New Method to compute the Sum of Geometric Series on Fractional Numbers

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Abstract: Nowadays, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations to solve today's scientific problems and meet challenges. In this paper, the author introduces the sum of geometric series on fractional numbers in a new way. This computational technique can enable the students and scholars to enhance their research knowledge in the domain of computing and mathematical sciences.

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1. Introduction

Geometric series [1-9] played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, and management. In this article, the author introduces the sum of geometric series on fractional numbers in a new method.

2. Computation of Geometric Series

In this section, the sum of geometric series is computed on factional numbers in a new way.

$$\frac{1}{2} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{2^2} + \frac{1}{2^2} \Rightarrow \frac{1}{2} = \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^n} + \frac{1}{2^n} \Rightarrow \sum_{i=2}^{n} \frac{1}{2^i} = \frac{1}{2} - \frac{1}{2^n} = \frac{2^{n-1} - 1}{2^n}.$$

Let x be a variable and it is assumed to be a positive integer. Then, $1/x$ is a fractional number. Now, let us compute the sum of geometric series using the equation $1/x = 1/x$.

$$\frac{1}{x} = \frac{1}{x} \Rightarrow \frac{1}{x} = \frac{x-1}{x^2} + \frac{1}{x^2} \Rightarrow \frac{1}{x} = \frac{x-1}{x^2} + \frac{x-1}{x^3} + \frac{x-1}{x^4} + \frac{x-1}{x^5} + \cdots + \frac{x-1}{x^{n-1}} + \frac{x-1}{x^n} + \frac{1}{x^n}$$

$$\Rightarrow \frac{x-1}{x^2} + \frac{x-1}{x^3} + \frac{x-1}{x^4} + \frac{x-1}{x^5} + \cdots + \frac{x-1}{x^{n-1}} + \frac{x-1}{x^n} = \frac{1}{x} - \frac{1}{x^n} = \frac{x^{n-1} - 1}{x^n}$$

$$\Rightarrow \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5} + \cdots + \frac{1}{x^{n-1}} + \frac{1}{x^n} = \frac{1}{x} - \frac{1}{x^n} = \frac{x^{n-1} - 1}{x^n(x-1)} \Rightarrow \sum_{i=2}^{n} \frac{1}{x^i} = \frac{x^{n-1} - 1}{x^n(x-1)}. \quad (1)$$

Here, $\sum_{i=2}^{n} \frac{1}{x^i} = \frac{x^{n-1} - 1}{x^n(x-1)}$ is the sum of geometric series on fractional numbers.

Next, let us compute the sum of the geometric series $\sum_{i=1}^{n} \frac{1}{x}$ using the equation $1 = 1$.

$$1=1\Rightarrow 1=\frac{x-1}{x}+\frac{1}{x}\Rightarrow 1=\frac{x-1}{x}+\frac{x-1}{x^2}+\frac{x-1}{x^3}+\frac{x-1}{x^4}+\cdots+\frac{x-1}{x^{n-1}}+\frac{x-1}{x^n}+\frac{1}{x^n}$$

$$\Rightarrow\frac{x-1}{x}+\frac{x-1}{x^2}+\frac{x-1}{x^3}+\frac{x-1}{x^4}+\frac{x-1}{x^5}+\cdots+\frac{x-1}{x^{n-1}}+\frac{x-1}{x^n}=1-\frac{1}{x^n}=\frac{x^n-1}{x^n}$$

$$\Rightarrow\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}+\frac{1}{x^5}+\cdots+\frac{1}{x^{n-1}}+\frac{1}{x^n}=\frac{x^n-1}{x^n(x-1)}\Rightarrow\sum_{i=1}^{n}\frac{1}{x}=\frac{x^n-1}{x^n(x-1)}. \qquad (2)$$

Here, $\sum_{i=1}^{n}\frac{1}{x^i}=\frac{x^n-1}{x^n(x-1)}$ is the sum of geometric series on fractional numbers.

Also, let us compute the sum of the geometric series $\sum_{i=0}^{n}\frac{1}{x}$ using Equation (2).

Adding 1 on both sides of Equation (2).

$$1+\sum_{i=1}^{n}\frac{1}{x}=\frac{x^n-1}{x^n(x-1)}+1\Rightarrow\sum_{i=0}^{n}\frac{1}{x}=\frac{x^n-1+x^{n+1}-x^n}{x^n(x-1)}\Rightarrow\sum_{i=0}^{n}\frac{1}{x}=\frac{x^{n+1}-1}{x^n(x-1)}. \qquad (3)$$

Therefore, $\sum_{i=1}^{n}\frac{1}{x^i}=\frac{x^{n+1}-1}{x^n(x-1)}$ is the sum of geometric series on the equation $1=1$.

3. Conclusion

In this article, the author has introduced the sums of geometric series on fractional numbers. This idea can enable the researchers to be involved in the scientific research on computing and mathematical sciences.

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