a factor of (1+z), according to this theory, the observed local flux F from the distant objects must be multiplied by $(1+z)^2$ while determining the luminosity distance D_L of standard candle objects such as Type Ia supernovae, which should be equal to the proper distance D measured through the cosmological redshift z, providing a falsifiable prediction for this theory.

$$D = \frac{c}{H_0}\left(1-\frac{1}{e^z}\right) = D_L = \sqrt{\frac{L}{4\pi F(1+z)^2}}$$, where L is intrinsic luminosity.

4. Variable physical constants G, h, ε_0, and k_B, and the change in rest mass

As the local speed of light is constant or invariant in all frames of reference, the local Planck length and Planck time are considered to be constants. Therefore, the product of the gravitational constant G and Planck constant h is considered to be constant. As the Planck charge $\left(q_p = \dfrac{e}{\sqrt{\alpha}}\right)$ is conserved,

https://doi.org/10.33774/coe-2022-68929-v42 Content not peer-reviewed by Cambridge University Press.

alpha (α), or the fine-structure constant, is considered to be constant. Therefore, the product of the Planck constant h and permittivity of free space ε_0 is considered to be constant. As the Planck temperature is considered to be constant, the product of the gravitational constant G and Boltzmann constant k_B is considered to be constant. As the above relations between the physical constants are applicable in all frames of reference according to this theory, we can observe how the relative values of the physical constants G, h, ε_0, and k_B change using the law of conservation of energy.

Cosmological redshift $z = \dfrac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}}$ or $\lambda_{obs} = \lambda_{emit}(1+z)$ **(9)**

As the energy is conserved in cosmological redshift, $E = \dfrac{hc}{\lambda_{obs}} = \dfrac{h_{t_p} c}{\lambda_{emit}}$ **(10)**

Here, h is the current Planck constant and h_{t_p} is the Planck constant when the age of the universe is the Planck time t_p.

$h = h_{t_p}(1+z)$ based on (9) and (10) **(11)**

As $hG = h_{t_p} G_{t_p}$, $h\varepsilon_0 = h_{t_p}\varepsilon_{t_p}$, and $k_B G = k_{B_{t_p}} G_{t_p}$

$$h = h_{t_p}(1+z) = h_{t_p} \ln\left(\frac{1}{H_0 t_p}\right) \text{ using (5)} \quad \textbf{(12)}$$

$$G = \frac{G_{t_p}}{(1+z)} = G_{t_p} \quad \textbf{(13)}$$

$$\varepsilon_0 = \frac{\varepsilon_{t_p}}{(1+z)} = \varepsilon_{t_p} \quad \textbf{(14)}$$

$$k_B = k_{B_{t_p}}(1+z) = k_{B_{t_p}} \quad \textbf{(15)}$$

Below are the relative values of G_{t_p}, h_{t_p}, ε_{t_p}, and $k_{B_{t_p}}$ in MKS units when the age of the universe is Planck time t_p, based on the current values of G, h, ε_0, and k_B for $H_0 = 77.7$ (km/s)/Mpc. Therefore, these constants vary proportionally with relativistic Hubble flow.

$h = 6.62607015\times10^{-34}$	$G = 6.67430\times10^{-11}$	$\varepsilon_0 = 8.8541878128\times10^{-12}$	$k_B = 1.380649\times10^{-23}$
$h_{t_p} = 4.69428003\times10^{-36}$	$G_{t_p} = 9.42090\times10^{-9}$	$\varepsilon_{t_p} = 1.2497863179\times10^{-9}$	$k_{B_{t_p}} = 9.781292\times10^{-26}$

As $E = h(1+z)\nu = m(1+z)c^2$, it follows that $m_0 = m_z(1+z)$ because of the constancy of the speed of light, where m_0 is the current mass and m_z is the original mass when light is emitted in the past at a cosmological redshift z. The mass increase due to the expansion of space-time could also be seen as an increase in the cosmological potential energy U_c.

$$U_c = -(m_z(1+z) - m_z)c^2 = -m_z z c^2 = -m_z \ln\left(\frac{c}{c - H_0 D}\right)c^2 \text{ using (6)}$$

Changes in the rest mass can be observed by converting m_0 and m_z into energy, which is the observed energy difference in the cosmological redshift. Therefore, the energy of a photon does not increase with space-time expansion, whereas the potential energy/rest mass increases with expansion, which explains the observed energy difference in the cosmological redshift. Similarly, for gravitational redshift, the change in rest mass is manifested as gravitational potential energy U. Similar to the cosmological redshift, the change in rest mass in gravitational redshift is associated with the change in physical constants G**,** h**,** ε_0**,** and k_B. Therefore, the energy of a photon does not change when it moves against gravity. However, the gravitational redshift z, which is the decrease in the frequency of the photon, is caused by an increase in the relative Planck constant [E = h(1+z)ν]. This proves that photons do not curve space-time by themselves but take a curved path around any matter (refer to Section 7.5). As photons do not curve the space-time in this theory, the annihilation of matter converts the cosmological potential energy, gravitational potential energy, and the rest mass to photons. Therefore, the gravitational potential energy U of mass m in the gravitational field can be expressed as follows:

$U = -(m(1+z) - m)c^2 = -mzc^2$, where m(1+z) is the relative rest mass at infinity **(16)**

$$U = -mzc^2 \quad \textbf{(17)}$$

$$\frac{U}{m} = \phi = -zc^2 \quad \textbf{(18)}$$

where z is the gravitational redshift and ϕ is the gravitational potential.

However, after factoring in the change in rest mass, the physical constants would appear to be locally constant or invariant (Table 5) in the cosmological and gravitational frameworks.

Table 2 (Relative change in the values of the physical constants)

Cosmological (From the Big Repulsion)	$\frac{G_{t_p}}{(1+z)}$	$h_{t_p}(1+z)$	$\frac{\varepsilon_{t_p}}{(1+z)}$	$k_{B_{t_p}}(1+z)$
Gravitational (From infinity)	$G(1+z)$	$\frac{h}{(1+z)}$	$\varepsilon_0(1+z)$	$\frac{k_B}{(1+z)}$

where z is the cosmological and gravitational redshift, respectively.

For example, the Planck constant due to the cosmological redshift ten years from now is h(1+δz) (12), where δz is the change in the cosmological redshift value ten years from now.

$\delta z = \ln\left(\frac{H_0}{H_{10}}\right)$ using (6), where H_{10} is the Hubble constant after 10 y.

δz values:

After 10 y : 7.95×10^{-10}
After 50 y : 3.97×10^{-9}
After 100 y : 7.95×10^{-9}

Similar changes in the relative values of the physical constants can be observed and calculated in the gravitational field using the factor (1+z), where z is the gravitational redshift.

In Table 3, the relative rest mass increases by a factor of (1+z) owing to the space expansion. Similarly, for space contraction, the relative rest mass decreases by a factor of (1+z). Here, m_0 is the relative rest mass, and m_z is the rest mass at the point of photon emission. However, locally, the rest mass remains invariant and is equal to the inertial mass (Table 5).

However, the absolute inertial frame cannot be uniquely identified in this theory based on the change in the physical constants because any change is compensated for by the change in the rest mass; hence, the local physical constants remain unchanged or invariant.

Table 3 (Relative change in the rest mass)

Cosmological	Gravitational
Space expansion: $m_0 = m_z(1+z) = m_z\left(1+\ln\left(\frac{c}{c-H_0 D}\right)\right)$	Space expansion: $m_0 = m_z(1+z) = m_z\frac{1}{\sqrt{1-\frac{2GM}{Rc^2}}}$
Space contraction: $m_0 = \frac{m_z}{(1+z)} = \frac{m_z}{1+\ln\left(\frac{c}{c-H_0 D}\right)}$	Space contraction: $m_0 = \frac{m_z}{(1+z)} = m_z\sqrt{1-\frac{2GM}{Rc^2}}$

5. Acceleration of the universe due to electrostatic repulsion

In this theory, the expansion of space-time is due to electrostatic repulsion between the Planck charges in the Planck volumes in the hidden three-dimensional space of the seven-dimensional universe. The local acceleration at the beginning of the Big Repulsion is $a(z) = cH(z) = \frac{c}{t_p} = a(p)$ based on (2), (5), and (6), which can be reformulated as $\sqrt{\rho_p G}$, where ρ_p is the local Planck energy density and G is the local gravitational constant when the age of the universe is the Planck time t_p. As the acceleration a(p) is directly proportional to $\sqrt{\rho_p}$, it proves that the proposed model of the universe has Planck charges in Planck volumes, which means that each cube (Planck volume ${l_p}^3$) in the hidden three-dimensional space is filled with Planck energy E_p.

Planck energy $E_p = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{\alpha(l_p)} = \sqrt{\frac{\hbar c^5}{G}}$, where l_p is the Planck length.

$$\rho_p = \frac{E_p}{{l_p}^3} = \frac{\sqrt{\frac{\hbar c^5}{G}}}{{l_p}^3} \quad ; \quad \rho_p G = \frac{\sqrt{\frac{\hbar c^5}{G}}}{{l_p}^3} G = \left(\frac{c}{t_p}\right)^2 = a(p)^2$$

Therefore, the local energy density of the hidden three-dimensional space when the age of the universe is Planck time can be expressed as follows:

$$\rho_p = \frac{c^2}{{t_p}^2 G} = \mathbf{4.63 \times 10^{113}\ J/m^3}$$

As energy is relative in this theory, the relative energy density of the hidden three-dimensional space when the age of the universe is Planck time with respect to the current epoch is given below, which is also equivalent to using the relative gravitational constant G(1+z) instead of G in ρ_p.

$$\frac{\rho_p}{(1+z)} = \frac{c^2}{{t_p}^2 G\left(1+\ln\left(\frac{1}{H_0 t_p}\right)\right)} = \mathbf{3.28 \times 10^{111}\ J/m^3} \text{ using (5)}$$

As the relative energy scale of the universe increases and Hubble parameter H(z) decreases with the expansion of the universe, the relative energy density of the past decreases. As the universe accelerates owing to electrostatic repulsion, the electrostatic potential energy in the hidden three-dimensional space is gradually transferred to the four-dimensional space-time grid and stored as potential energy. As the elastic space-time grid resists expansion, the local rate of acceleration gradually decreases to follow a(z) = cH(z) (2), which asymptotes to zero.

