Computation of Geometric Series on Numerical Expansions

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Abstract: Nowadays, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations for solving today's scientific problems and challenges. In this paper, the author introduces the sum of geometric series using numerical expansions. This idea can enable the students and scholars for further involvement in research and development.

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1. Introduction

Geometric series [1-6] played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, and management. In this article, the author gives a technique to create the sum of geometric series using numerical expansion.

2. Numerical Expansions & Geometric Series

Let us find the numerical expansions of 2^n and $\frac{1}{2^n}$ *and* its geometric series below:

$$2^n = 2^n \Rightarrow 2^n = 2^{n-1} + 2^{n-2} + 2^{n-3} + \cdots + 2^2 + 2 + 1 + 1 \Rightarrow \sum_{i=0}^{n-1} 2^i = 2^n - 1.$$

$$\frac{1}{2^n} = \frac{1}{2^n} \Rightarrow \frac{1}{2^n} = \frac{1}{2^{n+1}} + \frac{1}{2^{n+2}} + \frac{1}{2^{n+3}} + \cdots + \frac{1}{2^{n+k}} + \frac{1}{2^{n+k}} \Rightarrow \sum_{i=1}^{k} \frac{1}{2^{n+i}} = \frac{1}{2^n} - \frac{1}{2^{n+k}}.$$

Also, $2 = 2 \Rightarrow 2 = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} + \frac{1}{2^n} \Rightarrow \sum_{i=0}^{n} \frac{1}{2^i} = 2 - \frac{1}{2^n} = \frac{2^{n+1} - 1}{2^n}.$

$$1 = 1 \Rightarrow 1 = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^n} + \frac{1}{2^n} \Rightarrow \sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n} = \frac{2^n - 1}{2^n}.$$

$$\frac{1}{2} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^n} + \frac{1}{2^n} \Rightarrow \sum_{i=2}^{n} \frac{1}{2^i} = \frac{1}{2} - \frac{1}{2^n} = \frac{2^{n-1} - 1}{2^n}.$$

Similarly, we can create the sum of geometric series for other expansion.

$$\frac{1}{2^2} = \frac{1}{2^2} \Rightarrow \sum_{i=3}^{n} \frac{1}{2^i} = \frac{2^{n-2} - 1}{2^n}; \quad \frac{1}{2^3} = \frac{1}{2^3} \Rightarrow \sum_{i=4}^{n} \frac{1}{2^i} = \frac{2^{n-3} - 1}{2^n}; \ \cdots\cdots\cdots;$$

$$\frac{1}{2^{n-1}} = \frac{1}{2^n} + \frac{1}{2^n} \Rightarrow \sum_{i=n}^{n} \frac{1}{2^i} = \frac{1}{2^{n-1}} - \frac{1}{2^n} = \frac{2-1}{2^n}; \text{ and } \frac{1}{2^n} = \frac{1}{2^n} \Rightarrow 0 = \frac{1-1}{2^n}.$$

3. Conclusion

In this article, the author has introduced the sums of geometric series using numerical expansions. This idea can enable the researchers for further involvement in the scientific research.

References

[1] Annamalai, C. (2023) Computational Technique for Geometric Series with Radicals. *TechRxiv, IEEE*. https://doi.org/10.36227/techrxiv.24311773.v1.

[2] Annamalai, C. (2023) Computation of Algebraic Expressions and Geometric Series with Radicals. *TechRxiv, IEEE*. https://doi.org/10.36227/techrxiv.24311773.v2.

[3] Annamalai, C. (2018) Annamalai's Computing Model for Algorithmic Geometric Series and Its Mathematical Structures. *Journal of Mathematics and Computer Science*, 3(1),1-6 https://doi.org/10.11648/j.mcs.20180301.11.

[4] Annamalai, C. (2017) Analysis and Modelling of Annamalai Computing Geometric Series and Summability. *Mathematical Journal of Interdisciplinary Scienc*es, 6(1), 11-15. https://doi.org/10.15415/mjis.2017.61002.

[5] Annamalai, C. (2018) Novel Computation of Algorithmic Geometric Series and Summability. *Journal of Algorithms and Computation*, 50(1), 151-153. https://www.doi.org/10.22059/JAC.2018.68866.

[6] Annamalai C (2010) "Application of Exponential Decay and Geometric Series in Effective Medicine", *Advances in Bioscience and Biotechnology*, Vol. 1(1), pp 51-54. https://doi.org/10.4236/abb.2010.11008.