RANDOMNESS AND CAUSALITY IN QUANTUM PHYSICS

ELEVEN FAMOUS QUANTUM EXPERIMENTS

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ABSTRACT

Quantum Physics is usually defined as a theory that affirms a primary role of randomness and probability. Eleven well-known quantum experiments are examined and the result is the coexistence of both random and causal behaviour, necessary to describe experiments. Quantum Mechanics states the general overcoming of causality and this statement constitutes an unlimited generalization, not supported by experiments. Determinism and indeterminism are philosophical systems, that universalize causality or chance. The crucial point is the difference between epistemic and intrinsic randomness. In the first aspect, randomness does not have a fundamental meaning, in the second, randomness of an individual event is explained, but the detection of high stable regularities remains to be explained. The article also addresses the question of entanglement and various aspects of probability theory, including the law of large numbers, arriving at the thesis that many relevant questions are unresolved. A causal description is in quantum mechanics impossible in principle, given its assumptions. In order not to contradict experience, both points of view are necessary, causal and random. This is the state of the research, to return to and start again from.

INTRODUCTION

The title of the work is not at all intended to suggest that QM could be a non-indeterministic theory, which would be spectacularly unsustainable, but rather that it is necessary to analyze, beyond generic formulations, the actual role played in the theory by the *random* and the *causal*. When we talk about quantum physics, in writings and speeches of more or less every type, popular or research, it is usually defined as a theory based on *randomness* and on *probability*. Mostly, *sic et simpliciter* indeterministic. Reiterated that the essential role of randomness and probability in Quantum Mechanics is beyond question, it can nevertheless be demonstrated that things are a little more problematic. This constitutes the fundamental point of the paper. I add that we do not even intend to reduce the issue a priori to the concepts of random and causality, because the phenomenon of entanglement, before any other phenomenon, verified experimentally, has opened new chapters of research, introducing instant actions at a distance (spooky action, this is how Einstein characterized entanglement) and the question of correlations, that can be hypothesized in some class of phenomena, not describable in terms of cause-effect, but neither

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reducible to random behaviour. We mention the concept of *acausal correlations*, introduced into the debate on causality and randomness, with the Pauli-Jung correspondence and investigations into various phenomena in different fields. It seemed useful to include this aspect for completeness.

The relationship between *randomness*[1] and *causality*[2] constitutes one of the most debated and profound questions in the history of philosophical and scientific thought. We come from a long history of fierce and highly refined lines of thought. It will be enough to remember, starting from Leucippus[3], Democritus, Plato, Epicurus, Aristotle, Lucretius and continuing with T. Hobbes[4], G. Galilei[5], R. Descartes[6], B. Spinoza[7], G. W. Leibniz[8], D. Hume[9], I. Kant[10], P.S. Laplace, H. Poincaré[11] and so on. We like to remember Wittgenstein's thought, also for its particularity. He starts from an empiricism similar to the Humean approach (« A construction, according to which one thing must happen because another has happened, does not exist. There is only logical necessity»[12]) and he arrives at a position, for which the causal structure is included a priori in the logical form of the world, close to Kantian theory.

The definition of determinism and indeterminism is not at all univocal. These terms indicate two or more families of doctrines, historically rich in variations. There is also some ambiguity about the concepts[13]. Funny and profound, in this regard, the essay, on the probability of the improbable, by well-known statistician and mathematician D. J. Hand[14]. If you want to avoid sinking into quicksand, it is essential to define the concepts to be used. For example, if we took the extreme formulations, that of Laplace[15], which constitutes a clear conceptualization of mechanistic determinism, and that of Bohr[16] and Heisenberg[17], which proclaims the final defeat of causality, we would find ourselves faced with incompatible alternatives, without possible meeting points .

It would be equivalent to stating that the universe is either governed by an inexorable determinism or by an integral indeterminism. The topic of acausal correlations, as mentioned above, will lead us to ask whether it is correct to exclude a priori the possibility of detecting statistical correlations between phenomena, such as constant sequences, concomitant variations or functional dependencies, which in some cases may not be describable in terms of cause-effect connection. We are calling them, only conjecturally, *acausal correlations* to distinguish them conceptually both from random events and from causal links[18].

We are not referring here to 'synchronicity', which is outside of this article, and to topics, which seem to belong to the soft sciences rather than physics[19]. Also because someone might observe that, as Ruelle states, the analysis of the properties of triangles was more fruitful of the interpretation of dreams, for the scientific description of natural phenomena[20]. Conversely, Ruelle could be reminded of the importance of openness to the various areas of human experience.

B. Russell writes : «All our data, both in physics and psycology, are subject to psycological causal laws; but physical causal laws, at least in traditional physics, can only be stated in terms of matter, which is both inferred and constructed, never a datum. In this respect psycology is nearer to what actually exists»[21]. Physics however is, and we sincerely hope it remains, an experimental science.

Jung himself warns : «We must obviously be careful not to consider any event whose cause is unknown as acausal. The use of the concept of acausal is admissible only in cases in which a cause is not even conceivable»[22]. The problem is whether acausal correlations, that cannot be reduced to chance, are detectable and what their ordering principles might be.

It also seems useful to introduce the distinction between *events related* in causal ways (and possibly acasually) and *independent events*. The theoretical point is represented by the philosophical question, which remains in the background, of the interdependence or independence between events. One can in fact hypothesize that random events would preferably require to be unrelated, or independent to allow, in the space of alternatives, the equivalence of outputs, which could be thought of as the optimal condition for randomness.

One can ask oneself whether such matters are a scientific problem or a philosophical problem, we pragmatically prefer to maintain that it is a problem of human thought, scientific and philosophical, which cannot be evaded.
However, the theme of the work is well defined. We will try to examine and show what some very well-known and basic quantum experiments have to say about it. Having examined the experiments and outlined the debate, we will draw some provisional conclusions.

DISCUSSION

The concepts indicated will be used with an explicit definition, even if reduced to the essentials, without going into the philosophical dispute, which would involve explaining exactly the differences between determinism, mechanistic determinism (what happens is the product of pure mechanical causality, according to the prevailing vision of science in the 19th century), principle of causality, randomness, indeterminism, necessary causality, probabilistic causality , also acausality and so on in an endless undertaking. There are very valuable works , by philosophers and physicists, even recent ones, which face the difficult challenge of clarifyng the history and the perspective of concepts and terms. They are books and articles to meditate on, starting from The Oxford Handbook[23], to continue with papers which are very profitable to explore further[24].
The aim of this work is more limited. We will use *causality* in the elementary sense that what happens must have a cause or a constant relationship of succession occurs, which determines the predictability of the subsequent event and *randomness* in the sense of what happens without there being a reason or at least being able to indicate it. The expression *intrinsic* or *ontological* randomness and *apparent* or *epistemic* randomness are also used in the literature[25]. Random would mean that the probabilities that an event occurs or not, that is the alternatives, appear to be randomly distributed[26]. Let's insert here a quick excursus on the concept of *random*. It is a common idea of everyday language, but it does not have a precise and univocal meaning. Random and randomness have multiple and quite controversial meanings, for example : a) the concatenation of many causes, mostly unknown, which come together through the most varied coincidences; b) the absence of a purpose, i.e. the opposite of deliberate actions; c) the absence of meaning, that is, the fact that insignificant actions determine formidable or catastrophic effect. In the philosophical and scientific language it is necessary to use exact definitions. In the history of thought different definitions have been given, depending on the periods and cultures. Telling its story is not the task of this work. We could propose to also introduce, among others, a very rigorous definition of random as an event interdependent on other events, without high regularity being detectable or even as an independent event, also with the possible application of the passage to the limit in the usual way, that is, causing the quantities considered to tend towards zero. All of the above, I think, may be sufficient to make the following arguments understandable enough. These are quite clear definitions for the purposes of this paper. We do not believe, for example, it is necessary to introduce a difference between randomness and chance, as others do. Obviously it is different to affirm the absence of causes and not get to know them. It is also correct to make a distinction between causality and randomness on the one hand and determinism and indeterminism on the other hand. The first two constitute interpretative schemes of the relation between events, the second two are theories that generalize and organize the first, affirming or denying the universality of the causal principle. It should also be made clear that saying that an event has a cause is not the same as saying that it has meaning. These are different plans. The causes and their effects seem to be on the plan of what happens in nature, while the meaning belongs to the observer and does not concern nature at all.

To underline the scale and meaning of the debate between the 17th and 18th centuries, without following its course here, we can limit ourselves to reporting two highly authoritative positions. Leibniz summarizes like this : «Nihil est sine ratione sufficiente, cur potius sit, quam non sit» and clarifies that nothing happens without a determining reason, that is, something that explains a priori why what exists exists rather than not and why it exists in this way and not in another way[27]. Hume, for his part, states that all reasoning about cause and effect are based on experience and since everything that is based on experience constitutes a supposition that the course of reality will continue uniformly, we conclude that similar causes, in similar situations, will always produce similar effects[28].

