$$Collatz\ Sequence\ Proof\ (2nd\ Way)$$

$let\ Collatz\ Sequence\ of\ (n) = S(n), \&\ let\ loop\ of\ Collatz\ Sequence\ (n) = lS(n)$
$Collatz\ Sequence\ rules: n\ (Even) : \frac{n}{2}, or\ n\ (Odd): 3n+1$

$Proof:$

$let\ n, a, b, c, w, r \in N_+$

$if\ S(n) = \{a, b, c, \ldots, w\} \Rightarrow lS(n) = lS(a) = lS(b) = lS(c) = \cdots = lS(w) \ldots Fact1$

$i) \because S(1) = \{4,2,1\} \Rightarrow lS(1) = \{4,2,1\}.$

$S(2) = \{1, 4,2\} \Rightarrow lS(2) = \{4,2,1\}.$

$S(3) = \{10, 5,16,8,4,2,1\} \Rightarrow lS(3) = \{4,2,1\}.$

$S(4) = \{2,1,4\} \Rightarrow lS(4) = \{4,2,1\}.$

$S(5) = \{16,8,4,2,1\} \Rightarrow lS(5) = \{4,2,1\}.$

$ii)\ if\ S(r) = \left\{\frac{r}{2}\ or\ (3r+1), \ldots, 4,2,1\right\} \Rightarrow lS(r) = \{4,2,1\}$

$\therefore\ lS(n) = \{4,2,1\}, \forall n \in Set\ Z = \{1,2,3,4,5,6,7,8,9, \ldots, \frac{r+1}{2}, \ldots, r\}.$

$iii)\ is\ lS(r+1) = \{4,2,1\}?$

$Part\ a)$

$If\ (r+1) \in N_{even}\ \Rightarrow S(r+1) = \left\{\left(\frac{r+1}{2}\right), \ldots, ?\right\} \Rightarrow lS(r+1) =\ lS\ (\frac{r+1}{2}) \ldots Fact1$

$\because \frac{r+1}{2} \in Set\ Z \Rightarrow lS\ \left(\frac{r+1}{2}\right) = \{4,2,1\}.$

$\therefore\ lS(r+1) = \{4,2,1\} \ldots (substitution)$

$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{even}.$

$Part\ b)$

$If\ n \in N_{odd} \Rightarrow S(n) = \{3n+1, \ldots, ?\} \Rightarrow lS(n) = lS(3n+1) \ldots Fact1$

$\because (3n+1) \in N_{even}$

$\therefore lS(3n+1) = \{4,2,1\} \ldots by\ Part\ a$

$\therefore lS(n) = \{4,2,1\}\ \forall\ n \in N_{odd} \ldots (substitution)$

$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{+ \ldots} by\ Part\ a\ \&\ Part\ b.$

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