$$\rho_z G = a(z)^2 = \left(cH(z)\right)^2 \quad ; \quad \rho_z = \frac{\left(cH(z)\right)^2}{G} = \frac{\left(cH_0\left(1-\frac{H_0 D}{c}\right)^{-1}\right)^2}{G} \text{ using (6)}$$

The dark energy density ρ_z at distance D is not constant in this theory but decreases with the expansion of the universe. The current energy density ρ_0 of the hidden three-dimensional space responsible for the acceleration of the universe can be determined by setting D = 0 in the above equation.

$$\rho_0 = \frac{(cH_0)^2}{G} = \mathbf{8.54 \times 10^{-9}\ J/m^3}$$

$$a^2 = G\rho_0 \qquad \textbf{(19)}$$

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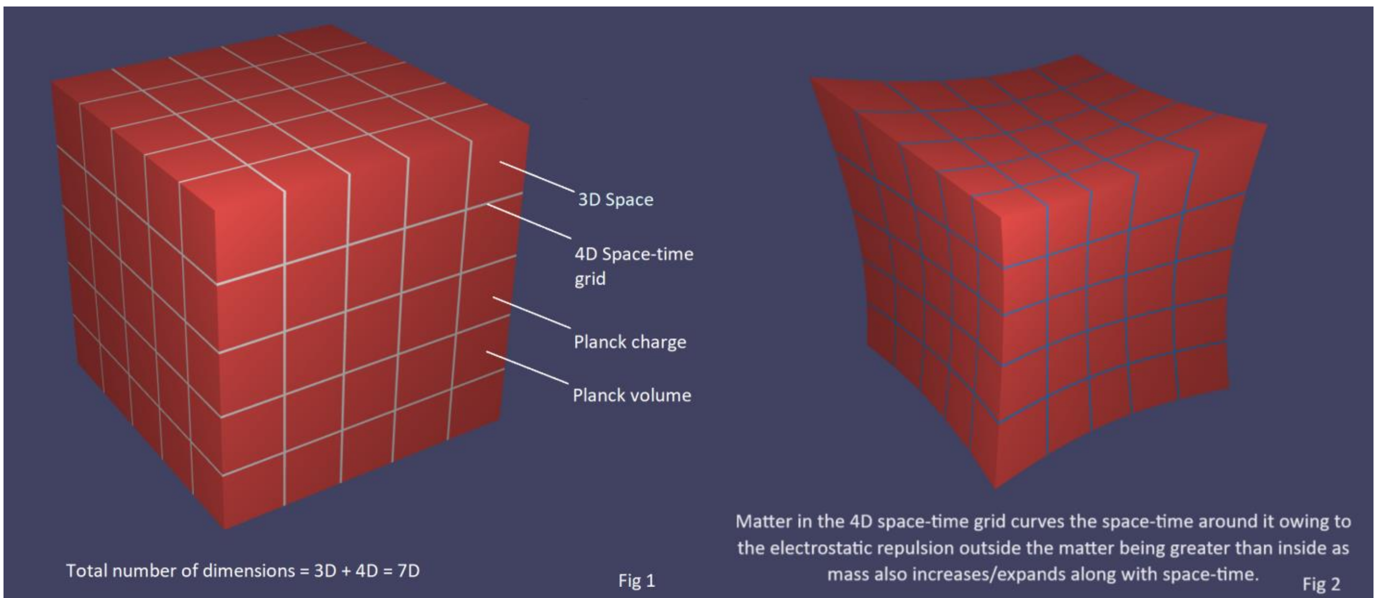
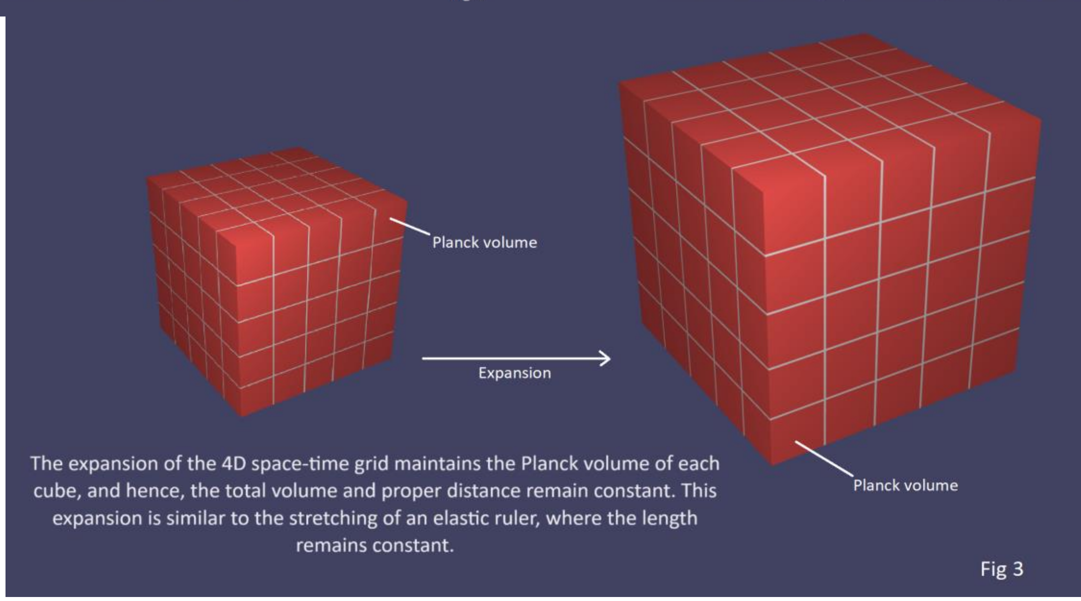
Therefore, the product of the net energy density of the hidden three-dimensional space and the gravitational constant is equal to the square of the acceleration of the universe.

a = Current acceleration of the universe, which is cH_0 (2)
ρ_0 = Net electrostatic energy density of the universe responsible for acceleration
G = Local gravitational constant, which always remains constant (Table 5)

As the total energy is conserved in the expansion of the universe, the potential energy density of the four-dimensional space-time grid ρ_g is given by the difference between the relative energy density $\left(\frac{\rho_p}{(1+z)}\right)$ of the three-dimensional space at the beginning of the universe and the current energy density ρ_0. Therefore, ρ_g continues to increase until infinity. $|\rho_g| = \left(\frac{\rho_p}{(1+z)} - \rho_0\right)$

Seven-dimensional space-time, curvature, and expansion





6. The absolute frame of reference

The four-dimensional space-time grid proposed in this theory acts as the absolute frame of reference or the absolute inertial frame. The relative values of the physical constants and rest mass/potential energy change with cosmological expansion and gravitational field according to the formulas given in Tables 2 and 3. Therefore, in the gravitational field, the absolute inertial frame is at an infinite distance from the stationary mass generating the gravitational field, and the relative values of the physical constants and passive gravitational/rest mass change as we move from infinity towards the active gravitational mass owing to the contraction of space. For example, the relative gravitational constant would be higher, and the relative Planck constant and rest mass would be lower near the mass than further away. In this theory, the universal time is the time in the absolute inertial frame, which is the Hubble time.

However, the absolute inertial frame cannot be uniquely identified in this theory based on the change in the physical constants owing to the expansion/contraction of space, because any change is compensated for by the change in the rest mass/potential energy, as shown in Table 5. Therefore, the local values of the physical constants and laws remain invariant owing to the change in the rest mass. For example, a light source emitting green photons will still emit green photons of the same frequency in an expanded universe or higher gravitational potential with an increased Planck constant owing to a proportional increase in the rest mass. Hence, the local values of the Planck constant and rest mass in all reference frames appear to be constant or invariant. As the absolute inertial frame cannot be uniquely identified, but is still mandated by this theory to explain dark energy, MONDian gravity, and the change in rest mass/potential energy, it can be assumed that the absolute inertial frame is the cosmic rest frame in which the cosmic microwave background radiation (CMBR) is isotropic.

In this theory, the acceleration due to Newtonian gravity is invariant in all inertial frames, except for relativistic time dilation or gravitomagnetic effects, as shown in Table 4. Gravitomagnetic effects are discussed in section 7.9. Consider two point masses M and m, which are the active and passive gravitational masses, respectively, separated by distance r and moving at velocity v in a stationary reference frame. The relativistic time dilation of the moving masses in the stationary reference frame compensates for the time dilation of the observer moving with the masses in the moving reference frame. According to this theory, only the proper rest mass M contributes to the active gravitational mass in all frames of reference as the associated kinetic energy, if any, does not produce additional curvature in the space-time grid; hence, the active gravitational mass is a Lorentz-invariant quantity.

Table 4 (Invariant proper acceleration due to Newtonian gravity)

	WRT stationary reference frame (without relativistic time dilation)	WRT moving reference frame
Masses aligned perpendicular to the direction of motion	$F=\gamma ma=\dfrac{GM(\gamma m)}{r^2}$; $a=\dfrac{GM}{r^2}=-\nabla\phi$	$a=\dfrac{GM}{r^2}=-\nabla\phi$
Masses aligned parallel to the direction of motion	$F=\gamma^3 ma=\dfrac{GM(\gamma m)}{\left(\dfrac{r}{\gamma}\right)^2}$; $a=\dfrac{GM}{r^2}=-\nabla\phi$	$a=\dfrac{GM}{r^2}=-\nabla\phi$

In this theory, physical laws, constants, and rest mass remain constant or invariant in the local inertial frame, although the relative values of constants and rest mass vary owing to the contraction/expansion of space in cosmological and gravitational frameworks.