Let's see the thought in the field of physics. Newton's[29] and Maxwell's[30] physics is entirely deterministic, although not in harmony with each other. Even Planck's[31] and Einstein's[32] physics is based on the causal principle. It is well known that, on the contrary, the quantum revolution affirms a central and ineliminable role of randomness, in an ontological sense, i.e. inherent in reality, not epistemic. «QM makes chance intervene in a new and intrinsic way»[33]. The central point is the fundamental randomness of the phenomena that occur in microscopic processes. QM addresses the causality – randomness problem in a completely new formulation. We could talk about a new paradigm. The denial of principle of causality is written in its presuppositions. Reading carefully, we have all studied it even in the great Landau : «Hence it is clear that, for a system composed only of quantum objects, it would be entirely impossible to construct any logically independent mechanics»[34]. In this regard will see Bohm's rigorous criticism below[35].

A very clear example is given by the 'half-life', that is, the time it takes for half of the nuclei of a radioactive substance to disintegrate. Can anyone predict whether a particular uranium nucleus will decay before dinner or in 5 million years? Nobody. We cannot have anything other than a probability of decay as a function of time. These themes were among the main terrains of the great battle between Einstein and the architects of the Copenhagen interpretation, led by Bohr, Heisenberg and, particularly on these issues, Born[36]. It is usually said that, for quantum physics, events happen to a large extent without there being a cause. Heisenberg states : «But what is wrong in the most drastic formulation of the principle of causality, 'when one knows the present precisely, one can predict the future', is not the conclusion but the premise. Even in principle we cannot know the present in every determining element»[37] and further says : the space-time description of events and the classical causal law represent two complementary aspects, therefore they are mutually exclusive[38]. Von Neumann claims apodictically : in macroscopic physics there is no experience that proves the principle of causality, because the apparent causal order of the macroscopic world has no other origin than that of large numbers and , therefore, there is no reason that allows us to affirm the existence of causality in nature and no experience can give us the trial[39]. Thus, it's not possible for quantum physics to accurately predict the outcome of observing a single system and only probabilities can be calculated. According to Heisenberg, Bohr, Born, Pauli[40], von Neumann etc., classical determinism would be set aside and the principle of causality would be surpassed as one of the essential cornerstones of the functioning of reality.

I would like to point out that indeterminism is not a result of quantum mechanics, but is actually written into its underlying assumptions. The decisive point is the relation between subject and object or, more precisely, between observer and observed. On this aspect a widening is essential. In quantum physics, the definition of a state requires the elimination of every external perturbation, but any interaction constitutes a perturbation, including observation. For a particularly clear exposition on the topic, please refer to N. Bohr, *Epistemological discussion with Einstein* and *The Quantum Postulate and the Recent Development of Atomic Theory*[41], and also to two famous articles of W. Pauli[42].

Bohr summarizes his explanation with a simple and brief formulation:

«…the description of an experimental device and the results of the observation must be made in an unambiguous terms, with the appropriate application of the terminology of classical physics… This crucial point implies the impossibility of a clear separation between the behaviour of atomic objects and their interaction with the measuring instruments that serve to define the conditions in which the phenomenon manifests itself. In fact, the individuality of typical quantum effects finds its expression in the fact that any attempt to divide the phenomena necessarily requires a change in the experimental device, and introduces new possibilities of interaction between objects and measuring instruments, which cannot be controlled in principle»[43].

The consequences for Bohr are logical and immediate : «…the data obtained in different experimental conditions… must be considered *complementary…* the attribution of traditional physical properties to atomic objects implies an essential element of ambiguity… … in the *indeterminacy relation*… it cannot be expressed with the same words that are used to describe classical physical images …»[44]. Bohr continues the reasoning thus : «… While the combination of these concepts [space-time concepts and conservation dynamic laws] into a single picture of causal chains of events constitutes the essence of classical mechanics…the study of complementary phenomena requires mutually exclusive experimental devices»[45].

The conclusion necessary follows. The unambiguous use of physical concepts and the uncontrollable interaction between objects and measuring instruments forces us to give up causal description. In other words, the complementary point of view can be considered «…as a rational generalization of the ideal of causality itself»[46].

Let's add some formulations by Pauli, of the same significance and perhaps even more lapidary. In two famous articles, Pauli summarizes his position on complementarity, space, time and causality. Let's read, among other things : a) Each of the exact measurements «implies a partially indeterminate and indeterminable interaction in principle between measuring instrument and measured object»[47] ; b) «The state can only be described with statistical information about the distributions of values of the results of possible position and momentum measurements in this state»[48]. c) In short, as consequence of the fact that a part of the interaction must always remain undetermined, there is a clear cut between object and instrument. Furthermore, this occurs on the basis, up to a certain point, of an arbitrary choice on the object to be measured and on the measuring instrument. d) Causality « … loses its univocal meaning as a consequence of the new epistemological situation originating from the need to distinguish measuring instrument and measured object and from the partial indeterminability of their interaction»[49]. Pauli is elsewhere even more explicit, if one can : every observation «…interrupts the causal connection between the phenomena that precede it and those that follow it»[50].

With the revolution brought about in particular by Bohr, Heisenberg[51], Pauli and Born, the games on the concept of causality seem to be over. More generally, assuming the assumptions mentioned above, i.e, the interpretation of the observation, the principle of uncertainty and the complementarity of Bohr, classical mechanics is out of play and, with it, the access to the individuality of the object and the concepts of continuity and causality. There was, clearly, no shortage of the unconvinced and the bearers of a critical position, starting from Einstein, de Broglie and Schrödinger. Respected, but cast aside, as nostalgic conservatives[52]. The scope of the quantum revolution, which pushes for a sort of deconstruction of the object and the space-time framework, is very clear even in Dirac. Let's read two short passages. Given that classical physics leads to the formation of 'mental picture in space and time of the whole scheme', he continues : «It has become increasingly evident in recent times, however, that the nature works on a different plan. Her fundamental laws do not govern the world as it appears in our mental picture in any very direct way, but instead they control a substratum of which we cannot form a mental picture without introducing irrilevancies. The formulation of these laws requires the use of the

mathematics of transformations»[53]. Dirac continues further on : «… it may be remarked that the main object of physical science is not the provision of pictures, but is the formulation of laws governing phenomena and the application of these laws to the discovery of new phenomena. If a picture exists, so much the better; but whether a picture exists or not is a matter of only secondary importance. In the case of atomic phenomena non picture can be expected to exist in the usual sense of the word 'picture', by which is meant a model functioning essentially on classical lines»[54]. On the topic of causality Dirac agrees with Bohr, Heisenberg, Pauli… : «A consequence of the preceding discussion is that we must revise our ideas of causality … Causality applies only to a system which is left undisturbed»[55]. Differential equations can express a causal connection only when they are used to describe an undisturbed system and are, in this case, in close correspondence with the equations of classical mechanics. «There is an unavoidable indeterminacy in the calculation of observational results, theory enabling us to calculate in general only the probability of our obtaining a particular result when we make an observation»[56].
Several years later, D. Bohm began trying to reopen the game. A great unorthodox thinker, who proposes a causal reinterpretation of QM, through the hypothesis of a subquantum mechanical level, containing hidden variables[57]. We can read his highly autonomous interpretation of quantum mechanics in his various works and, regarding causality and chance, particularly in the aforementioned specific work (1957)[58]. We can summarize his lesson in some fundamental points, from our defined visual angle:
A) Once the assumptions of QM have been accepted, «…one is no longer able to describe or even think about well-defined connections between the phenomena in a given instant and those in a previous instant»[59]. B) «The traditional interpretation of quantum theory requires abandoning the concepts of causality, continuity and objective reality of individual microobjects», and consequently «physics is implicity and inevitably limited to manipulating mathematical symbols…which only allow the calculation of the probable behavior of phenomena observable in the macroscopic field»[60].
C) «These important changes in the conceptual structure of physics are based on the assumption that certain characteristics of the current formulation of quantum theory, namely the uncertainty principle and the appearance of a set of 'complementary' pairs of behavioral modes are absolute and final properties of the laws of nature, which continue to apply, uncontradicted and without approximation, in every sector of physical investigation»[61]. D) Are QM assumptions demonstrable and demonstrated? Are we justified in attributing to them an unlimited validity ("absolute and final"), not limited to the atomic sphere? Bohm's thesis is that, between classical determinism and quantum indeterminism, we have "two dogmatic and arbitrary extremes"[62]. He extensively explains the " philosophy of mechanism" and critically identifies a transition from deterministic mechanism to indeterministic mechanism[63]. E) Determinism and indeterminism are philosophical systems, not physical theories[64]. F) «… Neither causal laws nor laws of chance can ever be completely correct because, inevitably, they neglect some aspects of what happens in broader contexts. … We do not assume, as is done in mechanistic philosophy, that all of nature can sooner or later be treated in a complete, perfect and unconditional way on the basis of only one of these aspects, so that the other is considered inessential»[65].

