Novel Method to compute the Sum of Geometric Series on Real Numbers

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Abstract: Nowadays, the growing complexity of mathematical and computational modelling demands the simplicity of mathematical equations to solve today's scientific problems and meet challenges. In this paper, the author introduces a novel technique to compute the sum of geometric series on real numbers. This computational technique can enable the students and scholars to enhance their research knowledge in the area of computing and mathematical sciences.

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1. Introduction

Geometric series [1-10] played a vital role in differential and integral calculus at the earlier stage of development and still continues as an important part of the study in science, engineering, and management. In this article, the author introduces the sum of geometric series on real numbers in a new way.

2. Computation of Geometric Series

Let x be a variable and it is assumed to a real number. Then, the sums of geometric series are computer on the real number in a novel way.

$$x = x \Rightarrow x = (x-1) + 1 \Rightarrow x = \frac{x-1}{x^0} + \frac{x-1}{x} + \frac{1}{x} \Rightarrow x = \frac{x-1}{x^0} + \frac{x-1}{x} + \frac{x-1}{x^2} + \frac{1}{x^2}.$$

We can further expand the above expression as follows:

$$x = x \Rightarrow x = \frac{x-1}{x^0} + \frac{x-1}{x} + \frac{x-1}{x^2} + \frac{x-1}{x^3} + \frac{x-1}{x^4} + \cdots + \frac{x-1}{x^{n-2}} + \frac{x-1}{x^{n-1}} + \frac{x-1}{x^n} + \frac{1}{x^n}.$$

$$x = x \Rightarrow \frac{x-1}{x^0} + \frac{x-1}{x} + \frac{x-1}{x^2} + \frac{x-1}{x^3} + \cdots + \frac{x-1}{x^{n-1}} + \frac{x-1}{x^n} = x + \frac{1}{x^n} = \frac{n^{n+1}-1}{x^n}.$$

$$x = x \Rightarrow \frac{1}{x^0} + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \frac{1}{x^4} + \frac{1}{x^5} + \frac{1}{x^6} + \cdots + \frac{x}{x^{n-2}} + \frac{1}{x^{n-1}} + \frac{1}{x^n} = \frac{n^{n+1}-1}{x^n\,(x-1)}.$$

$$\text{Therefore}, x = x \Rightarrow \sum_{i=0}^{n} \frac{1}{x^i} = \frac{n^{n+1}-1}{x^n\,(x-1)}.$$

Similarly, we can compute the sums of geometric series for others.

$$1 = 1 \Rightarrow 1 = \frac{x-1}{x} + \frac{1}{x} \Rightarrow 1 = \frac{x-1}{x} + \frac{x-1}{x^2} + \frac{x-1}{x^3} + \frac{x-1}{x^4} + \cdots + \frac{x-1}{x^{n-1}} + \frac{x-1}{x^n} + \frac{1}{x^n}$$

$$\Rightarrow \frac{x-1}{x} + \frac{x-1}{x^2} + \frac{x-1}{x^3} + \frac{x-1}{x^4} + \frac{x-1}{x^5} + \cdots + \frac{x-1}{x^{n-1}} + \frac{x-1}{x^n} = 1 - \frac{1}{x^n} = \frac{x^n-1}{x^n}$$

$$\Rightarrow \frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}+\frac{1}{x^5}+\cdots+\frac{1}{x^{n-1}}+\frac{1}{x^n}=\frac{x^n-1}{x^n(x-1)} \Rightarrow \sum_{i=1}^{n}\frac{1}{x^i}=\frac{x^n-1}{x^n(x-1)}.$$

$$\therefore 1=1 \Rightarrow \sum_{i=1}^{n}\frac{1}{x^i}=\frac{x^n-1}{x^n(x-1)}.$$

$$\frac{1}{x}=\frac{1}{x} \Rightarrow \frac{1}{x}=\frac{x-1}{x^2}+\frac{1}{x^2} \Rightarrow \frac{1}{x}=\frac{x-1}{x^2}+\frac{x-1}{x^3}+\frac{x-1}{x^4}+\frac{x-1}{x^5}+\cdots+\frac{x-1}{x^{n-1}}+\frac{x-1}{x^n}+\frac{1}{x^n}$$
$$\Rightarrow \frac{x-1}{x^2}+\frac{x-1}{x^3}+\frac{x-1}{x^4}+\frac{x-1}{x^5}+\cdots+\frac{x-1}{x^{n-1}}+\frac{x-1}{x^n}=\frac{1}{x}-\frac{1}{x^n}=\frac{x^{n-1}-1}{x^n}$$
$$\Rightarrow \frac{1}{x^2}+\frac{1}{x^3}+\frac{1}{x^4}+\frac{1}{x^5}+\cdots+\frac{1}{x^{n-1}}+\frac{1}{x^n}=\frac{1}{x}-\frac{1}{x^n}=\frac{x^{n-1}-1}{x^n(x-1)}.$$
$$\therefore \frac{1}{x}=\frac{1}{x} \Rightarrow \sum_{i=2}^{n}\frac{1}{x^i}=\frac{x^{n-1}-1}{x^n(x-1)}.$$