Table 5 (Invariant local physical laws, constants, and rest mass)

Physical constant	Physical constant due to space contraction	Physical constant due to space expansion	Invariant law, constant, and rest mass in the local inertial frame due to space expansion (z is the redshift due to expansion)
G	$G(1+z)$	$\frac{G}{(1+z)}$	$\left(F = m(1+z)a = \frac{G}{(1+z)}\frac{M(1+z)m(1+z)}{r^2}\right) \equiv \left(F = ma = \frac{GMm}{r^2}\right)$
h	$\frac{h}{(1+z)}$	$h(1+z)$	$\left(E = m(1+z)c^2 = h(1+z)\nu\right) \equiv \left(E = h\nu = mc^2\right)$
ε_0	$\varepsilon_0(1+z)$	$\frac{\varepsilon_0}{(1+z)}$	$\left(F = m_e(1+z)a = \frac{(1+z)}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right) \equiv \left(F = m_e a = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\right)$
k_B	$\frac{k_B}{(1+z)}$	$k_B(1+z)$	$\left(\langle E\rangle(1+z) = \frac{3}{2}Nk_B(1+z)T\right) \equiv \left(\langle E\rangle = \frac{3}{2}Nk_B T\right)$

As the rest mass increases with the expansion of the universe, a point mass m moving with velocity v in an inertial reference frame slows down to $\frac{v}{(1+z)}$ to conserve non-relativistic linear momentum. $mv = m(1+z)\frac{v}{(1+z)}$, where z is the cosmological redshift. The respective non-relativistic linear kinetic energy also decreases as part of the kinetic energy is converted into cosmological potential energy/rest mass, along with the energy contribution from the expansion of the universe. $KE = \frac{1}{2}mv^2 \neq \frac{1}{2}m(1+z)\left(\frac{v}{1+z}\right)^2$. Therefore, the total kinetic energy lost to the cosmological potential energy is $\left(KE - \frac{KE}{(1+z)}\right)$. However, as the local rest mass m is invariant (Table 5), the local linear momentum and kinetic energy would appear to be $m\left(\frac{v}{1+z}\right)$ and $\frac{1}{2}m\left(\frac{v}{1+z}\right)^2 = \frac{KE}{(1+z)^2}$, respectively, without violating the law of conservation of momentum and energy. As the local kinetic energy $\frac{KE}{(1+z)^2}$ is not sufficient to maintain the gravitationally bound point mass in its original circular orbit, gravitational potential energy is expended until a stable orbit is found. Therefore, in an expanding universe, any orbiting mass continuously loses its orbit and spirals down to the mass by which it is gravitationally bound, thereby providing falsifiable predictions. However, it remains uncertain whether this effect is associated with the spiral configuration of galaxies, particularly in their early developmental stages, and the observation that relative angular dimensions of galaxies appear larger beyond a specific cosmological redshift (z ≈ 1.5) [4] as the size of galaxies should decrease over time as matter within them spirals towards the center according to this theory.

7. The relativistic Newtonian theory of gravity

In this theory, the presence of mass increases the permittivity of the four-dimensional space-time grid and hence reduces the electrostatic repulsion within the grid enveloped by the mass compared to the electrostatic repulsion outside the mass. As the contraction of the space-time grid within the mass will be greater than outside owing to the permittivity difference and the net compressing force on the matter from the surrounding Planck charges, space-time becomes naturally curved around any mass. Consequently, the net compressive force at the center of mass between the two masses is zero. As this compressive force is neutralized at the center of mass between the two masses, they are mutually attracted to their center of mass due to the compressive forces acting upon the masses from the outer edges, thus resulting in the phenomenon of attractive gravitational force. Hence, the gravitational force is considered a real force rather than a fictitious force of the general theory of relativity. As gravity works on the space-time grid, the principle of locality is maintained.

The change in the gravitational constant G in the gravitational field (Table 2) is similar to the change in the gravitational constant due to cosmological space expansion. Therefore, we can use Newton's law of gravitation and introduce the variable G and the curvature of space to derive a relativistic law. In this theory, only the rest mass contributes to the active gravitational mass and the relativistic mass to the passive gravitational mass (Table 4).

$F = \dfrac{G(1+z)Mm_R}{\left(\dfrac{R}{1+z}\right)^2} = (1+z)^3\dfrac{GMm_R}{R^2}$, where G is the gravitational constant at infinity, and $m_R = \dfrac{m}{(1+z)}$ is the relative rest mass or passive gravitational mass at a distance R from the active gravitational point mass M with M ≥ m, and m is the point rest mass at infinity or the invariant local rest mass (Table 5). As the space-time grid contracts and becomes curved as we move from infinity towards mass M, R is divided by (1+z) to factor in the local curvature of space, which is also the length contraction factor, as observed from infinity. Because the relative rest mass is equal to the passive gravitational mass, the gravitational acceleration g is given as follows:

$g = \dfrac{G(1+z)M}{\left(\dfrac{R}{1+z}\right)^2}$. Therefore, $\dfrac{d\phi}{dR} = g = (1+z)^3\dfrac{GM}{R^2} = \left(1-\dfrac{\phi}{c^2}\right)^3\dfrac{GM}{R^2}$ using (18).

$$\int_0^{\phi}\left(\frac{1}{c^2-\phi}\right)^3 d\phi = \frac{GM}{(c^2)^3}\int_{\infty}^{R}\frac{1}{R^2}dR \quad ; \quad \frac{1}{2(c^2-\phi)^2}-\frac{1}{2(c^2)^2} = -\frac{GM}{(c^2)^3}\frac{1}{R} \quad ; \quad \frac{1}{2(1+z)^2}-\frac{1}{2} = -\frac{GM}{c^2}\frac{1}{R}$$

$$1+z = \sqrt{\frac{1}{1-\dfrac{2GM}{Rc^2}}} \qquad \textbf{(20)}$$

As the gravitational redshift (20) matches the redshift from the Schwarzschild solution of the Einstein field equations without using the weak-field approximation, it validates the concept of variable physical constants. Additionally, the formulas below for the gravitational force and gravitational potential energy are not approximations but complete solutions that work in both strong and weak gravitational fields for non-spinning stationary spherical masses without using any weak field approximation of the general theory of relativity.

The gravitational redshift z becomes infinite at $R = \frac{2GM}{c^2}$ in (20), which yields the Schwarzschild radius R_S, without using a weak-field approximation.

Gravitational force F:

$$F = (1+z)^3 \frac{GMm_R}{R^2} = \left(1 - \frac{2GM}{Rc^2}\right)^{-\frac{3}{2}} \frac{GMm_R}{R^2} \quad \textbf{(21)}$$

Here, the proper gravitational potential ϕ (23) and rest mass M, which is the active gravitational mass, are Lorentz-invariant quantities (Table 4). Gravitomagnetic effects are discussed in section 7.9. In addition, the Newtonian instantaneous action-at-a-distance is eliminated, as (21) reduces to the regular Newtonian equation for gravity when the speed of light, which is the speed of gravity, is set to infinity ($c = \infty$).

Gravitational potential energy U:

$$U = -m_R z c^2 = -m_R \left(\left(1 - \frac{2GM}{Rc^2}\right)^{-\frac{1}{2}} - 1 \right) c^2 \quad \textbf{(22)}$$

Equating the gravitational potential energy U with the relativistic kinetic energy $m_R(\gamma - 1)c^2$ yields the Schwarzschild radius $R_S = \frac{2GM}{c^2}$ for escape velocity c. For weak gravitational fields, (22) reduces to the regular Newtonian gravitational potential energy and hence validates this theory.

$$U = -m_R \left(\left(1 - \frac{2GM}{Rc^2}\right)^{-\frac{1}{2}} - 1 \right) c^2 \approx -m_R \left(1 + \frac{GM}{Rc^2} - 1\right) c^2 = -\frac{GMm_R}{R}$$

Gravitational potential ϕ:

$$\phi = \frac{U}{m_R} = -zc^2 = -\left(\left(1 - \frac{2GM}{Rc^2}\right)^{-\frac{1}{2}} - 1 \right) c^2 \quad \textbf{(23)}$$

The formula for the orbital velocity is invariant at all radii greater than R_S for the relativistic passive gravitational mass γm_R orbiting around mass M ($M \gg m$) in a circle.

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$(1+z)^3 \frac{GM(\gamma m_R)}{R^2} = \frac{(\gamma m_R)v^2}{\left(\frac{R}{1+z}\right)}$; $v = (1+z)\sqrt{\frac{GM}{R}}$, where z is the gravitational redshift at radius R, γm_R is the relativistic passive gravitational mass, and $\frac{R}{(1+z)}$ represents the curvature of the space at radius R. The path taken by mass m_R around mass M is equivalent to the geodesic path of the Schwarzschild metric.

The length contraction and time dilation at radius R in the gravitational field are given by $L = \frac{L_R}{(1+z)}$ and $T = T_R(1+z)$, respectively, where L_R and T_R are the proper length and time at radius R, respectively, and L and R are the coordinate length and time, respectively, at infinity **(24)**

As the proper length L_R, time T_R, and relative rest mass m_R become zero $\left(m_R = \frac{m}{\infty} = 0\right)$ at the Schwarzschild radius in this theory, the space-time terminates at the event horizon of a black hole (Figure 5), unlike the general theory of relativity, in which the space-time is extended into the black hole. Blackhole singularities do not exist in this universe model, as the mass is converted to equivalent informational entropy at the event horizon and stored on the surface of the black hole, which continues to curve the space-time. Additionally, charged black holes do not exist in this model of the universe, as the permittivity of free space ε_0 becomes infinite (Table 2) at the event horizon. Therefore, stationary black holes can only have two properties: informational (entropic) mass and angular momentum, and hence only partially satisfy the no-hair theorem.

7.1. Special relativity in inertial frames

The total energy E_{tot} (Rest + Potential) of a point mass m_R when taken from radius R to infinity from mass M can be expressed as follows:

$E_{tot} = m_R c^2 + m_R z c^2 = m_R (1+z)c^2 = mc^2$, where $m = m_R(1+z)$ is the rest mass at infinity.

Therefore, the increase in the gravitational potential energy is equal to the energy equivalent of the increase in the rest mass. Because the total energy is conserved in the free-fall motion of an object in the gravitational field, the velocity of the object can be derived from the total energy E_{tot}. Through dimensional analysis based on (20), the energy at any point during freefall must be of the form $m_R\left(\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}\right)c^2$. Therefore, $1+z = \gamma = \sqrt{\frac{1}{1-\frac{v^2}{c^2}}} = \sqrt{\frac{1}{1-\frac{2GM}{Rc^2}}}$ and $v = \sqrt{\frac{2GM}{R}}$ which is the proper velocity of the object at radius R in a freefall motion from the point of maximum potential energy or infinity, which is also equal to the escape velocity v_e. Because the freefall motion is inertial, the Lorentz factor γ is directly obtained from energy conservation. The kinetic energy gained from losing gravitational potential energy $\left|m_R z c^2\right|$ (17) in the freefall motion from infinity is equal to relativistic kinetic energy $(\gamma - 1)m_R c^2$ at radius R and the transverse relativistic redshift $(\gamma - 1)$ of a free-falling object observed at radius R is equal to the gravitational redshift z. Hence, this theory upholds special relativity in local inertial frames.