We have outlined a profoundly diversified landscape, up to the threshold of statistical mechanics which we will briefly mention below, but we aim to show that things are more complicated, watch out, even for quantum theory itself, that is, for the conduction and description of quantum experiments by experimental quantum physicists themselves. If God plays dice, he does not, however, seem to always play dice in an absolute and universal way, including every step of quantum experiment. The description of the experiments requires, as we will see better below, classical devices and terminology. To definitively and completely overcome the cause-effect connection, at all levels of experience, truly strong and probative arguments must be made. The causal link is derived from an universally consolidated experience of the macroscopic world. Of course, even at the macro-physical level a lot of things happen, the causes of which we don't know, but we are led to think with a certain reasonableness that there are reasons, even when we don't know them. Furthermore, it should be kept in mind that determinism does not imply predictability, likewise, unpredictability does not necessarily imply randomness. Chaotic systems, e.g., they are deterministic, but are predictable only within the limited horizon of predictability. Popper's[66] position should be underlined on the relationship and difference, which he places emphasis on, between determinism and predictability. Mathematicians have thoroughly studied the consequences of non-linearity and of the chaotic character that can be assumed by deterministic dynamics.

The quantum world may be different, the same laws that apply elsewhere do not operate in the atom. This is what emerges from the experiments.

In a while , we will find ourselves examining experimental situations, in which we will see *random elements together with causal elements* or, in other words, *a certain quantity of randomness* coexisting with *a certain quantity of causality.* M. Born, who formalized the probabilistic interpretation of the wave function, spoke of 'a deterministic theory of probability', as J. Bell recalls in 'Speakable and unspeakable in quantum mechanics'[67].

Aristotle himself already observed[68] : «Luck and chance are also causes» and we know that an event can depend on the results of an inconceivably large number of accidental events».

To follow Heisenberg, it can already be noted that 'knowledge of the present' and 'prediction of the future' are on the epistemic level, that is, they concern not the phenomena, but our knowledge of the phenomena and their predictability. Also Heisenberg, however, was consistently very clear, as Bohr and Pauli : the concept, expressed in various writings, is that with QM the invalidity of the law of causality is definitively established.

From the examination of the experiments, solid arguments will emerge on the more complex and subtler character of quantum theory. For these reasons we will argue that certain representations of the theory can be seen as arbitrary generalizations. The coexistence of the two aspects constitutes a line of interpretation to be tested. Moreover, we can begin to glimpse it in the Schrödinger equation itself, which governs the evolution of states in a deterministic way, even if it only concerns the probability of predictions about states. It can also be observed, regarding the quantum measurement process, that there is upstream an arbitrary choice of the observer and that the outcome of the measurement is probabilistic and not predictable, but that nevertheless it is this choice that determines the normalized state, i.e., the reduction of the wave packet.

Having made a brief reference to the history of the discussion on our topic, it may useful, to facilitate understanding, to introduce a distinction between a *strong form* and a *weak form*, applicable both for causality and for randomness. We will indicate : a) as *strong form* the theories which respectively assume the cause-effect connection, on the hand, and the absence of it, on the other hand, as universally valid concepts; b) as *weak form* the theories which consider both the constant succession of events (or the concomitant variation or functional dependence) and the random occurrence of events to be a product of experience, without making them principles of universal validity.

In this context we can clearly place the studies of the 20th century, with a notable development in recent decades, on statistical physics for the understanding and description of stochastic phenomena, on the nature of induction and probability and also the concept of probabilistic causality[69].

ELEVEN WELL-KNOWN EXPERIMENTS

Experiments 1), 2), 3) concern the phenomenology of photons, from 4) onwards they can be extended to other particles (electrons, protons, neutrons, atoms…) and other properties like spin, depending on the case.

Be careful, we will have to consider the entire experimental structure, including the preparation, apparatus and execution of the experiments. It is the observer/experimenter who prepares the device, the system and everything. The entire procedure inevitably requires a sequence of operations characterized by a high degree of causal behaviour.

The experimental apparatus work according to the laws of classical physics and the description of each experiment must be clearly done in classical terms. There are no possible alternatives, as Bohr makes clear : « …althoug the phenomena may transcend the explanatory possibilities of classical physics, the exposition of each experiment must be done in classical terms[70].

In the analysis of the experiments we will not use, except in special cases, the terms 'deterministic-indeterministic', or 'causal-random', which presuppose generalizing theories, one or the other, but rather descriptive expressions, such as 'casual type behaviour' or 'causal type link'.

1) Let's start the series by examining a very simple experiment. A beam of light impinges on a semi-reflective mirror (beam splitter): half of the light passes through the mirror, half is reflected, as recorded by the installed detectors. We cannot know where the single photon will go, but we know at least two things with certainty: a) if we send a beam of light and this affects the mirror, we have put in place a chain of events that generate effects of a passing/reflecting of the beam; b) we discover that there is a regularity, with approximately 50% reflection and approximately 50% passing, and that this rule *always* works, approximately 100%, under the same circumstances. We are, perhaps, not entirely in the realm of randomness, but we will have to investigate further. On the case of the experiment, with two possible outcomes, which repeated a very large number of times gives 50 % and 50%, we propose to return to the end of the article. What happens using just one photon? This cannot be divided into two, we record that the two outcomes occur with the same probability of approximately 50%. It is not possible in any way to predict the behaviour of the single photon, but only the statistical distribution. The experiment attests to the role played by *casual type behaviour* in fundamental steps and also the emergence of a regularity and, furthermore, by analyzing the entire operation the action of a chain, the phenomena that are calling *causal type link*. Randomness and causality seem to coexist. We could express ourselves with the words that Melville , in Moby Dick makes Ishmael say : «… chance, free will and necessity, far from incompatible, intertwining, all work together»[71].

2) The light from a lamp, with a mixture of a multiplicity of possible polarizations, passing through a polarizer, prepared in such a way as to let only electric fields oriented along a certain direction pass, assumes according to the setting of the instrument a polarization, let's say, vertical . We know what happens if we install a second polarizer. If the two devices are parallel, all the light *always* passes through , if they are perpendicular none passes under any circumstances , if they are oriented obliquely, part of it *always* passes, naturally depending, In accordance with the law of Malus, on the angle between them (it varies with the square of the

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cosine of the angle, but we don't care now). What matters to us here is to verify what presents a random type behaviour and what, in reverse, a causal type behaviour. We install a first polarizer, we prepare it to produce a certain effect, then we install a second polarizer and prepare it to have a multiplicity of orientations; each orientation produces a different effect, but with an absolute regularity, except for experimental errors. The structure of the experiment presents unequivocally casual type aspects (what the single particle does in the case of oblique orientation), but also causal type aspects (with parallel devices everything passes, with orthogonal devices none). The latter compatible, indeed compliant, with the causal type connection.

3) Let us now consider a source that emits two particles at a time, for example photons. The directions are not predetermined, but the experiment is set up so that the directions have the constraint of being diametrically opposite. The source emits , barring experimental errors, particles that go in exactly opposite directions. So far everything causal type behaviour? Not at all. The conditions we impose on the experiment sometimes hide pitfalls. The reference is to the Horne[72]-Zeilinger[73] thought experiment of 1985. Here the casual type behaviour appears. The particles, in fact, can have any position and the directions, at the exit of the slit, can be, according to Heisenberg's principle, all possible ones. It is not important now to describe the continuation of the experiment (it is easily found in the papers and in a beautiful book by Zeilinger[74]), but to try to understand how casual and causal type behaviour are intertwined in this experiment. The thesis, which I put forward and which needs to be showed in this work, is that in quantum theory and in the related experiments we see, in components well-defined, casual type behaviour and, above all although not exclusively, general components, based on causal type link, operating together.

4) Let's look at Bohm's[75] famous thought experiment (1952). A source emits pairs of particles prepared so as to always have opposite spin. The original particle, in fact, is prepared with zero spin and decays into two particles. The spin, on the chosen rotation axis, can only take on one value. The value is fixed and only the direction can vary, i.e. the spin can be directed up or down, without intermediate positions. Since the angular momentum remains constant and was originally zero, the sum of the angular momentum of the two particles must equal zero. The particles do not have a spin before the measurement, but if one is measured in any direction and is found, at random, in one of the two possibilities, the other will take approximately 100% the opposite direction. It's the so-called entanglement. The following two observations can be made: a) the measurement of the spin of the single particle has a random type result, but a constant correlation is detected, i.e., given the spin value of one particle, we can instantly and correctly predict the other's spin; in short, there is no causal correlation, but a rule operates. b) It is always stated that in quantum mechanics it is not possible to make predictions on the behaviour in a single measurement, but only statistical predictions. Is this statement really and completely corresponding to what happens in this experiment? One can legitimately doubt, whether this is entirely the case. Everyone knows that quantum mechanics makes extremely precise predictions about the behavior of ensembles, but here the case seems different. In fact, in the case under consideration, if one particle has spin up, the other will certainly have spin down, i.e., we are making *correct predictions on individual cases*. Probabilistic predictions, but approximately 100% correct.

Can the behaviour found, in points a) and b), be explained by resorting to the macroscopic character of the device and terminology? The burden of proof is on those who support it, not on us. The demonstration seems far from easy. We will return to this point later.

5) The 'acid test', one may imagine, could be given by single photon experiments. These are experiments that have been carried out for a long time and have now become almost routine.

