$$Collatz\ Sequence\ Proof\ (1st\ Way)$$

$Abstract$:

$Collatz\ Sequence\ rules: n\ (Even): \frac{n}{2}, or\ n\ (Odd): 3n+1.$

$With\ enough\ repetition, do\ all\ positive\ integers\ converge\ to\ 1?$

$Proof$:

$let\ \ n,x,y,z,t,u,v,k,r,h \in N_+, let\ \ x \in lS(n) = loop\ of\ Collatz\ Sequence\ (n),$

$let\ S(n) = \left\{\frac{n}{2} or\ (3n+1), \dots, ?\right\}, or\ S(n) = \left\{\frac{n}{2} or\ (3n+1), \dots\right\}$

$I\ have\ to\ prove\ lS(n) = \{4,2,1\}, \forall n \in N_+\ and\ proof\ it\ is\ the\ only\ loop.$

Clouds of Collatz Sequence's, $r\ is\ number\ of\ elements\ of\ lS(n), \&\ x \in\ lS(n)$

Using Collatz even and odd rules on x (the 1st element of $lS(n)$*) to find all clouds.*

Make each cloud = x to find value of x, and if $x \in N_+ \Rightarrow Collatz\ Sequence\ has\ \ lS(n)$.

$r = number\ of\ elements\ of\ lS(n)$ x = e or odd →

$r = 1$	→ $\frac{x}{2}$ = e or odd →	→ $3x+1$ = e →

$r = 2$	→ $\frac{x}{4}$ =e or odd →	→ $\frac{3x+2}{2}$ =e →	→ $\frac{3x+1}{2}$ =e or odd →

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$r = 3$	→ $\frac{x}{8}$ =e or odd →	→ $\frac{3x+4}{4}$ =e →	→ $\frac{3x+2}{4}$ =e or odd →	→ $\frac{3x+1}{4}$ =e or odd →	→ $\frac{9x+5}{2}$ =e →

$r = 4$	→ $\frac{x}{16}$ =e or odd →	→ $\frac{3x+8}{8}$ =e →	→ $\frac{3x+4}{8}$ =e or odd →	→ $\frac{3x+2}{8}$ =e or odd →	→ $\frac{3x+1}{8}$ =e or odd →	→ $\frac{9x+10}{4}$ =e →
→ $\frac{9x+7}{4}$ =e →	→ $\frac{9x+5}{4}$ =e or odd →					

$r = 5$	→ $\frac{x}{32}$ =e or odd *	→ $\frac{3x+16}{16}$ =e or odd $	→ $\frac{3x+8}{16}$ =e or odd !	→ $\frac{3x+4}{16}$ =e or odd @	→ $\frac{3x+2}{16}$ =e or odd #	→ $\frac{3x+1}{16}$ =e or odd %
→ $\frac{9x+20}{8}$ =e @@	→ $\frac{9x+14}{8}$ =e &	→ $\frac{9x+11}{8}$ =e ((	→ $\frac{9x+10}{8}$ =e or odd {{	→ $\frac{9x+7}{8}$ =e or odd [[	→ $\frac{9x+5}{8}$ =e or odd **	→ $\frac{27x+19}{4}$ =e ***

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$r = 6$	* $\frac{x}{64}$	* $\frac{3x+32}{32}$	$ $\frac{3x+16}{32}$	! $\frac{3x+8}{32}$	@ $\frac{3x+4}{32}$	# $\frac{3x+2}{32}$	% $\frac{9x+19}{16}$
% $\frac{3x+1}{32}$	$ $\frac{9x+64}{16}$	{{ $\frac{27x+38}{8}$	@ $\frac{9x+28}{16}$	# $\frac{9x+22}{16}$	% $\frac{3x+1}{32}$	$ $\frac{9x+64}{16}$	! $\frac{9x+40}{16}$
@@ $\frac{9x+20}{16}$		& $\frac{9x+14}{16}$	(($\frac{9x+11}{16}$	{{ $\frac{9x+10}{16}$	[[$\frac{9x+7}{16}$	[[$\frac{27x+29}{16}$	** $\frac{9x+5}{16}$
** $\frac{27x+23}{8}$		*** $\frac{27x+19}{8}$					

$A)\ let\ lS(n) = \{x\}, \forall n \in N_+$

$Cloud$	$Cloud = x$
$\frac{x}{2}$	$\frac{x}{2} = x \Rightarrow x = 0 \notin lS(\mathrm{n})$
$3x+1$	$3x + 1 = x \Rightarrow x = -\frac{1}{2} \notin lS(\mathrm{n})$

$\therefore x \notin N_+ \Rightarrow lS(\mathrm{n}) \neq \{x\}, \forall n \in N_+$

$B)\ let\ lS(n) = \{x, y\}, \forall n \in N_+$

$Cloud$	$Cloud = x$	y
$\frac{x}{4}$	$\frac{x}{4} = x \Rightarrow x = 0 \notin lS(n)$	….
$\frac{3x+2}{2}$	$\frac{3x+2}{2} = x \Rightarrow 3x + 2 = 2x \Rightarrow x = -2 \notin lS(n)$	….
$\frac{3x+1}{2}$	$\frac{3x+1}{2} = x \Rightarrow x = -1 \notin lS(n)$	….