7.2. Space-time interval (ds^2)

The invariant line element (ds^2) in the Minkowski space-time in spherical coordinates is given by $ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$.

Based on (3) and (24), the coordinate time dt can be replaced with $\frac{dt}{(1+z)}$ and the coordinate length dr can be replaced with $dr(1+z)$ in the line element.

$$ds^2 = -\frac{c^2dt^2}{(1+z)^2} + dr^2(1+z)^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad \textbf{(25)}$$

Using (20) and (18), we obtain the Schwarzschild metric equivalent for this theory.

Schwarzschild metric equivalent:

$$ds^2 = -c^2dt^2\left(1-\frac{2GM}{Rc^2}\right) + dr^2\left(1-\frac{2GM}{Rc^2}\right)^{-1} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

OR

$$ds^2 = -c^2dt^2\left(1-\frac{\phi}{c^2}\right)^{-2} + dr^2\left(1-\frac{\phi}{c^2}\right)^{2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Once the (1+z) factor $\left(1-\frac{\phi}{c^2}\right)$ is identified by solving the relativistic Poisson equation (32), it can be substituted into (25) to obtain the space-time interval equation for this theory. For any inertial or non-inertial reference frame except the absolute reference frame, the respective coordinate length, time, and gravitational redshift z up to the reference point in the frame must be used in (25) to make the line element (ds^2) invariant in all reference frames. However, the non-relativistic or classical Doppler redshift, if any, must be excluded from the gravitational redshift z.

At the event horizon of the black hole, proper time $d\tau = 0$, coordinate length $dr = 0$, and coordinate time $dt = \infty$ (24). Therefore, this theory eliminates singularity, unlike the general theory of relativity, where proper time continues to pass normally at the event horizon, and custom coordinate systems are used to extend the space-time up to the singularity at the center.

Applying the cosmological length contraction and time dilation (6) in (25), we obtain the equivalent of the FLRW metric of the flat space-time for this theory.

FLRW metric equivalent:

$$ds^2 = -c^2dt^2\left(1+\ln\left(\frac{c}{c-H_0r}\right)\right)^{-2} + dr^2\left(1+\ln\left(\frac{c}{c-H_0r}\right)\right)^{2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

7.3. Innermost stable circular orbit (ISCO)

The ISCO, which is the smallest stable circular orbit around a non-rotating stationary black hole, can be identified using the rate of change in the angular momentum of an orbiting test particle with respect to its radius. The relativistic angular momentum of a circular orbit is given by $L = \gamma m \cdot v \cdot R$, where γ is the Lorentz factor ; $v = \frac{cL}{\sqrt{R^2c^2m^2 + L^2}}$.

The centripetal acceleration of the test particle in the curved space-time around mass M is $\frac{v^2}{\left(\frac{R}{1+z}\right)}$, where the factor (1+z) accounts for the local curvature of the space.

$(1+z)^3 \frac{GM}{R^2} = \frac{v^2}{\left(\frac{R}{1+z}\right)}$ (21) ; $(1+z)^2 \frac{GM}{R} = v^2$; $L = \frac{mcR\sqrt{(Rc^2 - 3GM)GM}}{Rc^2 - 3GM}$ using (20)

ISCO is the orbit where the rate of change in the angular momentum $\frac{dL}{dR} = 0$, as any orbit below ISCO, will be unstable, as the angular momentum required to maintain a stable orbit steeply increases until the Schwarzschild radius. Alternatively, the ISCO radius can be derived using the effective potential U_{eff}.

Solving for $\frac{dL}{dR} = 0$, we obtain the ISCO radius, as predicted by the general theory of relativity.

$$R_{isco} = \frac{6GM}{c^2} = 3R_S \qquad \textbf{(26)}$$

The velocity of the orbiting test particle at R_{isco} is $v = (1+z)\sqrt{\frac{GM}{R_{isco}}} = \frac{c}{2}$.

7.4. Innermost bound circular orbit (IBCO)

The IBCO, which is the smallest bound circular orbit around a non-rotating stationary black hole, can be identified by the orbiting velocity of the test particle where it is equal to the escape velocity v_e. Solving the following, we obtain the IBCO radius, as predicted by the general theory of relativity.

$v = (1+z)\sqrt{\frac{GM}{R_{ibco}}} = v_e = \sqrt{\frac{2GM}{R_{ibco}}}$ (refer to Section 7.1)

$$R_{ibco} = \frac{4GM}{c^2} = 2R_S \qquad \textbf{(27)}$$

The velocity of the orbiting test particle at R_{ibco} is $v_e = \sqrt{\frac{2GM}{R_{ibco}}} = \frac{c}{\sqrt{2}}$.

7.5. Photon sphere and refractive index of curved space-time

The photon does not curve the space-time by itself in this theory, but only takes the curved path due to optical refraction owing to curved space-time. The path taken by a photon owing to refraction in curved space-time is equivalent to the null geodesic path of the general theory of relativity. The photon sphere at radius R_{ph} (Figure 4) surrounding a non-rotating stationary black hole results from the continuous refraction of light at the critical angle, beyond which total internal reflection occurs, causing photons to spiral toward the event horizon of the black hole. The refractive index η of the curved space-time at radius R with respect to the flat space-time or space-time at infinity is given below, based on (24) and (4). Here, the refraction of light in the curved space-time does not produce the rainbow effect because the local speed of light c is invariant for all wavelengths.

$$\eta(R) = \frac{c}{v} = \frac{c}{\left(\frac{c}{(1+z)^2}\right)} = (1+z)^2 = \left(1 - \frac{2GM}{Rc^2}\right)^{-1} \approx 1 + \frac{2GM}{Rc^2} \qquad \textbf{(28)}$$

As the gravitational redshift z = 0 in flat space-time, the respective refractive index $\eta = (1+z)^2 = 1$. Snell's law for a radially varying refractive-index medium is given by $\eta(r) \cdot r \cdot \sin(\theta)$ = constant, which is applicable for both strong and weak gravitational lensing.

As $\theta = \frac{\pi}{2}$ at R_{ph} (Figure 4), $\eta(r) \cdot r = \text{constant}$

Photon sphere

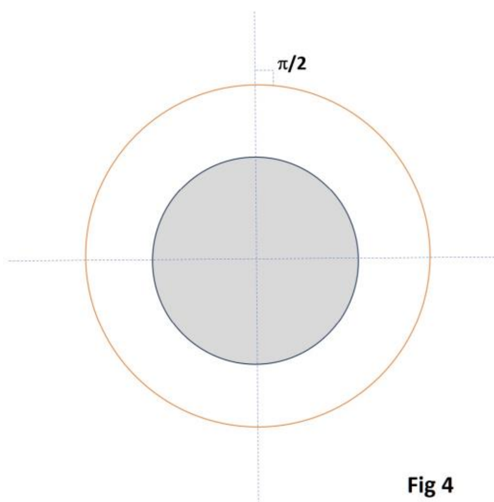

Fig 4

$\left(1 - \frac{2GM}{Rc^2}\right)^{-1} R = \text{constant}$, differentiate both sides with respect to R.

$$\left(1 - \frac{2GM}{R_{ph}c^2}\right)^{-1} - \frac{2GM}{R_{ph}c^2}\left(1 - \frac{2GM}{R_{ph}c^2}\right)^{-2} = 0$$

$$R_{ph} = \frac{4GM}{c^2} = 2R_S = R_{ibco} \qquad \textbf{(29)}$$

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However, if the photon is considered to curve the space-time by itself, like matter, which does not happen in this theory, the radius of the photon sphere obtained is the same as that predicted by the general theory of relativity.

The centripetal acceleration of the photon in the curved space-time around mass M is $\frac{c^2}{\left(\frac{R}{1+z}\right)}$,

where the factor (1+z) accounts for the local curvature of the space.

$(1+z)^3 \frac{GM}{R^2} = \frac{c^2}{\left(\frac{R}{1+z}\right)}$, by solving using (20), we obtain the radius of the photon sphere $R = \frac{3GM}{c^2}$.

7.6. Gravitational lensing

As $\sin(\theta) \approx \theta$ for small deflection angles, Snell's law for a radially varying refractive index medium is given by $\eta(r) \cdot r \cdot \theta$ = constant, and the respective angle of deflection Δθ of a photon due to refraction around a point mass M for very small angles in the weak lensing regime is as follows:

$\Delta\theta \approx 2\int_{r_c}^{\infty} \frac{d}{dr}\left(\frac{1}{n(r)}\right) dr$, where r_c is the radius of the closest approach to point mass M.

By solving the above equation for the refractive index profile in (28), we obtain the deflection angle Δθ, as predicted by the general theory of relativity.

$$\Delta\theta \approx \frac{4GM}{R_c c^2} \tag{30}$$

7.7. Perihelion precession of Mercury

From Kepler's second law, $dA = \frac{1}{2} r^2 d\theta$, where dA is the change in the area swept out by the orbiting mass, dθ is the change in the angle, and r is the radius. Let dA' and dθ' be the local changes in area and angle, respectively. Divide the above equation on both sides by the local time dt' and apply length contraction (24) to factor in the local curvature of space-time.

$$\frac{dA'}{dt'} = \frac{1}{2}\left(\frac{r}{1+z}\right)^2 \frac{d\theta'}{dt'}$$

Apply time dilation using (24). The curvature stretches out the space-time fabric, and hence, the local time dilates to dt' = dt(1+z), where dt is the local time without the space-time curvature.

$$\frac{dA'}{dt'} = \frac{1}{2}\left(\frac{r}{1+z}\right)^2 \frac{d\theta'}{dt(1+z)} = \frac{1}{2} r^2 \frac{d\theta'}{dt} \frac{1}{(1+z)^3}$$

As the angular momentum is conserved, the above equation can be compared with the non-relativistic equation without the space-time curvature to obtain the following equation:

$d\theta' = d\theta(1+z)^3 = d\theta\left(1-\frac{2GM}{Rc^2}\right)^{-\frac{3}{2}} \approx d\theta\left(1+\frac{3GM}{Rc^2}\right)$ by ignoring higher-order terms.