However, worth examining. Well, let’s choose a very simple experiment. A single photon passes through the first polarizer and will come out (approximately 100%) polarized according to the preset orientation. Let's install a second polarizer and see what happens. If the polarizer is oriented parallel to the first, the photon (approximately 100%) passes through it, if it is oriented at a right angle, the photon (approximately 100%) is absorbed, if it is oriented at an oblique angle, for example halfway, i.e. 45°, the photon, which is an indivisible something, can only either pass through or be absorbed. This is where chance comes into play . The probability of passing or being absorbed is approximately 50%, no more can be said about the behaviour of the single photon. Every single photon will be transmitted or reflected in a random behaviour, but if we are talking about an ensemble of photons, what we can say is that approximately half will pass and half will be absorbed. It happens that only one of the two detectors records a photon, never both at the same time. The realization of one or the other possibility, passing or absorption of photons, cannot be explained in any way. One might conclude that we do not know the cause of the photon's behaviour or that there is no cause at all or again that another type of rule operates, which produces a correlation. The answer of quantum mechanics is the second. It is legitimate to keep the discussion open? We’ll come back to it later. Whatever the answer, we can already see, also in this experiment, except for the case of the oblique angle, that a fundamental random type component and a set of steps governed by causal type links are at work. As in the cases previously analyzed.
The explanation given by QM rests on the superposition of two or more possibilities and on collapse of the wave function, pillars of quantum theory. The collapse of the wave function resolves the quantum superposition, but I am the experimenter who, having brought a particle into superposition of paths, decides whether to detect the trajectory or maintain the superposition. There is something else besides chance. Can all this be explained with classical apparatus and terminology, which are nevertheless necessary? Isn’t clear enough that there is an intertwining of random type behaviour and causal type link in the entire structure of the experiment? An arbitrary choice produces necessary effects.

6) A particularly interesting experiment is the one, this too with a single photon, performed by installing two polarizing optical beam splitters. The photon is sent to the first PBS (Polarizing Beam Splitter), with the superposition of two paths, which are made to converge towards the second PBS. The result is that both constituents of the polarization exit from the 2° PBS in a single direction, while in the other direction no photon exits. The experiment is described in numerous texts, among which, for example, we mention the very careful examination by Zeilinger[76]. What is interesting to underline here is that, detected without any doubt the random type behaviour at the exit from the first PBS, practically everything else seems to unfold according to the usual causal type rules*.* In the first path there is a vertical polarization that passes (approximately 100%) the second PBS and in the other path the deflected component, with horizontal polarization, which is reflected (approximately 100%) also by the second PBS, with the result that the photon is always detected in a single direction. It must be concluded, even in this experiment, that we find a well-defined random type behaviour and that everything else seems to show causal type behaviour*.*

7) Let’s also briefly mention the double slit experiments[77], which concern the wave/corpuscolar character of light and matter and have a long and extremely beautiful history, starting from Youngs[78] original one in 1801, rich in variations both in the technical apparatus (polarizers, PBS, interferometer…) and in the types of experiment (particle beams, one single particle at a time…) or particles used (photons, electrons, neutrons, fullerene, tetraphenylporphyrins, atomic radii…). These experiments are also, rightly, commonly taken to demonstrate the role of randomness in physical phenomena*.* To a certain extent this is surely correct. Which slit the

particle passes through or even neither or both at the same time, whether or not the interference effect is produced : in the actual behaviour among the various possibilities there is a fundamental random type behaviour. We remember furthermore, that trajectory and interference figure are complementary. Therefore, we cannot talk about trajectory if the specific experiment is not carried out and, once the trajectory has been determined, the interference figure regularly disappears. Likewise, the interference pattern can regularly be observed or not, depending on whether the information on the trajectory is available or not. Also in these cases it is the experimenter's choice to decide whether one wants to know the trajectory or detect the interference pattern. One or the other choice produces opposite effects. We have used the term *regularly,* but we could use the term *always*, provided that one point is clear : there are intermediate stages and that trajectory and interference are mutually exclusive, always, only if we want to know one of the two, with absolute precision. We can now detect that also in double slit experiments there is a certain combination of random type behaviour and causal type behaviour, similarly to the experiments already examined. The analysis of double slit experiments is generally focused on another aspect, namely on *interference* and the *decoherence* phenomenon , and mainly on the perturbation due to the act of measurement and on the distinguishability of the paths (the *which way* problem), but also the point of view, considered here, of the random-causal behaviour involves an essential aspect.
We are bound to imagine a clear separation between randomness and causality, but it could happen differently in phenomenal reality, as we are seeing in the considered experiments.

Let us now apply the same method to three fundamental and truly famous experiments:

A) the Stern-Gerlach experiment, which has as its main content the indivisibility of the quantum;
B) the Mach-Zehnder experiment, on quantum superposition;
C) the experiment Zhou-Wang-Mandel (1991) and the experiment Hong- Ou-Mandel (HOM 1987).

Subsequently, we will briefly broaden our attention to two topics, also to be examined below the profile random-causal relationship : entanglement and decoherence.

8) We will see that even in the Stern-Gerlach experiment[79], analyzed from our specific point of view, we can highlight that a random type component and a set of causal type concatenations are together at work. The experiment, using mainly atomic beams and electrons, concerns the deflection of particles[80] and constitutes one of the main evidential tests of quantization. In fact, if the particle enters a non-homogeneous magnetic field, a continuous distribution of possible spin values is not obtained. These, which previously could have any orientation, when they are measured, can only place themselves up or down with respect to the direction of the magnetic field. More precisely, when a particle passes through the device it cannot do anything other than take on the values $+\frac{\hbar}{2}$ or $-\frac{\hbar}{2}$. Also in this case, we register the presence of a random type component, as it is in no way possible to predict whether the measurement of the single particle will give spin up or down, and a series of causal type behaviours. The 'rules' we encounter are the following: a) not disparate spin, with any orientation, come out of the device, but only one value between two possible, with a precise orientation, because the axis is given by the direction of the magnetic field; b) the measurement of a particle automatically fixes the state of the other particle and the measures must basically always be opposite; c) if the original axis and the measurement axis are orthogonal, the result will be approximately 50% up and 50% down, that is, the single measurement, any of the two, is unpredictable and clearly random type, but the second will assume always the exact opposite direction; d) the measurement results of the two particles are approximately 100% correlated; e) ultimately, we see a combination of random type behaviour (the single measurement) and a set of causal type concatenations, including automatic correlation between measurements of two separate particles, regardless of distance. This experiment leads to addressing the question of the relation between quantum theory and local realistic theories and therefore it connects us to the Bell's formidable theorem.

9) The experiment with the Mach-Zehnder interferometer[81] is carried out with 4 mirrors, 2 of which are normal, which reflect all the light, and 2 beam splitters, which reflect exactly half of it and let the other half pass. The experiment is also applicable to particles other than photons, including fullerenes. A source emits photons, which can be detected, if the two branches are both active, at the two possible outputs. The beam from the source on the left arrives at the 1st beam splitter and is split in half between the upper and lower trajectories. Both arrive at the two normal mirrors, which reflect completely, and then flow into the 2nd beam splitter. In a classical explanation each of the two beams should come out with half the light. But it doesn't happen this way, due, according to the quantum interpretation, to the interference produced by the superposition of the partial waves, which is created in both trajectories and in both beams. Now, given that the partial waves of the upper and lower trajectories are not equal, since the length is the same, but the number of reflections is different, the interference will in one case be constructive (they are in phase) and in the other destructive (they are in phase opposition), we note in fact that the partial wave, towards the right, passes twice, the other, directed upwards, is reflected twice. The consequence is that all the light coming from the source (i.e. all the photons) will go into only one of the two outputs. If, however, we proceed not with a coherent light beam, but with a luminous flux reduced to a photon-by-photon emission, we will see that, if both trajectories are open, the photon will have only one exit, while if in one trajectory an obstacle of absorbing material is inserted, both outputs, repeating the emissions, will bring light, each ¼, then added ½, of the photons emitted (approximately 50% will in fact have been blocked by the obstacle). It's an amazing result.
It can be observed that from the point of view of this work, which is the relation between randomness and causality , we have nothing new compared to the cases already examined, in the sense that we find the combination of a well-defined random type behaviour and for the rest a functioning of the causal type links in the theoretical and experimental structure. The disconcerting aspect is another. The Mach-Zehnder experiment, in its various versions, with a coherent light beam, with low intensity photon by photon, in the standard configuration or in the one with an obstacle, highlights the insufficiency and dissatisfaction of any classical but also quantum interpretation. There is no interpretation that does not raise conceptual problems. The very tenacious Zeilinger himself concludes that we must limit ourselves to a 'truly minimalist' interpretation: the waves are probability waves, a mathematical aid for calculating probabilities, what we have is an interference of completely abstract probability waves and also the wave is simply our tool for calculating probabilities[82]. The puzzle can be profitably pursued with the Elitzur A.-Vaidman L. 'defused bomb' thought experiment[83]. Furthermore, the interferometer-based experimental device can be modified with an additional detector, which allows measurements such as which way, resulting in the disappearance of interference. With this we fully run into the 'mysterious' problem of the relationship between the distinguishability/indistinguishability of the paths and the appearance/disappearance of the interference figures, which is at the center of the following two experiments.