$\therefore x \notin N_+ \Rightarrow lS(n) \neq \{x, y\}, \forall n \in N_+$

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$C)\ let\ lS(n) = \{x, y, z\}, \forall n \in N_+,$

$Cloud$	$Solve\ equation: Cloud = x$	y	z
$\frac{x}{8}$	$\frac{x}{8} = x \Rightarrow x = 0 \notin lS(n)$	---	---
$\frac{3x+4}{4}$	$\frac{3x+4}{4} = x \Rightarrow x = 4 \in lS(n)$	$\frac{x}{2} = 2$	$\frac{x}{4}$ =1
$\frac{3x+2}{4}$	$\frac{3x+2}{4} = x \Rightarrow x = 2 \in lS(n)$	$\frac{x}{2} = 1$	$\frac{3x+2}{2} = 4$
$\frac{3x+1}{4}$	$\frac{3x+1}{4} = x \Rightarrow x = 1 \in lS(n)$	$3x + 1$ =3(1)+1=4	$\frac{3x+1}{2}$ =2
$\frac{9x+5}{2}$	$\frac{9x+5}{2} = x \Rightarrow x = -\frac{5}{7} \notin lS(n)$	---	---

$\therefore x \in N_+ \Rightarrow lS(n) = \{x, y, z\} = \{4,2,1\}, \forall n \in N_+$

$\therefore S(n) = \left\{\frac{n}{2}\ or\ (3n+1), \ldots, 4,2,1\right\}, \forall n \in N_+$

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$D)\ let\ lS(n) = \{x, y, z, t\}, \forall n \in N_+$

$Cloud$	$Solve\ equation: Cloud = x$	y	z	t
$\frac{x}{16}$	$\frac{x}{16} = x \Rightarrow x = 0 \notin lS(n)$	---	---	---
$\frac{3x+8}{8}$	$\frac{3x+8}{8} = x \notin lS(n)$	---	---	---
$\frac{3x+4}{8}$	$\frac{3x+2}{4} = x \notin lS(n)$	---	---	---
$\frac{3x+2}{8}$	$\frac{3x+1}{4} = x \notin lS(n)$	---	---	---
$\frac{3x+1}{8}$	$\frac{9x+5}{2} = x \notin lS(n)$	---	---	---
$\frac{9x+10}{4}$	$\frac{9x+10}{4} = x \notin lS(n)$	---	---	---
$\frac{9x+7}{4}$	$\frac{9x+7}{4} = x \notin lS(n)$	---	---	---
$\frac{9x+5}{2}$	$\frac{9x+5}{2} = x \notin lS(n)$	---	---	---

$\therefore x \notin N_+ \Rightarrow lS(n) \neq \{x, y, z, t\}, \forall n \in N_+$

From Clouds of Collatz Sequence's above

When $r = 5$ *elements* $\Rightarrow x \notin N_+ \Rightarrow lS(n) \neq \{x, y, z, t, u\}, \forall n \in N_+$

When $r = 6$ *elements* $\Rightarrow x \in N_+ \Rightarrow lS(n) = \{x, y, z, t, u, v\} = \{1,2,4,1,2,4\} = \{1,2,4\}, \forall n \in N_+$

Pattern of Collatz Sequence

$r = 1 = 3(0) + 1$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{1-0}} \Rightarrow x = 1, but\ y = 4 \notin lS(n) \Rightarrow lS(n) \neq \{x\},\ \forall n \in N_+$
$r = 2 = 3(0) + 2$ $k = 0$	$x = \frac{3^0 x + 2^0}{2^{2-0}} \Rightarrow x = \frac{1}{3} \notin N_+ \Rightarrow x, y \notin lS(n) \Rightarrow lS(n) \neq \{\ x, y\},\ \forall n \in N_+$
$r = 3 = 3(1)$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{3-1}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{4,2,1\},\ \forall n \in N_+$
$r = 4 = 3(1) + 1$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{4-1}} \Rightarrow\ x \notin N_+, \forall n \in N_+$
$r = 5 = 3(1) + 2$ $k = 1$	$x = \frac{3^1 x + 2^0}{2^{5-1}} \Rightarrow\ x \notin N_+, \forall n \in N_+$
$r = 6 = 3(2)$ $k = 2$	$x = \frac{3^2 x + 3(2^0) + 2^2}{2^{6-2}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow\ lS(n) = \{x, y, z\},\ \forall n \in N_+$
$r = 9 = 3(3)$ $k = 3$	$x = \frac{3^3 x + 3^2(2^0) + 3^1(2^2) + 2^4}{2^{9-3}} \Rightarrow x = 1, y = 4, z = 2 \Rightarrow lS(n) = \{1,4,2\},$ $\forall n \in N_+$
$r = 12 = 3(4)$ $k = 4$	$x = \frac{3^4 x + 3^3(2^0) + 3^2(2^2) + 3(2^4) + 2^6}{2^{12-4}} \Rightarrow x = 1, y = 4, z = 2, \forall n$ $\in N_+$
$r = 15 = 3(5)$ $k = 5$	$x = \frac{3^5 x + 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8}{2^{15-5}} \Rightarrow x = 1, y = 4,$ Z=2 $\forall n \in N_+$