The polar equation of the ellipse is $R = \frac{a(1-\varepsilon^2)}{1-\varepsilon\cos\theta}$.

$\theta' = \int_0^{2\pi}\left(1+\left(\frac{3GM}{c^2}\frac{1-\varepsilon\cos\theta}{a(1-\varepsilon^2)}\right)\right)d\theta$; $\theta' = 2\pi + \frac{6\pi GM}{c^2 a(1-\varepsilon^2)}$, the second term provides the precession angle $\Delta\phi$ of perihelion per revolution, where M is the mass of the Sun, a is the semi-major axis of Mercury, and ε is the orbital eccentricity.

$$\Delta\phi = \frac{6\pi GM}{c^2 a(1-\varepsilon^2)}$$

By solving the above equation, we obtain a perihelion precession of 43”/century, which is the same as that predicted by the general theory of relativity [5].

7.8. Shapiro time delay

We can calculate the gravitational time delay of light passing near a mass such as the Sun. As the gravitational length contraction and gravitational time dilation go together in this theory, we can calculate the effective speed of light, as observed by a stationary observer on Earth (B), as light from a distant object (A) passes near the Sun and reaches Earth. The speed of light is constant at every point in space in the gravitational field, because the length contraction is compensated for by time dilation.

Therefore, $\frac{\frac{\text{Distance}}{(1+z)}}{\frac{Time}{(1+z)}} = \frac{\text{Distance}}{Time} = c$ (speed of light) remains constant. However, a stationary observer on Earth would see a change in the speed of light as it passes through the gravitational field near the Sun. Here, time expansion is applied instead of contraction, as a time delay is observed in the frame of reference of the observer, whose local time is faster than the time near the Sun. Therefore, the length contraction and time expansion do not cancel out in the observer’s reference frame, and hence produce a time delay according to this theory, which is called the Shapiro time delay.

Therefore, the reduced speed of light observed on Earth is $\frac{\left(\frac{\text{Distance}}{(1+z)}\right)}{\text{Time}(1+z)} = \frac{c}{(1+z)^2}$ (24).

$\frac{dx}{dt} = \frac{c}{(1+z)^2} = c\left(1 - \frac{2GM}{Rc^2}\right)$ using (20), where R is the distance between the Sun and the traveling photon. Neglecting the deflection of light near the Sun, the path of light from A to B is a straight line.

$dt = \frac{1}{c}\left(1 - \frac{2GM}{Rc^2}\right)^{-1} dx \approx \frac{1}{c}\left(1 + \frac{2GM}{\sqrt{x^2+b^2}c^2}\right)dx$ by ignoring higher-order terms, where b is the impact parameter. Integrating the left side from T_A to T_B and the right side from X_A to X_B.

$$\int_{T_A}^{T_B} dt = \int_{X_A}^{X_B} \frac{1}{c}\left(1 + \frac{2GM}{\sqrt{x^2+b^2}c^2}\right)dx \quad ; \qquad T_B - T_A = \frac{X_B - X_A}{c} + \frac{2GM}{c^3}\ln\left(\frac{X_B + \sqrt{X_B{}^2 + b^2}}{X_A + \sqrt{X_A{}^2 + b^2}}\right)$$

The second term provides the additional one-way time delay Δt of the light coming from point A, as observed by an observer at point B [6], where $X_B - X_A$ is the distance between A and B.

$$\Delta t = \frac{2GM}{c^3}\ln\left(\frac{X_B + \sqrt{X_B{}^2 + b^2}}{X_A + \sqrt{X_A{}^2 + b^2}}\right) = \frac{2GM}{c^3}\ln\left(\frac{(R_B + X_B)(R_A - X_A)}{b^2}\right)$$

where R_A and R_B are the radial distances from the Sun to A and B, respectively.

7.9. Relativistic Poisson and gravitoelectromagnetism (GEM) equations

Relativistic Poisson and Maxwell-like GEM equations that function in strong gravitational fields and at relativistic velocities can be derived by applying the Laplace operator (∇^2) to the derived gravitational potential ϕ. The relativistic GEM equations can provide explanations for phenomena such as the frame-dragging effect, orbital precession, geodetic effect, and gravitational waves. The relativistic GEM and Poisson equations, when combined, can constitute the equivalent of the Einstein field equations for this theory, with the potential for further generalization.

As $z = -\frac{\phi}{c^2}$ (18), $(1+z) = \left(1 - \frac{\phi}{c^2}\right)$.

7.9.1. Relativistic Poisson equations

The gravitational potential ϕ in Cartesian coordinates based on (23) can be expressed as follows:

$$\phi = -\left(\left(1 - \frac{2GM}{\left(\sqrt{x^2+y^2+z^2}\right)c^2}\right)^{-\frac{1}{2}} - 1\right)c^2$$

The relativistic Poisson equation outside the point mass can be generated by applying the Laplace operator to the gravitational potential.

$$\nabla^2\phi = -\frac{3G^2M^2x^2(1+z_n)^5}{(x^2+y^2+z^2)^3c^2} - \frac{3G^2M^2y^2(1+z_n)^5}{(x^2+y^2+z^2)^3c^2} - \frac{3G^2M^2z^2(1+z_n)^5}{(x^2+y^2+z^2)^3c^2}$$
$$-\frac{3GMx^2(1+z_n)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}} - \frac{3GMy^2(1+z_n)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}} - \frac{3GMz^2(1+z_n)^3}{(x^2+y^2+z^2)^{\frac{5}{2}}} + \frac{3GM(1+z_n)^3}{(x^2+y^2+z^2)^{\frac{3}{2}}}$$

$\nabla^2\phi = -2\pi G\rho\left[(1+z_n)^5 - (1+z_n)^3\right]$, where $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$ is the density of the point mass.

Outside the point mass: $$\nabla^2\phi = -2\pi G\rho\left[\left(1-\frac{\phi}{c^2}\right)^5 - \left(1-\frac{\phi}{c^2}\right)^3\right] \approx 4\pi G\rho\left(\frac{\phi}{c^2}\right) \quad \textbf{(31)}$$

The relativistic Poisson equation at the point mass can be generated by considering only the positive terms and ignoring the negative terms in the Laplacian ($\nabla^2\phi$), because the positive terms represent the positive divergence or source of the potential gradient $\nabla\phi$ flux. Here, the proper gravitational potential ϕ and active gravitational mass density ρ are Lorentz-invariant quantities (Table 4).

$\nabla^2\phi = \dfrac{3GM(1+z_n)^3}{(x^2+y^2+z^2)^{\frac{3}{2}}} = 4\pi G\rho(1+z_n)^3$, where ρ is the density of the point mass.

At the point mass: $$\nabla^2\phi = 4\pi G\rho\left(1-\frac{\phi}{c^2}\right)^3 \quad \textbf{(32)}$$

When the speed of light is set to infinity ($c = \infty$) or when the gravitational redshift z_n is negligible ($z_n \approx 0$), (31) and (32) are reduced to the Laplace and non-relativistic Poisson equations, respectively, as shown below:

Outside the point mass: $\nabla^2\phi = -2\pi G\rho\left[(1+z_n)^5 - (1+z_n)^3\right] \approx -2\pi G\rho[1-1] = 0$

At the point mass: $\nabla^2\phi = 4\pi G\rho(1+z_n)^3 \approx 4\pi G\rho[1] = 4\pi G\rho$

7.9.2. Relativistic GEM equations

Relativistic Maxwell-like GEM equations can be generated using (32) and (31) at the point mass/current and outside point mass/current, respectively, where E_G is the gravitoelectric field, B_G is the gravitomagnetic field, ϕ is the proper gravitational potential due to the gravitoelectric effect, J is the mass current density, and the speed of gravitational waves is the speed of light c. Here, the proper gravitational potential ϕ and active gravitational mass density ρ are Lorentz-invariant quantities. Similar to Maxwell's equations for electromagnetism, the proposed relativistic GEM equations uphold special relativity in local inertial frames, Lorentz covariance, gauge invariance, and locality.

At the point mass/current:

$$\nabla.\mathrm{E_G} = -4\pi G\rho\left(1-\frac{\phi}{c^2}\right)^3$$

$$\nabla\times\mathrm{B_G} = -\frac{4\pi G\left(1-\frac{\phi}{c^2}\right)^3}{c^2}J + \frac{1}{c^2}\frac{\partial E_G}{\partial t}$$

$$\nabla.\mathrm{B_G} = 0$$

$$\nabla\times\mathrm{E_G} = -\frac{\partial B_G}{\partial t}$$

Outside the point mass/current:

$$\nabla.\mathrm{E_G} = 2\pi G\rho\left[\left(1-\frac{\phi}{c^2}\right)^5 - \left(1-\frac{\phi}{c^2}\right)^3\right]$$

$$\nabla\times\mathrm{B_G} = \frac{2\pi G\left[\left(1-\frac{\phi}{c^2}\right)^5 - \left(1-\frac{\phi}{c^2}\right)^3\right]}{c^2}J + \frac{1}{c^2}\frac{\partial E_G}{\partial t}$$

$$\nabla.\mathrm{B_G} = 0$$

$$\nabla\times\mathrm{E_G} = -\frac{\partial B_G}{\partial t}$$

The Lagrangian density $\mathcal{L}$ and the respective action $\mathcal{S}$ for the relativistic GEM equations can be established using GEM tensors and four-vectors, similar to those in electromagnetism. The GEM equivalent of the Lorentz force of electromagnetism can be expressed as $F = \gamma m(E_G + v_m \times B_G)$, where γm is the relativistic passive gravitational mass and v_m is the respective velocity.