10) The 1991 experiment[84] (Zhou X.Y., Wang L.J., Mandel L.) and the 1987 experiment[85] (Hong Chou Ki, Ou Zhe-Yu, Mandel L.) represent, especially the second, refined examples of the arcana of quantum physics. For the specific topic we are dealing with, i.e. the randomness-causality relation, we could limit ourselves to repeating what was said above about the Mach-Zehnder experiment. The now usual combination between a defined presence of random type events and a general causal type concatenation presents itself again. But how do you avoid the temptation to mention these astonishing experimental results? The 1991 experiment, on coherence and indistinguishability in optical interference, tested two possible processes, using non-linear crystals capable of producing pairs of photons, when excited by a laser, one indistinguishable (A) and the other distinguishable (B). In A, there is interference between the two trajectories, in B, it is possible to distinguish the two cases and only one of the photons reaches the detector. The meaning is that interference occurs only if the apparatus cannot provide any information about which of the two processes occurred. The experiment is in agreement with those with the double slit, in which there is information 'which way', even if the measurement does not concern the photon that arrives at the detector, but only the secondary photon. If there is the possibility of distinguishing the path of the photon, no interference appears. What should be underlined is this rather shocking element, whereby it is sufficient that the information is in principle available in the system to cause the disappearance of the interference.

11) The 1987 experiment, an effect in quantum optics of interference with two photons, HOM[86] for short, examines what happens when two photons in the same state, therefore indistinguishable, are sent to the two inputs of a beam splitter, with two detectors installed at the outputs. The experiment is very famous and anyone can find a description of it, so it is not necessary to give a detailed description. In a nutshell, there will be four possibilities of equal probability, according to a classical prediction: both transmitted or both reflected, to detector 1 or to detector 2 (therefore in both cases two photons on the same side) or one reflected and the other transmitted alternatively (therefore in the two cases one at detector 1 and the other at detector 2). The test disproves the classical framework of expectations, except when the photons are somehow distinguishable. The arrival times can be varied by modifying the position of the device and therefore it is possible to test the effects of the gradual increase/decrease of the distinguishability level. When the arrival times get closer, interference begins to appear and when the times tend to coincide, the probability that the photons end up in two different detectors becomes less than 50%, to the point of being zero for completely indistinguishable photons. Always two photons on the same side, on one side or the other. That is, in the two cases, in which total indistinguishability of the paths occurs (same wavelength, same polarization, precision of a few femtoseconds), the picture is this : a) in the first case superposition and, therefore, interference occurs; b) in the second case a coincidence never occurs , i.e., one photon per output, but always two from the same output, one or the other (the other two possibilities are eliminated by destructive interference); c) in the third case the photons emerge, mind you, always together from the same side, but which side is random ; d) in the fourth case, the two detectors will each continue to count approximately 50% of the photons emitted and, therefore, it is not the behaviour of the single photon that changes (approximately 50 and 50%), but that of the pair of photons; e) in the fifth case not all particles behave the same way. In fact, a 1998 experiment, with electrons and a potential barrier instead of a beam splitter, shows an increase in coincidences at the two outputs, up to the suppression of the probability that the electrons arrive together at the same output. In short, the photons, as good bosons, end up in the same exit, while the electrons, which are fermions, behave in the opposite way. Photons go to an output randomly, but always together, electrons always go alone in different outputs. The mathematical explanation lies in the *exchange rule*. In extreme summary : if two processes differ only for the exchange of two indistinguishable fermions, the associated phasors must be subtracted and not added,

while for the exchange of indistinguishable bosons they must be added. The experiment has, for photons, yet another development. As mentioned, by gradually decreasing the level of indistinguishability, the interference gradually disappears and the number of cases in which the photons exit the two different exits progressively increases. There is more. What happens if the photons are entangled, e.g., for polarization? If they are entangled and the measurement will be performed after the two photons have left the beam splitter, the result will be that they will end up in the same exit and the same polarization, let's say horizontal, will be verified when it is measured. But, as we know, in entanglement the polarizations can be different and even mutually orthogonal. The experiment was performed by Zeilinger in Innsbruck in 1996[87].
The two entangled photons impact, as above, on a beam splitter and, if the entanglement is such that they are always orthogonally polarized, the result obtained is that the photons will *always* come out one per exit , with mutually orthogonal polarization. In this case, Zeilinger concludes the photons, which are bosons, behave like fermions. So, if the two photons are entangled to have a different polarization, they will behave like fermions, ending up in different outputs. From the point of view of experimenters, when two photons go to different outputs it means that they are entangled in the aforementioned way and this clearly has important experimental consequences (e.g. for quantum computers, for teleportation...). This fascinating experiment doesn't suggest anything new on the topic of randomness-causality, compared to what we found previously, however it seemed useful to include it in the review of experiments. Now, one thing must be said, events are events and experimental data are experimental data, but it cannot be denied that this last group of experiments is a little disconcerting and also that the theoretical explanations may appear sometimes to be *handmade*. In particular, experiments in which the introduction, even if only in principle, of an observer or a detector changes the behavior of a system, raise serious questions. It is the theme of the relationship between path distinguishability and interference, which we find also in the double slit experiments. Furthermore, the question does not only concern the paths , but the *processes* or *stories,* through which an initial state evolves into a final state. Taking the example of the path of a signal photon, which is not way varied in the setup of an experiment, we see that if the apparatus is able to distinguish the path, through the control photon, the result is different. Therefore, not only the actual acquisition of information on a quantum system, but rather the mere availability of information changes its evolution. Now, it can be understood well, and shared without difficulty, that in quantum mechanics the instruments are internal to the model and that a change (the addition of a detector, a shift or any variation...) can modify the system and the results of the measurements, but it must also be highlighted in which difficult waters we are swimming or sailing. The even possible existence somewhere of information about *history,* as well as about the path, destroys the interference. What reason would there be to think that the behaviour of nature has preferences based on the presence or not of information for a possible observer about the distinguishability or indistinguishability of paths and stories? It is more reasonable to think that nature, beyond the areas and levels of interaction, does what it wants, regardless of the observer. By putting together the underlying indeterminacy of the properties of the quantum system and the condition, even just in line of possibility, of the indistinguishability of paths and stories , the risk is the loss of all descriptive power and all predictive power, if everything is applied with logical rigor, not only the superposition and interference, but also the possibility of the experiment itself! Zeilinger himself, regarding such riddless, writes : « … when a particle finds itself in a situation for which it has no instructions, it must behave completely randomly. It can be said that nature is not rich enough to have already determined a priori the answers to all the questions. Many questions – and if you think about it, they are the majority - must therefore remain without a certain answer»[88].

On these topics, in addition to the scientists mentioned, many others also made a great contribution. They must certainly be remembered, for example, J. A. Wheeler and A. Aspect, with its team of collaborators, for the extraordinary theoretical and experimental work. In this paper, however other experiments have been preferred, as the specific purpose is to study the question of causality and randomness, which is only one piece of the whole.

There are quite a few of these puzzles to solve, along with others. We can do nothing but examine and evaluate the explanations that are given. For now, this is what we have.

Can we say, however, that we can legitimately doubt whether the explanations given so far are the definitive ones?

We have summarized eleven of the most famous quantum experiments. We could propose many others, in various sectors of quantum experimentation, to strengthen the thesis proposed, but it is not essential. Wherever you go, you can find the coexistence of channels classical and quantum, type causal behaviour and type random behaviour[89]. If we deny this point, we encounter logical and interpretative inconsistencies, because a profound contradiction arises between theory and experience.

RANDOMNESS AND CAUSALITY IN ENTANGLEMENT AND DECOHERENCE

In the cases examined, we have already encountered the phenomenon of *entanglement* (the term was coined by Schrödinger) several times . We are talking about experiments, first thought experiments and then carried out, of decisive importance, also from the point of view developed in our reasoning. Indeed, it can be stated that entanglement is not a hypothesis, but a fact. Historically there have been many variations and different modalities, starting with the fact that you can measure polarization, spin or other properties. The entanglement of two particles was the subject of the famous article EPR[90], as well as Bohm's[91] thought experiment. In these first experiments we find: 1) a random type behaviour in the act of measurement of a property, whatever it may be, of a particle A; 2) a constant concatenation regarding the measurement result between the entangled particles A and B. For example, if the spin of A is up, the spin of B will be down. Correlations between incompatible observables (by the Heisenberg principle) can be predicted, even at a distance and not subject to interaction with each other. Can a random outcome be so stably regular? The phenomena do not seem describable either in causal terms or in terms of pure chance. In fact, there is no causal effect, but not even pure random behaviour in this correlation. Obviously the interpretations are subject to verification entrusted to ongoing experiments[92]. If there were randomness, the spin of B could be again up, as can happen analogously to the faces of a coin. However, this does not happen. If two or more particles or quantum objects are entangled, the correlation works necessarily and instantaneously, anywhere in the universe. It seems quite plausible that this isn't pure chance and that there must be more to it than that. In fact, there is a clear regularity, which allows precise predictions. A random result would contradict experience. The correlation could perhaps depend on the Pauli exclusion principle or on the conservation of the total spin of the system, so the sum of the spins before the interaction remains equal to the sum after it. In each case, we see a rule, or law, at work, not chance. The prediction is practically certain, even, in certain cases, for a single particle.