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$Equation\ of\ finding\ x\ value\ in\ the\ Collatz\ Loop$

$Part\ A$:

$i)\ if\ r = 3(k)$

$$let\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} = 1 \Rightarrow$$

$lS(n) = \{1,2,4\}\ , \forall n\ \in N_+\ when\ r \in \{3,6,9,12, \ldots ,3(k)\}$.

$Note$: $3^{k-1}(\ 2^0), 2^2, 2^4, \ldots , 2^{2(k-2)}, 2^{2(k-1)}, i, e\ 2^m, and\ m \in N = \{0,2,4,6,8, \ldots\}$

$ii)\ if\ r = 3(k+1) = 3k + 3,$

$$is\ x = \frac{3^{k+1}x + 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}}{2^{r-(k+1)}} = 1\ ?$$

$Proof$:

$$x = \frac{3^{k+1}x + 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}}{2^{r-(k+1)}} \Rightarrow$$

$$\left(2^{r-(k+1)} - 3^{k+1}\right)x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k} \Rightarrow$$

$$(2^{3k+3-k-1} - 3^{k+1})x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k} \Rightarrow$$

$$(2^{2k+2} - 3^{k+1})x = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k} \ldots eq1$$

$$is\ (2^{2k+2} - 3^{k+1}) = 3^k(2^0) + 3^{k-1}(2^2) + 3^{k-2}(2^4) + \cdots + 3^{k-k+1}(2^{2k-2}) + 2^{2k}?$$

$let\ k = 4\ \Rightarrow$

LHS = $(2^{10} - 3^5) = 781$, &

$RHS =\ 3^4(2^0) + 3^3(2^2) + 3^2(2^4) + 3(2^6) + 2^8 = 781$

$$\therefore eq1: (781)x = 781 \Rightarrow\ x = 1 \Rightarrow\ y = 4, z = 2 \Rightarrow$$

$lS(n) = \{4,2,1\}\ , \forall n\ \in N_+\ when\ r \in \{3,6,9,12, \ldots ,3(k),3(k+1), \ldots\}$

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$Part\ B$:

$let\ \ r = 3(k) + h, h < 3, k, h \in N_+$

$i) if\ \ r = 3(k) + 1$, *when h=1*

$$let\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow x \notin N_{+,}$$

$\Rightarrow lS(n) = \emptyset, \forall n\ \in N_+\ when\ r \in \{1,2,4,5, \ldots, [3(k) + 1]\}$

$ii) if\ \ r = 3(k) + 2$, *when h=2*

$$is\ x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow\ x \notin N_+?$$

Proof:

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{r-k}} \Rightarrow$$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{3k+2-k}}$$

$$x = \frac{3^k x + 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)}}{2^{2k+2}}$$

$$(2^{2k+2} - 3^k)x = 3^{k-1}(2^0) + 3^{k-2}(2^2) + 3^{k-3}(2^4) + \cdots + 3^{k-k+1}\left(2^{2(k-2)}\right) + 2^{2(k-1)} \ldots eq1$$

$let\ k = 4$

$\therefore\ eq1: (2^{10} - 3^4)x = 3^3(2^0) + 3^2(2^2) + 3^1(2^4) + 2^6 \Rightarrow$

$943\ x = 175 \Rightarrow x \notin N_+ \Rightarrow\ lS(n) = \emptyset, \forall n \in N_+\ when\ r \in \{1,2,4,5, \ldots, [3k + 1], [3k + 2], \ldots\}$

$\because N_+ = \{3,6,9,12, \ldots, 3(k), 3(k + 1), \ldots\} \cup \{1,2,4,5, \ldots, [3k + 1], [3k + 2], \ldots\}$

$\therefore by\ part\ A\ \&\ part\ B \Rightarrow lS(n) = \{1,2,4\} \cup \emptyset = \{1,2,4\}, \forall n \in N_+$