8. MOND (Modified Newtonian dynamics)

We can generate a relativistic MOND formula using the (1+z) factor, similar to relativistic Newtonian theory of gravity. However, the relativistic effect in the deep-MOND regime is not significant because of lower acceleration. This theory proposes that the MOND regime can only be present around a black hole but not around ordinary masses such as gas and stars. In the absence of black holes, the gravitational field around ordinary masses is always Newtonian at all accelerations. Therefore, the cutoff acceleration for the MOND regime, which is a_0 ($\sim 1.2 \times 10^{-10}$ m/s^2) [7], is only applicable for the area around a central black hole but not for ordinary masses.

Each cell in Figure 5 of the curved space around the black hole represents the Planck volume with Planck charge. The radial length becomes zero at the event horizon, but each Planck volume continues to expand as we move away from the black hole. In addition, the number of Planck volumes on the circumference remains constant at any radius from the event horizon.

Curved space around the black hole

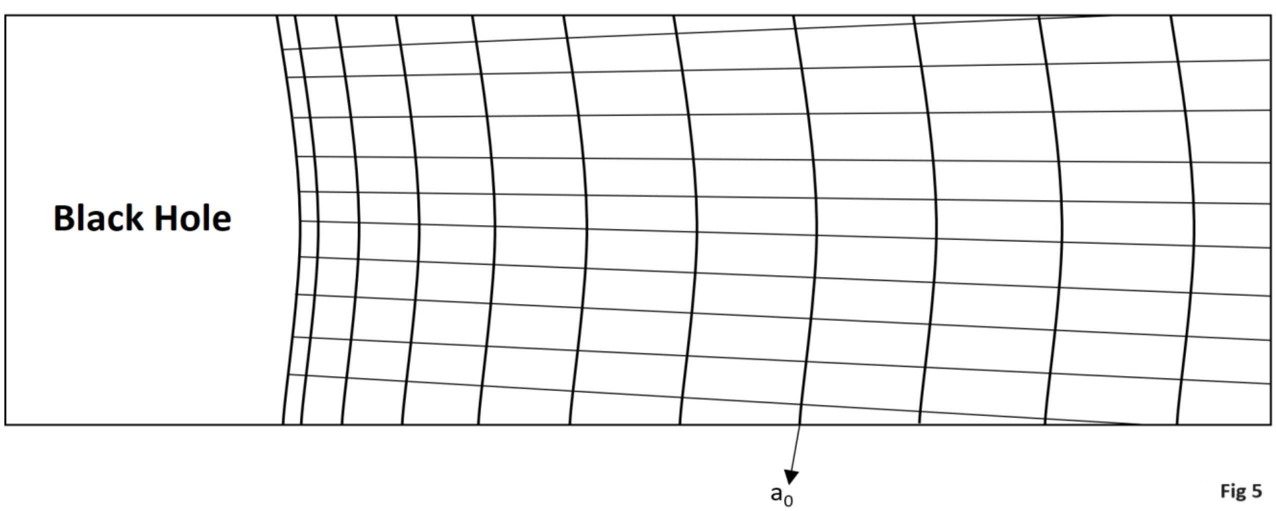

Fig 5

To understand the reason for the transition from the Newtonian regime to the MOND regime at a_0, we can flatten the above curved space, as shown in Figure 6. As the relative Planck volume cannot increase beyond the maximum allowed by the expansion of the universe, the relative size of the Planck volume should remain constant after a_0 radius, as shown in Figure 6.

Hypothetical flattened space around the black hole

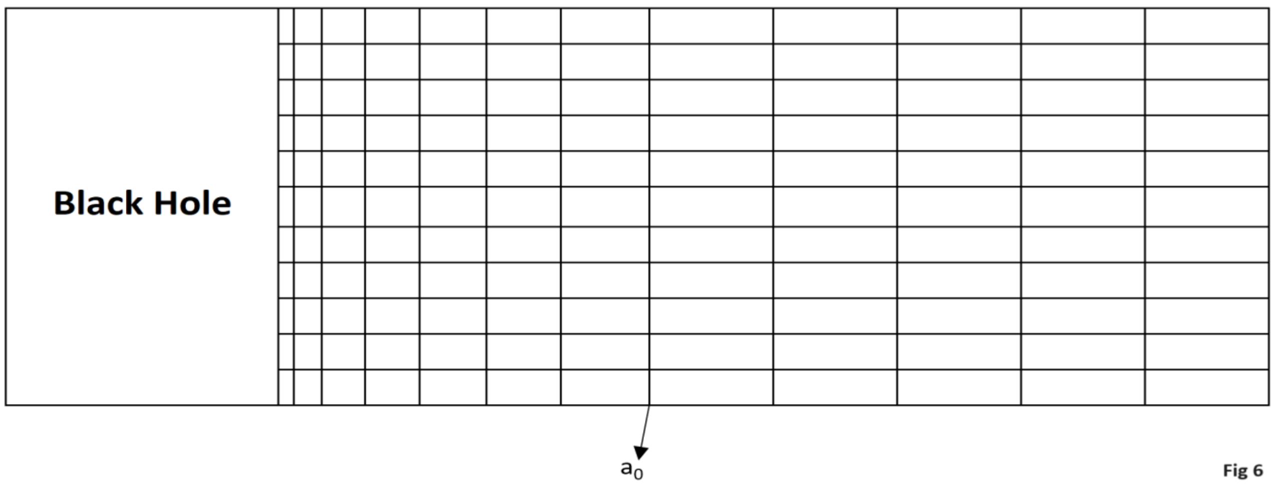

Fig 6

Once the space becomes curved, as is the case around the black hole, the Planck volumes beyond the a_0 radius also become curved; hence, the relative Planck volumes also gradually increase as the stretching of space-time continues to increase beyond the a_0 radius owing to the curvature, as the circumference farther from the a_0 radius should stretch more than the one closer to it. However, the rate of change in the Planck volume for $a < a_0$ is less than the rate of change for $a > a_0$. Therefore, the Newtonian law of gravitation switches to the MOND law at a_0 radius around the black hole.

Let us consider an elastic rubber band stretched around a cylinder; the radial force exerted by the rubber band on the cylinder is $\frac{1}{2\pi}$ times the tension in the rubber band, owing to the circumference of the cylinder, given by $2\pi r$. As the current acceleration of the universe is cH_0 (2) per this theory, the radial acceleration at a_0 should be $\frac{cH_0}{2\pi}$ [8], which is ~1.2×10^{-10} m/s^2 per the above analogy. Here, the entire universe acts like a rubber band wrapped around the black hole at a_0 radius. Additionally, we can see that the space-time grid ends at the event horizon of the black hole and does not extend into it, as in the general theory of relativity.

Figure 7 shows the distortion of the space-time grid around ordinary matter. As the matter is present only in the four-dimensional space-time grid, it is depicted in red in the middle of Figure 7. Each cube in Figure 7 is a Planck volume with Planck charge. As the number of circumferential Planck volumes and the respective area increase radially as we move away from the matter, cutoff acceleration such as a_0 is not applicable, and Newton's law of gravitation can be applied at any acceleration without using any modification, such as MOND, in the absence of black holes. However, the gravity of ordinary matter will still switch to MOND at a_0 if it is present within the a_0 radius of a black hole owing to the geometry of the space-time as shown in Figures 5 and 6. This could explain why the expected gravitational lensing is not observed around the gaseous part of the Bullet cluster (1E 0657-56) [9], but around the galactic matter as it is subjected to MONDian gravitational lensing because of the presence of black holes at the centers of the galaxies.

Curved space around ordinary matter

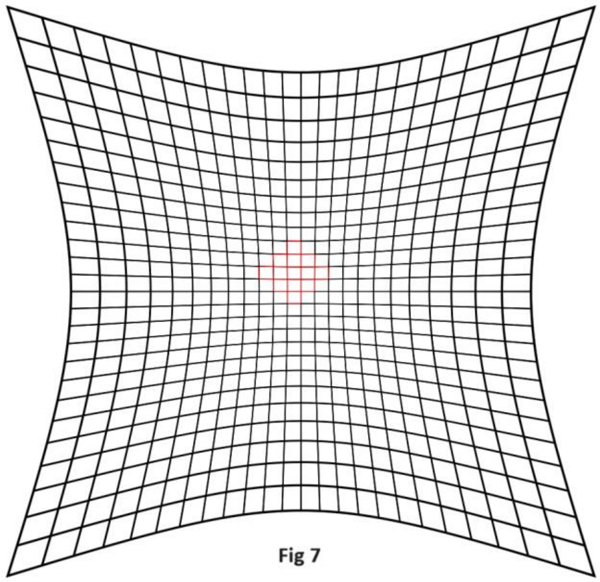

Fig 7

As dark matter has not been detected in the galaxy NGC 1052-DF2 [10], this theory predicts that galaxies that do not require dark matter to explain the rotation curves beyond the a_0 orbit should not have black holes at their centers or anywhere in the galaxies.

The MONDian gravitational law is expressed as follows:

$F = \frac{GMm}{r^2} f\left(\frac{r}{r_0}\right)$ [11], where f(x) → 1 for $x \ll 1$, f(x) → x for $x \gg 1$, and r_0 is the radius at which the acceleration due to gravity $g = a_0$.

We can use the above MOND formula and introduce the variable G and the curvature of space to develop a relativistic law around a non-spinning stationary black hole, similar to the relativistic Newtonian theory of gravity.

$F = \frac{G(1+z)M_0 m_R}{\frac{R_0}{(1+z)}\frac{R}{(1+z)}} = (1+z)^3 \frac{GM_0 m_R}{R_0 R}$, where R_0 is the radius at which the acceleration due to gravity is a_0, z is the gravitational redshift (33) at $R > R_0$, M_0 is the hypothetical point mass located at the center of the black hole, which is the sum of all masses (including the black hole) located within a_0 radius ($R \le R_0$), $m_R = \frac{m}{(1+z)}$ is the relative rest mass or passive gravitational mass at radius R and m is the point rest mass at radius R_z, where the gravitational potential is zero ($\phi = 0$), which can be appropriately chosen for the specific problem being solved.

$$\frac{d\phi}{dR} = g = (1+z)^3 \frac{GM_0}{R_0 R} = \left(1 - \frac{\phi}{c^2}\right)^3 \frac{GM_0}{R_0 R} \text{ using (17)}$$

$$\int_0^{\phi} \left(\frac{1}{c^2 - \phi}\right)^3 d\phi = \frac{GM_0}{R_0 (c^2)^3} \int_{R_z}^{R} \frac{1}{R} dR \quad ; \quad \frac{1}{2(c^2-\phi)^2} - \frac{1}{2(c^2)^2} = \frac{GM_0}{R_0 (c^2)^3} \ln\left(\frac{R}{R_z}\right)$$

$$\frac{1}{2\left(1-\frac{\phi}{c^2}\right)^2}-\frac{1}{2}=\frac{GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right) \quad ; \quad \frac{1}{2(1+z)^2}-\frac{1}{2}=\frac{GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right)$$

$$1+z=\sqrt{\frac{1}{1+\frac{2GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right)}} \qquad \textbf{(33)}$$

where $R_0 < R \le R_z$ and the MONDian gravitational redshift z ranges from radius R to R_z.