In the theoretical and experimental evolution of entanglement, formulations that were more attentive to objections emerged, starting from EPR, after the versions mentioned. We are now

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no longer referring to two particles, but two components of a single and indivisible system. Entanglement really means non-separability of the system. The measurement concerns the entire system and there is no information that travels between the two components. Each of the two components has two possible states in quantum superposition. The measurement instantly fixes the state in each component, but a concomitant connection always occurs between the two fixed states. So A can give P or Q, but if A gives P, B gives Q and vice versa. This correlation is instantaneous, constant and independent of distance. We cannot speak of cause-effect relationship, because the system is one; certainly P or Q measurement appears to be random type, but the concomitant correlation, i.e. the opposite behaviour of the two components, does not appear to be random. It is not a question of causality, but a rule emerges, which governs the experiment.
EPR aims to demonstrate the incompleteness of QM, but the critical reasoning is very acute and attacks quantum non-locality and non-separability. It raises the question of space. This explains the theoretical persistence of EPR, beyond the variants. Separability is «the possibility of separating two different objects or parts of an object and consider each one as an entity in itself, at least in principle»[93]. On separability and locality, please refer to the beautiful work of G. Musser. We limit ourselves to observing, as regard the relation with Special Relativity, that to guarantee compliance it is not enough to respect the velocity limit *c*, as several scholars state, when the concepts of separability and spatial distance are substantially called into question[94]. Experiments in this field are still in full development. Among the recent experiments we suggest that of Colciaghi and colleagues from the University of Basel[95].
The experiment is carried out with two clouds of hundred of rubidium-87 atoms, released by a Bose-Einstein condensate, in which the entanglement between the isospins was caused.
By examining the entire structure of the experiment, it is possible to detect, from our specific point of view, random type elements and causal type sequences. Not only the former nor only the latter. It would seem that nature mixes both.
Among the most promising experimental lines are the experiments relating to interferometry with neutrons, even those not specifically on entanglement, but relevant to the topics covered here. Let's just mention a significant example, dating back to 1985. Thus the result is summarized by the authors : «… the distribution of intensity in the neutron beams leaving the device and the changes in the intensity distribution induced by phase-shifting elements placed between the crystals depend sensitively on the coherence properties of the neutron radiation as it enters the interferometer»[96].
To be very rigorous and subtle, we would discover that mechanism, we highlighted in entanglement, also operates in the similar way to that of measurement process and reduction of wave packet. We use, for brevity and clarity, Ghirardi's formal description. Let's take the simplest case, with only two possible outcomes, mutually exclusive[97], as in the measurement of the polarization of a photon along a given direction *Q*, characterized by a state of flat polarization in the *P* plane, which forms an angle θ with *Q* plan. The one or the other outcome is a genuinely type random and is characterized by probabilities, defined by the decomposition of the plane polarization state *P* , on the polarization states along *Q* and *R* (perpendicular to Q). That is :

$$|P\rangle = \cos\theta\, |Q\rangle + \sin\theta\, |R\rangle \qquad 1)$$

The probabilities of the outcomes will be : a) $\cos^2\theta$ b) $\sin^2\theta$ 2)
Identifying with the observable *W* the one that corresponds to passing the test, with value +1, and to failing, with value -1,
the observable can take only one of two values, +1 or -1. Let's carry out the measurement and assume that *W = +1.* Ghirardi continues : «Se ripetiamo immediatamente la medesima

misura sul sistema dobbiamo pretendere…che si ottenga di nuovo con certezza il risultato +1»[98]. It is a basic concept, which we find masterfully expressed by Landau : the typical setting of the quantum mechanics problem «… consists in predicting the result of a subsequent measurement from the known results of previous measurements»[99]. Even if we skip a few simple steps, the process is clear. The expected result $W = +1$ is obtained only if we eliminate the part $sin\theta$ IR> and replace $cos\theta$ with +1.
The formalism instantly transforms the system, that before the measure was in state 1), in state IQ > 3)

Ghirardi summarizes : «Il repentino cambiamento dello stato del sistema per effetto del processo di misura, un cambiamento casuale che dipende dall'esito della misura stessa, viene indicato tecnicamente come riduzione del pacchetto d'onde : lo stato, dopo la misura deve coincidere con uno stato che garantisce che il sistema dia, se assoggettato di nuovo alla stessa misura, lo stesso risultato»[100]. It is necessary to be cautious in such a fine mechanism, but perhaps an analogy can be seen with what was noted above for entanglement. In the measure process we have clearly random type behaviour, but at the same time we note the emergence of a rule, that is, a behaviour that does not appear to be pure randomness. If the new state determination were random, it could be different from 3), at least in a significant proportion, but this contradicts experience. The experiments give results with a high degree of regularity. Could these be random results? If they were, how could they repeat themselves in a highly stable way?
We are not attributing it to Ghirardi, the conceptualization is ours.

Decoherence, in turn, works according to a causal type law. If an interaction occurs between a quantum system and the environment or the observation and measurement apparatus or other, there is no uncertainty as to whether the system can lose or retain its quantum coherence. If there is interaction, coherence is instantaneously destroyed and the story ends there. If interaction occurs, the superposition of trajectories and the interference in the process of measurement are not 100% possible. The prediction is certain, even in relation to a single system. How much randomness is there? Chance does not appear to play a role in this aspect of the theory[101].

A FURTHER PROBLEM

A theory, any theory, consists of a structure of regularity, even partial and evolving, in which to frame the phenomena and experimental data. Quantum physicists, starting from Bohr and Heisenberg, have greatly emphasized the role of chance in theory and in nature and they have reached the point of affirming the definitive elimination of cause-effect connection. We have shown so far that in quantum theory, in all its phases and variations, there are many aspects that do not fit within this formulation and that are sometimes downright discordant. Even certain phrases or sayings belong to the language, which have a dissonant meaning. For example, we have seen: 'one or the other will *certainly* occur ', 'there is an interaction and the interference disappears', 'a measurement is made and the wave function collapses' and so on. I think we can say that quantum mechanics is not exactly the realm of pure randomness.

It is a more refined and more complex theory, beyond the representation given by many of its exponents.

Randomness in the ontological sense, strictly speaking, should mean the absence of regularities with a certain degree of stability, that could imply the predictability of events, of any regularity endowed with this property. Unless, having declared the principle of causality inapplicable, different principles ordering events are defined.

One of the most precise representations of random happening is constituted by the QRNG, the quantum random number generator[102]. Research is rapidly expanding, with positions ranging from “nothing is random, only uncertain” (Marsaglia[103]) to even opposite positions, such as, for example, interpretations, in which both classical and quantum mechanics contain a fundamental randomness[104] . As far as it seems plausible to us, there cannot be a logical theory of regularity in random sequences given by a QRNG.

On the other hand, if by carrying out the same measurements in identical experimental situations, always or frequently different results come out, we could even close laboratories and universities and go to the seaside!

A theory can certainly contemplate and contain chance, but it must necessarily define well and delimit its field. If random events were prevalent in nature or constitute the ordinary rule, the possibility of a theory (and also of complex forms of life, if not of any form of life) perhaps would not exist and, among other things, it would not even be possible to design and carry out an experiment. This is a difficulty not easy to circumvent on a logical level.

Anyway, randomness, probability[105] and statistical regularity take on a primary role in the discussion. B. Russel’s aphorism is very well known, according to which : «Probability is the most important concept in modern science, especially as nobody has the slightest notion what it means[106]. Laplace had attributed to the weakness of the human mind ‘ l’une des mathematique théories les plus fines et les plus ingenieuses, la science du hasard et des probabilités’[107].

Examined several historically and theoretically significant quantum experiments, we are faced with a further problem. Let’s take the problem first in the most simplified case.

We have two possibilities A and B (trajectories, spin or others properties), that is, there are two possible outcomes, such as an ideal coin (we are referring to mutually equivalent possibilities, for the so-called principle of indifference) and it is assumed that, when we operate with a large number of particles or perform a large number of repetitions of the experiment, the result tends statistically towards 50% A and 50% B. It almost seems like an inevitable natural phenomenon or law of nature. Clearly, the situation is different and more complex, as we will see better, when the possibilities are unequal, not equivalent, for example the throwing of an irregular solid or a ball in a pinball machine, or when the possible results are within a continuous range of values. We must evidently make use of the procedures and techniques of probability calculation. Not being specialists in the specific discipline, we leave the more complete and formalized discussion to others, but it is inevitable to discuss, at least from a qualitative point of view, some basic aspects. It is clear that when the 'space of alternatives' consists of only two equivalent possibilities, the relative frequency is ½. The relative frequency, when the number of experiments increases, tends to the limit and the probability is represented, in fact, by the limit of the relative frequency, as demonstrated by von Mises, essential figure in probability theory[108]. Now, even in relation to the most elementary case we must ask ourselves : is this result the product of a logical-mathematical demonstration? Or is it an empirical result? If you look into the matter carefully, the mathematical or logical demonstration can appear, or at least that’s how it seems to us, very difficult. Chance by definition does not follow rules. Furthermore, randomness has no

memory, otherwise it would fall back into causality and deny itself. The result of every single test is random and nothing prevents you, based on logical-mathematical point of view, from doing 1000 tests and getting the result 750 times A and 250 times B or any other result. Each event could give A or B, totally independently of the previously obtained sequence.
For the electron, for the photon or for the spin, every time is the first time.
Evidently, we must keep in mind that there are ensembles and that we must measure with them, theoretically and experimentally.
An important aspect is whether we should imagine that there is interdependence between events or not. if there were no interdependence between events, no event would influence another, that is, every event would be independent of any other. It is a typology of phenomena, a system with N non-interacting particles, studied in statistical mechanics.
It can perhaps be observed in the first instance, to be explored further, that if the events were independent of each other, it would be very difficult, maybe temerarious, to look for *a logical foundation* of the stability of regularities. It is easier to think in terms of interdependence between events. Even in this framework, there is no shortage of conceptual difficulties.
There are, in fact, serious arguments to support that, in a strict theory of chance, the connection between possibility and probability, i.e., between the distribution of possibilities and the probability distribution, would be cut. In other words, it is justified to ask whether a theory of probability can, on a logical level, be derived from a rigorous theory of chance, in ontological sense, or whether they are mutually compatible. Staying on logical ground, if the phenomenon is random, the probability of getting approximately 50% A and 50% B could be the same as getting any other probability distribution. Unless we can rigorously demonstrate that in the absence of causes and reasons, that influence the result, statistical regularity works as if it were a physical law. Regularity, the one with a high degree of stability and even more so those close to 100%, which are often encountered in quantum experiments, explains a lot, but it must also be explained. If the behaviour is random, why should outcomes show regularity with a high degree or stability? How is regularity formed, on a purely logical level, in going from a single measurement to a large number of particles or repetitions?
If we register a high degree of stable regularities we must ask ourselves what it is.
Let's now look at the case of unequal possibilities. It is quite natural to think about thermodynamic probability, for example in the kinetic theory of gases and Boltzmann entropy, where overall regularities emerge at the macroscopic level, mostly independent of the particular irregular movements of the atomic level. The Maxwell-Boltzmann distribution function is one of the three, as is well known, together with the Fermi-Dirac and Bose-Einstein statistics, which constitute its quantum update. The Maxwell-Boltzmann statistic allows the prediction, from a classical point of view, of overall average properties of the system, at the macroscopic level. We are using it as an analogy, expressed in a non-rigorous, but explanatory form, to think about the case of unequal possibilities. Let us therefore take the trend towards macrostates (thermodynamic states), in which a greater number of microstates are available[109].
Nature moves towards the macrostate, which corresponds to the maximum number of possible microstates. In short, it is as if nature, when it acts outside the causal order, does what presents the most possibilities. It is impossible to describe and make predictions in the microscopic dimension, but description and prediction are possible in the macroscopic one.
The analogy is strong and could suggest an explanation of 'random' behaviour in the case of non-equivalent possibilities, which we are now examining. However, one observation seems well founded, in that case we would have found a plausible conjecture : there would be something more than chance, there would be a rule or rather a law, powerful and of broader scope. At the state, however, we cannot say whether this could be an explanation.

Randomness does not seem to be able compressed into logical rules. If we follow this approach, even important points of statistical mechanics, such as Liouville's[110] theorem or the *equal a priori probability postulate*[111], could be considered of problematic application to quantum phenomena. In a strictly indeterministic theory, why a certain distribution in the phase space at time t_0 maintain the same configuration at time t_1? So too, if we examine, from logical point of view, the essential steps of equal a priori probability postulate, we immediately see that they are connected by causal links. In short, a system in equilibrium has no preference for any of the possible microstates, *therefore* each microstate has equal possibility, *therefore* the state, that can result from the greatest number of microstates, is the most probable macrostate of the system.
All the reasoning, done so far, is purely on the logical plan. Obviously, there is another path to follow, namely the *empirical method*. The explanation can be an observational and experimental nature. A rich, vast and multifaceted experimental activity has been accumulating, over decades, on the phenomena of atomic and subatomic level. Regularities of various kinds and various sizes are usually detected. Regularities should not be read as equal to *always*, but rather as equal to *variable and determinable degree*. Regularities, in fact, can be detectable, measurable and also quantifiable for the purpose of making predictions, not on individual events, which are not feasible, but on statistical regularities, that is, probabilistic predictions. The quantification of regularities, therefore, makes practicable probability theory and probabilistic calculation of predictability. Then we must be consequent.
This means that the counting must be actually performed, not inferred.
If the experiments confirmed the hypothesis just presented, as very plausible, it would be necessary to analyze the compatibility of the theory of randomness, in the ontological sense, with respect to experimental data. Anywhere, the trouble is that to reach empirical evidence you have to deal with really large numbers. Here comes the calculation.
Probability theory has also come a long way. The beginning of the theory is usually indicated in the correspondence between Pascal[112] and Fermat[113] on the questions posed by the Chevalier de Méré, a well-known gambler. But Huygens[114] also wrote on the topic and, if we wanted to be more complete, they should also be reread G. Cardano[115], G. Galilei[116] and L. Pacioli[117], which are chronological earlier. The reasoning gradually focuses on the measurement of the possibility of a random event occurring. We owe in particular to Bernoulli[118] the definition of probability and the law of large numbers (LLN). It is not necessary to recall here the contributions of many other thinkers, among them de Moivre, Laplace, Gauss, Legendre, Quételet, up to the formulation with Kolmogorov[119] of an axiomatic theory of probability. It had been known for some time that, by increasing the number of trials, progressively better probability values are obtained, now with Kolmogorov it is reconfirmed, consolidated and formalized that random phenomena, studied on a large scale, produce a regularity, which makes disappear randomness. The order appears with repetitions. But working with large numbers, especially if the ground is given by induction, it is anything but simple. At this point, fortunately, we have the concept of relative frequency and in general the mathematical treatment of empirical data, with the tools of probability calculation. In addition to the number of repetitions or tests, the number of possible options or outputs is important. For example, we could use a 'quantum random number generator' (QRNG) program, of which we are interested here in the purely theoretical aspect and not in the modern technological use of random numbers and the problems involved, taking the output of odd and even numbers as a parameter. You can run the experiment with a million 'launches' or with 100 million or a billion or however many you want, you just need the necessary computing power. We could also complicate the experiment by increasing the number of possibilities, choosing, for example, p

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numerical sequences, whose relative frequencies are to be verified in an interval q. Then, we have seen that according to Bernoulli's law, reiterated by Kolmogorov, generally accepted and applied in the most various disciplines, random events on large numbers reveal regularities. Logical evidence and empirical result are obviously not equivalent. An empirical law has the proper value of an empirical result, that is, it must be confirmed without exception by experimental data. The first point is repeat and verify the experiments, with many variations : how many identical tests, how many possible options, from here the relative frequencies and so on. We assume that with a very high probability the experiments, even with ensembles as large and complex as desired, give the expected regularities. This requires very rigorous examination. Some interesting works show that for infinite sequences, unlike finite ones, randomness is a binary property. That is, it may or may not be random[120]. Proceeding with the reasoning : «As a measure of randomness (or, more exactly, of nonrandomness) of a finite sequence, we consider the specific deficiency of randomness δ. In the second part… we prove that the function

$$\delta \,/\, l_n(1/\delta) \qquad 4)$$

characterizes the connection between randomness of a finite sequence and the extent to which the law of large numbers is satisfied»[121].

In the literature we can read a series of appreciated contributions, some of which we are mentioning, which give an extremely stimulating, but also somewhat controversial landscape. I may be wrong, but I doubt that we have reached an adequate experimentation and a satisfactory interpretation on this question. It should be perhaps noted that neither the axiomatic definition, nor the concepts of distribution, probability space, nor even the theorems of total probability, composite probability and a priori and a posteriori probability, can give us a definitive verdict.

The question seems like this to us : a large number of random events show regularities , with high stability, from which probabilities and probabilistic predictions are derived. Then, one of the two. Either the random events, beyond the fluctuations of precise details, are not really random, and the random appearance is due to our lack of knowledge, or an explanation must be given for the resulting regularity, that allows predictability, beyond the single event. It is not enough to simply say : it happens. Where does the regularity originate from? These results do not appear to be random. If it is randomness, there is no rule or predictability, if there are rules and predictability, it would be a very strange randomness. On these points, experimental research and theoretical discussion have been very open in recent years. You can see, for example, the beautiful review by Nath Bera and other co-authors[122], where you can read : «While the presence of randomness cannot be proven without making some assumptions about the systems, these assumptions are constantly weakened and it is an interesting open research problem to identify the weakest set of assumption sufficient to certify the presence of randomness».

Even Bohm's lesson on this issue is still alive and many researchers are working on it productively[123]. We summarize it briefly : the applicability of probability theory «… depends only on the objective existence of certain regularities which are characteristic of the systems and processes in question, regularities which imply that the long-term, or average, behaviour of a large set of objects or events is approximately independent of precise particulars, which determine exactly what will happen in each individual case»[124]. According to Bohm, in short, chance concerns 'precise details', beyond which objective regularities lie. What is at work, rather, is a tendency towards the annulment of 'casual fluctuations', that is, random contingencies that 'derive from outside the context in question'[125]. Even more precisely : «statistical regularities obtained on the basis of the large-scale annulment of random fluctuations»[126]. We would therefore have, before us, two opposing arbitrary generalizations. One attributes absolute and unlimited validity to the causal principle, the other affirms the unlimited validity of randomness. Unlimited extrapolation to all sets of possible conditions and phenomena is not based on theoretical and experimental development. Both are instead purely philosophical assumptions. The terms *philosophy* and *philosophical* are used by him in a descriptive sense, not in critical sense.