Similar to the relativistic Newtonian theory of gravity, this theory upholds special relativity in all inertial frames of the MONDian regime (refer to Section 7.1). The above equation (33) can be substituted into (25) to obtain the Schwarzschild metric equivalent for the MONDian regime.

Schwarzschild metric equivalent:

$$ds^2=-c^2dt^2\left(1+\frac{2GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right)\right)+dr^2\left(1+\frac{2GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right)\right)^{-1}+r^2(d\theta^2+\sin^2\theta d\phi^2)$$

MONDian gravitational force F:

$$F=(1+z)^3\frac{GM_0m_R}{R_0R}=\left(1+\frac{2GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right)\right)^{-\frac{3}{2}}\frac{GM_0m_R}{R_0R} \qquad \textbf{(34)}$$

MONDian gravitational potential energy U and potential ϕ:

$$U=-m_Rzc^2 \quad ; \quad \phi=\frac{U}{m_R}=-zc^2 \qquad \textbf{(35)}$$

$$\phi=-zc^2=-\left(\left(1+\frac{2GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right)\right)^{-\frac{1}{2}}-1\right)c^2\approx-\left(1-\frac{GM_0}{R_0c^2}\ln\left(\frac{R}{R_z}\right)-1\right)c^2=\frac{GM_0}{R_0}\ln\left(\frac{R}{R_z}\right)=\sqrt{GM_0a_0}\ln\left(\frac{R}{R_z}\right),$$

which is similar to the regular MONDian gravitational potential ϕ(r) [12], where $a_0=\frac{GM_0}{R_0^2}$.

According to this theory, any mass present beyond the a_0 radius ($R > R_0$) of a black hole should exert Newtonian gravity. Therefore, any uniformly and spherically distributed mass beyond a_0 radius can be considered as a point mass located at the center of the galaxy exerting Newtonian gravity at $R > R_0$ for any acceleration without requiring MOND. Therefore, the total gravitational force F_T experienced at radius $R > R_0$ for a galaxy with a central black hole can be expressed as follows:

$$F_T=F_M+F_N \qquad \textbf{(36)}$$

where F_M is the MONDian force due to mass M_0 and F_N is the Newtonian force due to the mass present between R_0 and R, ignoring the external field effect (EFE). Therefore, according to this theory, even for accelerations greater than a_0 at radius $R > R_0$ owing to gravitational force F_T, there can still be a contribution from MONDian gravity, as opposed to the current understanding of MOND, where it is applicable only for accelerations less than a_0.

8.1. MONDian gravitational lensing

We can solve the equation for the deflection angle $\Delta\theta_M$ for very small angles in the weak lensing regime of the deep-MOND regime for the refractive index profile $\eta(R)$ based on (28) and (33).

$$\eta(R) = (1+z)^2 = \left(1 + \frac{2GM_0}{R_0 c^2} \ln\left(\frac{R}{R_z}\right)\right)^{-1}$$

$\Delta\theta_M \approx 2\int_{r_c}^{r_x} \frac{d}{dr}\left(\frac{1}{n(r)}\right) dr$, where r_c is the radius of the closest approach.

Consider a uniformly and spherically distributed mass M_N between R_0 and R ($R > R_0$) around a non-spinning stationary black hole. The total deflection angle θ_T at $R > R_0$ can be obtained by summing the MONDian $\Delta\theta_M$ and Newtonian deflection angle $\Delta\theta_N$ (30) based on (36), which can be applied to both individual galaxies and galaxy clusters (refer to Section 8.4), but is not limited to, where M_0+M_N is the total mass, R_x is the reference radius from which the deflection angle needs to be determined, and $R_0 < R_x \le R_z$.

$$\theta_T = \left(\Delta\theta_M + \Delta\theta_N\right) \approx \left(\frac{4GM_0}{R_0 c^2} \ln\left(\frac{R_x}{R_c}\right) + \frac{4GM_N}{R_c c^2}\right) \quad \textbf{(37)}$$

8.2. MONDian Poisson equation

The MONDian gravitational potential ϕ_M in Cartesian coordinates based on (35) can be expressed as follows:

$$\phi_M = \left(\frac{GM_0}{R_0} \ln\left(\frac{\sqrt{x^2+y^2+z^2}}{R_z}\right)\right)$$

The non-relativistic Poisson equation for the MONDian regime can be generated by applying the Laplace operator to the gravitational potential ϕ_M.

$$\nabla^2\phi_M = -\frac{2GM_0 x^2}{(x^2+y^2+z^2)^2 R_0} - \frac{2GM_0 y^2}{(x^2+y^2+z^2)^2 R_0} - \frac{2GM_0 z^2}{(x^2+y^2+z^2)^2 R_0} + \frac{3GM_0}{(x^2+y^2+z^2) R_0}$$

$$\nabla^2\phi_M = \frac{GM_0}{(x^2+y^2+z^2) R_0} = \frac{GM_0}{R^2 R_0} = \frac{a_0 R_0}{R^2}$$

As any point mass present beyond the a_0 radius ($R > R_0$) of a black hole exerts Newtonian gravity in this theory, we can sum the MONDian and Newtonian Poisson equations based on (36) to obtain the final non-relativistic MONDian Poisson equation.

MONDian Poisson equation:

$$\nabla^2 \phi_{eff} = \mu\left(\nabla^2 \phi_M\right) + 4\pi G\rho = \mu\left(\frac{a_0 R_0}{R^2}\right) + 4\pi G\rho \quad \textbf{(38)}$$

where $\mu(x) \rightarrow x$ for $R \gg R_0$, $\mu(x) \rightarrow 0$ for $R \ll R_0$, ϕ_{eff} is the effective gravitational potential, and ρ is the density of the point mass. The Poisson equation for MOND (38) in this theory is equivalent to the following Poisson equation [13] based on AQUAL ("A QUAdratic Lagrangian").

$\nabla.\left[\mu\left(\frac{|\nabla\phi|}{a_0}\right)\nabla\phi\right] = 4\pi G\rho$, where $\mu(x) \rightarrow 1$ for $x \gg 1$ and $\mu(x) \rightarrow x$ for $x \ll 1$.

8.3. MONDian Lagrangian density

The non-relativistic Lagrangian density $\mathcal{L}$ for this theory, which is equivalent to AQUAL, and the respective action $\mathcal{S}$ based on (38) are given below.

$$\mathcal{L} = -\frac{(\nabla\phi)^2}{8\pi G} - \frac{\mu\left(\nabla^2\phi_M\right)}{4\pi G}\phi - \rho\phi \quad \textbf{(39)}$$

where $\mu(x) \rightarrow x$ for $R \gg R_0$, $\mu(x) \rightarrow 0$ for $R \ll R_0$; $\mathcal{S} = \iint \mathcal{L}\, d^3r dt$

8.4. MOND in galaxy clusters

The need for dark matter is not completely eliminated when MOND is applied to galaxy clusters [14], as it is applied only for accelerations less than a_0. As there can be a contribution from MONDian gravity, even for accelerations greater than a_0, as per this theory (36), it can be applied to galaxy clusters to eliminate the need for dark matter. For a simple gravitationally bound system such as a point mass m rotating around mass M with $M \gg m$, the total kinetic energy T is given by $T = \frac{1}{2}mv^2$, Newtonian potential energy U is given by $U = -\frac{GMm}{R}$, and rotational velocity v is given by $v = \sqrt{\frac{GM}{R}}$. By substituting v into T, we obtain the regular Newtonian virial theorem:

$$\langle T \rangle = -\frac{1}{2}\langle U \rangle \ [15]$$

However, when both Newtonian and MONDian dynamics are involved, the regular virial theorem cannot be applied to a system that is gravitationally bound or in hydrostatic equilibrium with the binding potential, such as a galaxy cluster, and hence the need for dark matter to fit the cluster dynamics into the regular virial theorem. We can eliminate the need for dark matter in galaxy clusters using the updated virial theorem with both Newtonian and MONDian gravity included.

$F_T = F_M + F_N = \dfrac{GM_0}{RR_0} + \dfrac{GM_N}{R^2}$ (36), where M_0 is the mass within R_0, M_N is the spherically distributed mass between R_0 and R (R > R_0), and M_0+M_N is the total baryonic mass.

$\dfrac{GM_0}{RR_0} + \dfrac{GM_N}{R^2} = \dfrac{v^2}{R}$; $v = \sqrt{\dfrac{GM_0}{R_0} + \dfrac{GM_N}{R}}$; $T = \dfrac{1}{2}m\left(\dfrac{GM_0}{R_0} + \dfrac{GM_N}{R}\right)$, which is the updated virial theorem based on this theory for a simple gravitationally bound system with contributions from both Newtonian and MONDian gravity. The updated virial theorem can be applied to a galaxy cluster using two point masses located at the center of the cluster: the first point mass M_N exerts Newtonian gravity, and the other point mass M_0 exerts MONDian gravity. The sum of all baryonic masses within the a_0 radius of the individual galaxies in the cluster becomes the mass of the point mass M_0 exerting MONDian gravity, and the sum of the rest of the baryonic masses in the cluster becomes the mass of the point mass M_N exerting Newtonian gravity in the absence of intergalactic black holes.

In galaxy clusters, it is assumed that approximately one-tenth of the total baryonic mass is present within the a_0 radius of the individual galaxies with central black holes, that is, $M_N = 9M_0$. The additional dynamic mass (dark matter) predicted by the regular Newtonian virial theorem can be identified by equating it with the updated (Newtonian + MONDian) virial theorem.