In this paragraph of the work we are discussing this aspect, both from a logical and an empirical point of view. Let's try to sum it up. Proving the law of large numbers from a logical point of view involves difficult problems, on the other hand explaining experimental data still remains a work in progress. It would be necessary to have a more detailed and complete catalogue of the experiments carried out. This would be essential to advance less conjectural interpretations.

It's, clearly, completely evident to us that the quantum procedure corresponds to the experimental results and above all that it is through statistical methods that a large part of scientific knowledge is built. The results of QT and of technologies based on quantum physics are formidable, as everyone knows, as well as in microphysics also in macroscopic phenomena, such as superconductivity, superfluidity, Bose-Einstein condensation etc.

The problem, discussed here, is not whether there are random type phenomena or whether science can be done very well with statistical methods, all of this is evident, the problem is whether QT is right when it applies its own assumptions to nature without limits.

Among other conjectures, one can also plausibly hypothesize that QT should be regarded as a simple means, very effective as is quite evident, of explaining the average behaviour of a large number of atomic systems.

However, it should be underlined that we are not talking about chance and probability in banal problems, but on the functioning of the fundamental components of reality and the relationship between phenomenal world and human knowledge.

The question remains difficult and intricate to unravel. Kolmogorov himself, for example, claims that there is no such things as a random sequence and that any particular sequence has equal probability of exactly zero[127]. Also because two issues must be kept in mind :

1) it would be necessary to rigorously demonstrate that the possible events and options can be perfectly identical to each other, but this is very difficult to ascertain. On this point, see for example, the position of Keynes, who criticizes the *principle of indifference*, assumed to treat probabilities as necessarily equal. The question, in any case, goes much beyond the character of this article; 2) probabilistic statements can only be transformed into certain statements if complete information is available on the events considered[128]. In the absence of complete information it does not seem possible to affirm the genuine random or causal nature of a process. As Ghirardi notes, randomness is implied by the assumption that theory is complete and that the theoretical description is exhaustive, that is : «… la specificazione del vettore di stato rappresenta l'informazione più completa che si può avere (in linea di principio e non solo pratica) su un sistema fisico»[129]. However, the completeness of the information about a physical system comes into conflict with other aspects of quantum theory, which we have analyzed previously[130].

CONCLUSIONS

The conclusions of this work are :

1) What did the analyzed experiments show us? The central data that emerges from the analysis of the eleven quantum experiments, considering the entire structure of the experiments, is the coexistence, side by side, better yet intertwined, in the actual experience of both random type and causal type behaviour. Neither one alone nor the other alone are able to describe the experimental data. We need both to describe the experience data. You can't easily escape chance[131] or, we could add, causality.
2) QM (Bohr, Heisenberg, Pauli, Born, Dirac and the most quantum physicists) states the general overcoming, with unlimited validity, of the principle of causality. This proclamation, according to which the causal principle would have been swept away, constitutes a universal generalization, not supported by the quantum experiments, as we have also shown above. One could say, without wanting to be disrespectful, that they declare the bridge collapsed, on which continue to run across, at least in some ways. Now, the fundamental role of random type behaviour of microscopic processes, that is in the atomic and subatomic sphere, is clearly evident, but it is another thing to affirm the liquidation of cause-effect link and the assertion of indeterminism, as a general rule of functioning of reality. Furthermore, causal type links and random type behaviours are patterns of relationship between events, observed in experience and subject to experimentation, instead determinism and indeterminism are on a different level. They are theories that universalize one causality, the other randomness. Theories must be demonstrated and verified. QT states more than we have been able to verify and more than is necessary to describe the experimental results. The experiments do not show that indeterminism or determinism, understood as doctrines that universalize the absence or action of the principle of causality, therefore in the *strong form*, can explain experimental phenomena, each alone on his behalf. Instead, we see both types of behaviour, those of causal type and those of random type, operating together.
3) Finding causal type connection not necessarily lead to an organically deterministic theory, just as finding random type behaviour does not justify an organically indeterministic theory.
   We have analyzed in the previous pages whether universalistic statements on the principle of cause or its negation are achievable logically and theoretically or experimentally.
   It should also be underlined that determinism and indeterminism in the *strong form* cannot be asserted without complete knowledge of the events. The concept can be extended, beyond determinism/indeterminism, also to the discernment between causal and random behaviour.
   The affirmation of the randomness of an event would require, as well the assertion of the causal link, a complete knowledge, with arbitrary precision, of the event and its occurrence. But QM does not allow the possibility of complete knowledge of an event (initial condition, boundary conditions, properties and precise evolution). Let's not now go into the topic of the distinction between pure quantum state, which is represented by a state vector, and mixed quantum state, represented by density matrix. It can be said that, as noted above, ultimately emerges an unresolved conceptual difficulty of QM, which on the one hand affirms the death of the principle of causality and on the other declares the non possibility of complete knowledge of an event or system[132]. It should be clear that if do not have complete knowledge, except the claimed completeness of the wave function, position not shared, e.g., by Born, randomness can only be epistemological, that is, related to the degree of information available. QM deduces the indeterministic theory from it, but this is a speculative conjecture. Exactly like the opposite one of determinism. If the data of a random system are effectively random, it would follow that no equation or prediction can contain the evolution of the system. Therefore, without complete knowledge of the events considered,

randomness or causality could consist of a subjective illusion. Both philosophical interpretations involve conceptual difficulties that cannot currently be overcome. To widening on this topic, please refer to two previous papers[133].

4) On entanglement : the detection of instantaneous and even remote correlations, not subject to the transmission of information, between components of an indivisible quantum system, cannot be explained in causal terms, but at the same time it cannot be explained in terms of randomness either, since the emergence of a rule is detected, which allows the predictabily of correlations.

5) Neither the causal nor the casual description, as we have shown, are capable of dealing with all the variety and richness of experimental phenomena. The present work, as you can see, has a point of affinity with Bohm's approach, on chance and causality. Bohm's *pars destruens* is fierce, the *pars costruens* of a quantum alternative to the widely prevailing interpretation requires a process of theoretical and experimental verification of a level much higher than the intentions and strengths of the author of this contribution. There is certainly no shortage of physicists, who consider Bohm's mechanics as a supplemented quantum theory[134].

6) The crucial point to keep in mind is the difference between epistemic randomness, due to our lack of information, and non-epistemic, i.e. intrinsic. In the first aspect, randomness would not have a fundamental meaning, in the second the randomness of the individual event is explained, but the presence of highly stable regularities, such as to allow predictability in ensembles, would remain to be explained.

7) On some aspects of probability theory, including the LLN, used not in a motivated and indispensable descriptive function of the empirical results, but in a foundational and demonstrative function of real laws, which constitute obviously different levels, several relevant questions remain open, in the sense shown above, both from a logical point of view and for the actual completion of the experimental program and for the interpretation of the regularities detected and verified. Even regarding probability theory, and the LLN itself, the conclusion is that there is still something be understood better.

8) On the topic of the coexistence of the random and causal behaviour, after having examined :
a) the eleven quantum experiment, b) the entanglement, c) the LLN question, we have introduced another line of research, that is, d) the reduction of the wave packet, which involves an essential concept of quantum theory.

The study of causality and chance in QT requires to be integrated with a formal analysis of the topic, within the framework of the three fundamental statistics, that of Maxwell-Boltzmann, which however falls within the laws of classical physics, and those of Fermi - Dirac and Bose - Einstein and therefore within the framework of QFT. This is the task scheduled for the development of the work, with the next step.

I would like to propose, in addiction to the conclusions set out above, three more brief final points, arising from the analysis conducted.

A) Not being able to have complete knowledge of a system, the cause-effect or chance question could be undecidable.

B) There is still a problem : without the causal scheme (we are not talking about determinism as a philosophical vision), that is, without being able to establish a causal connection or constant succession/concomitance or functional dependence between events of the macroscopic world, is it possible to practically reason or rather is it actually possible to survive? The issue also goes far beyond this work.

C) As a final stimulus for further research, it could be interesting to resume and develop the concept of probabilistic causality[135].

The profound conviction of the fact that even our fundamental theories are probable non-definitive, not final and non-complete has spread widely among physicists and among epistemologists and philosophers attentive to the work of physicists, theorists and experimentalists. We must be proud of quantum theory, the Standard Model, general relativity, the formidable advances in cosmology and so on, even if many things remain hypothetical. It is a study to raise a problem, first of all, among theoretical and experimental quantum physicists and philosophers. I hope that this work can contribute to updating the reflection on the topic addressed.

Pauli writes that «…after the unique original natural philosophy, which was however pre-prescientific … the conditions finally arose for a renewed agreement between physicists and philosophers regarding the epistemological foundations of the scientific description of nature»[136]. It was a wish, of which there is no sign, 70 years later, neither among physicists and philosophers, nor among the physicists themselves.

It is also already a good job to remove constraints that are not well motivated theoretically and experimentally , when they tend to take on the status of non questionable statements.

The games, however, still seem to be open.

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