$T = \dfrac{1}{2}m\left(\dfrac{GM_0}{R_0} + \dfrac{GM_N}{R}\right) = x\dfrac{1}{2}m\dfrac{G(M_0+M_N)}{R}$, where x is the dark-matter factor and m is the gravitationally bound point mass orbiting the cluster at its edge.

For $M_N = 9M_0$, $T = \dfrac{1}{2}m\left(\dfrac{GM_0}{R_0} + \dfrac{9GM_0}{R}\right) = x\left(\dfrac{1}{2}m\dfrac{10GM_0}{R}\right)$; $a_0 = \dfrac{GM_0}{R_0^{\ 2}}$; $R_0 = \sqrt{\dfrac{GM_0}{a_0}}$

For the Coma cluster, solving the above quadratic equation for the dark matter factor x with the Newtonian gravitational mass $x(10M_0) \approx 6.2\times10^{15}\ M_\odot$, the radius $R \approx 6.17\times10^{23}$ (~20 Mpc) [16], and $a_0 = 1.2\times10^{-10}$ in MKS units, we get $x \approx 7.24$. Therefore, the regular Newtonian virial theorem predicts that the total gravitational mass of the Coma cluster is ~7.24 times the total baryonic mass which means that approximately 86 percent of the total gravitational mass in the Coma cluster is identified as dark matter.

For the Virgo cluster, with the Newtonian gravitational mass $x(10M_0) \approx 7.4\times10^{14}\ M_\odot$ and the radius $R \approx 2.4\times10^{23}$ (~7.4 Mpc) [17] in MKS units, we get $x \approx 8.74$. Therefore, approximately 88 percent of the total gravitational mass in the Virgo cluster is identified as dark matter.

For the Fornax cluster, with the Newtonian gravitational mass $x(10M_0) \approx 3.32\times10^{14}\ M_\odot$ and the radius $R \approx 1.59\times10^{23}$ (~5.18 Mpc) [18] in MKS units, we get $x \approx 8.66$. Therefore, approximately 88 percent of the total gravitational mass in the Fornax cluster is identified as dark matter.

As the percentages of dark matter predicted in the galaxy clusters (Coma, Virgo, and Fornax) are in line with the standard model of cosmology and are completely accounted for by the updated virial theorem using only the baryonic mass, this theory does not require dark matter to explain the missing mass in galaxy clusters. Therefore, according to this theory, the generalized virial theorem for any gravitationally bound system with both MONDian and Newtonian dynamics based on (33) and (35) is given below, where R is the virial radius, U_0 is the MONDian gravitational potential energy owing to mass M_0, and U_N is the Newtonian potential energy owing to mass M_N.

Virial theorem:

$$\langle T \rangle = \frac{1}{2}\left(\frac{\langle U_0 \rangle}{\ln\left(\frac{R}{R_z}\right)} - \langle U_N \rangle \right)$$

9. The temperature of the universe

The internal energy U of black-body photon gas is given by $U = \left(\frac{8\pi^5 k^4}{15h^3c^3}\right)VT^4$ [19]

where k = Boltzmann constant, h = Planck constant, V = Volume, and T = Temperature.

As the number of photons $N = \left(\frac{16\pi k^3 \zeta(3)}{h^3c^3}\right)VT^3$, $U = \frac{\pi^4 NkT}{30\zeta(3)}$.

The maximum possible temperature T_{tp} of the universe when the age of the universe is t_p is given below by upholding the law of conservation of energy and considering the CMBR as black-body radiation. The energy U and the number of photons N remain constant with the expansion/contraction of the universe. However, owing to the time dilation or decreased cosmological potential energy, the local photons in the past appear to be more energetic (blue-shifted) owing to the increase in frequency ν. Therefore, the energy U in the past appears to be U(1+z) locally with the invariant local Boltzmann constant k, because the energy is relative in this theory.

$U(1+z) = \frac{\pi^4 NkT_{t_p}(1+z)}{30\zeta(3)}$; $T_{t_p} = T(1+z)$, which can be generalized as follows:

$$T = T_0(1+z) \tag{40}$$

Alternatively, using the relative Boltzmann constant $\frac{k}{(1+z)}$ with respect to the current epoch instead of k in the energy U yields the same equation (40) by upholding the law of energy conservation. As the current CMBR temperature T_0 = 2.725 K and the maximum possible cosmological redshift z is 140.15 (5), T_{tp} = 384.64 K. As the CMBR was emitted after the initial t_p of the Big Repulsion, the original temperature of the CMBR when it was first emitted should be less than 384.64 K. The future temperature of the CMBR can also be calculated using the equation (40). For example, the CMBR temperature after 10^7 y should be 2.723 K. In addition, the kinetic energy of the baryon particles and the respective temperature would also be higher in the past (refer to Section 6).

As the entropy of a photon gas $S = \frac{4U}{3T}$, $S = \frac{S_0}{(1+z)}$ (40), where S_0 is the current entropy of the CMBR and S is the entropy at the cosmological redshift z.

As the interaction of the electrostatic expansion energy with the quantum vacuum fluctuations to create matter at the time of Big Repulsion will be the same throughout space-time, the created primordial elementary particles, atoms, and their attributes, such as temperature and density, should be uniform throughout space-time without having the particles interact with one another or without being causally connected, eliminating the need for cosmic inflation, hence this theory solves the horizon problem, the flatness problem, and explains the uniformity of the CMBR. However, minor fluctuations in the density of the created particles due to the randomness of the quantum vacuum fluctuations and the subsequent concentration of matter due to gravity could explain temperature anisotropy. However, the angular power spectrum of the CMB is yet to be explained by this theory. According to this theory, the MONDian gravity around the primordial black holes, if any, in the CMB should behave like dark matter, as the a_0 radius should be closer to the black holes because of the high MONDian cutoff acceleration $\left(\frac{cH(z)}{2\pi}\right)$, where H(z) is the decreasing Hubble parameter from the time of primordial black holes formation. In addition, black holes can act as seeds for galaxy formation, as per a recent study using the James Webb Space Telescope (JWST) [20].

10. Atomic structure

In this theory, the local Bohr radius a_0 does not change with an increase in the gravitational potential energy or cosmological potential energy owing to the expansion of matter with space-time.

$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{4\pi\frac{\varepsilon_0}{(1+z)}\left(\hbar(1+z)\right)^2}{m_e(1+z)e^2}$ based on Tables 2 and 3, where $m_e(1+z)$ is the relative rest mass of the electron owing to the gravitational and/or cosmological redshift z. However, the relative energy of the Bohr atom increases to $E_{nz} = E_n(1+z)$ as the relative Planck constant increases by the same factor [$\Delta E = h(1+z)\nu$].

$$E_n = -\frac{Z^2e^4m_e}{32\pi^2\varepsilon_0{}^2\hbar^2n^2} \quad ; \quad E_{nz} = -\frac{Z^2e^4m_e(1+z)}{32\pi^2\left(\frac{\varepsilon_0}{1+z}\right)^2\left(\hbar(1+z)\right)^2n^2} = -(1+z)\frac{Z^2e^4m_e}{32\pi^2\varepsilon_0{}^2\hbar^2n^2} = E_n(1+z)$$

We observe a gravitational and/or cosmological redshift z as ΔE increases, owing to the expansion of matter with space-time. However, the local numerical value of the energy of the Bohr atom remains invariant, as shown below, because the local physical laws, constants, and rest mass remain invariant (Table 5).

$$\left(\left|E_n(1+z)\right| = (1+z)\frac{Z^2e^4m_e}{32\pi^2\varepsilon_0{}^2\hbar^2n^2} \equiv \left(h(1+z)\nu = m(1+z)c^2\right)\right) \equiv \left(\left|E_n\right| = \frac{Z^2e^4m_e}{32\pi^2\varepsilon_0{}^2\hbar^2n^2} \equiv \left(h\nu = mc^2\right)\right)$$

11. Conclusions

This theory demonstrates that both space-time and matter expand at the relativistic Hubble rate analogous to an elastic ruler with constant proper distance and volume, establishes the relativistic acceleration of space-time by proposing cosmological time dilation analogous to gravitational time dilation, and substantiates the relativistic Hubble law. The relative values of the physical constants are shown to vary due to the expansion/contraction of space in the cosmological and gravitational frameworks but are compensated for by the proportional change in the relative rest mass/potential energy. A relativistic Newtonian theory of gravity has been proposed, which adheres to special relativity in local inertial frames, Lorentz covariance, gauge invariance, and locality. The derived Schwarzschild radius, ISCO, IBCO, gravitational deflection, perihelion precession of Mercury, and Shapiro time delay concur with the general theory of relativity, with the exception of the photon sphere. Relativistic GEM equations are derived to substantiate the existence of gravitational waves and gravitomagnetic effects. The MONDian gravity model proposed in this theory is applicable only in the vicinity of black holes and not in the presence of ordinary matter. This theory postulates length contraction/expansion, time dilation, and varying potential energy in both cosmological and gravitational frameworks. It eliminates black hole singularities, precisely defines the scalar gravitational potential, and maintains universal momentum and energy conservation. In contrast, the general theory of relativity lacks a precise definition of gravitational potential, and universal space- and time-translation symmetries are not generally applicable. The general theory of relativity does not elucidate the mechanism of dark energy that accelerates the expansion of the universe and necessitates dark matter that has not yet been discovered, whereas this theory successfully explains the dark energy and dark matter in galaxy clusters using an updated virial theorem. In summary, this theory proposes a hidden extradimensional electrostatic background to gravity to elucidate dark energy, relativistic cosmology, relativistic gravity, and dark matter in galaxy clusters, which eliminates cosmic inflation, cosmic event horizon, cosmic scale factor (a), critical density (Ω), cosmological constant (Λ), and black hole singularities. However, the angular power spectrum of the CMB remains to be explained by this theory.

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Methods:
The computations in this paper were performed by using Maple™. Maple 2020.2. Maplesoft, a division of Waterloo Maple Inc., Waterloo, Ontario. Maple is a trademark of Waterloo Maple Inc.

Data availability statement:
Data sharing is not applicable to this article as no new data were created or analyzed in this study.