Also, $\frac{1}{x^2}=\frac{1}{x^2} \Rightarrow \sum_{i=3}^{n}\frac{1}{x^i}=\frac{x^{n-2}-1}{x^n(x-1)}$; $\frac{1}{x^3}=\frac{1}{x^3} \Rightarrow \sum_{i=4}^{n}\frac{1}{x^i}=\frac{x^{n-3}-1}{x^n(x-1)}$; $\cdots$;
$\frac{1}{x^{n-1}}=\frac{(x-1)}{x^n}+\frac{1}{x^n} \Rightarrow \frac{(x-1)}{x^n}=\frac{1}{x^{n-1}}-\frac{1}{x^n} \Rightarrow \sum_{i=n}^{n}\frac{1}{x^i}=\frac{x-1}{x^n(x-1)}$; and $0=\frac{1-1}{x^n(x-1)}$.

***Theorem* 2.1**: $\sum_{i=0}^{n}(i+1)\times\frac{1}{x^i}=\frac{n^{n+1}-n}{x^{n-1}(x-1)^2}, x\neq 1.$

Proof. Let us prove this theorem as follows:

$$\sum_{i=0}^{n}\frac{1}{x^i}+\sum_{i=1}^{n}\frac{1}{x^i}+\sum_{i=2}^{n}\frac{1}{x^i}+\cdots+\sum_{i=n}^{n}\frac{1}{x^i}=\frac{x^{n+1}-1}{x^n(x-1)}+\frac{x^n-1}{x^n(x-1)}+\cdots+\frac{x-1}{x^n(x-1)}+\frac{1-1}{x^n(x-1)}$$
$$\Rightarrow \sum_{i=0}^{n}(i+1)\times\frac{1}{x^i}=\frac{1}{x^n(x-1)}\{(1+x+x^2+x^3+\cdots+x^{n-1}+x^n+x^{n+1})-(n+1)\}$$
$$\Rightarrow \sum_{i=0}^{n}(i+1)\times\frac{1}{x^i}=\frac{1}{x^n(x-1)}\left\{\sum_{i=0}^{n}x^i-(n+1)\right\}, \text{where } \sum_{i=0}^{n+1}x^i=\frac{n^{n+2}-1}{x-1}.$$
Then, $\sum_{i=0}^{n}(i+1)\times\frac{1}{x^i}=\frac{1}{x^n(x-1)}\left\{\frac{n^{n+2}-1}{x-1}-(n+1)\right\}=\frac{1}{x^n(x-1)}\left\{\frac{n^{n+2}-1-nx+1}{x-1})\right\}.$
$$\therefore \sum_{i=0}^{n}(i+1)\times\frac{1}{x^i}=\frac{n^{n+1}-n}{x^{n-1}(x-1)^2}, x\neq 1.$$

3. Conclusion

In this article, the author has introduced the sums of geometric series on real numbers in a novel way. This idea can enable the researchers to be involved in the scientific research on computing and mathematical sciences.

References

[1] Annamalai, C. (2018) Annamalai's Computing Model for Algorithmic Geometric Series and Its Mathematical Structures. *Journal of Mathematics and Computer Science*, 3(1),1-6 https://doi.org/10.11648/j.mcs.20180301.11.

[2] Annamalai, C. (2017) Analysis and Modelling of Annamalai Computing Geometric Series and Summability. *Mathematical Journal of Interdisciplinary Scienc*es, 6(1), 11-15. https://doi.org/10.15415/mjis.2017.61002.

[3] Annamalai, C. (2018) Novel Computation of Algorithmic Geometric Series and Summability. *Journal of Algorithms and Computation,* 50(1), 151-153. https://www.doi.org/10.22059/JAC.2018.68866.

[4] Annamalai C (2010) "Application of Exponential Decay and Geometric Series in Effective Medicine", *Advances in Bioscience and Biotechnology*, Vol. 1(1), pp 51-54. https://doi.org/10.4236/abb.2010.11008.

[5] Annamalai, C. (2024) Computation of the Sum of Geometric Series on Numerical Expression. *Cambridge Open Engage, Cambridge University Press.* https://doi.org/10.33774/coe-2024-sb8lc-v2.

[6] Annamalai, C. (2023) Computational Technique for Geometric Series with Radicals. *TechRxiv, IEEE*. https://doi.org/10.36227/techrxiv.24311773.v1.

[7] Annamalai, C. (2024) Computation of Novel Binomial Series and Theorems using Bivariable Geometric Series based on Algebraic Expression. *Cambridge Open Engage, Cambridge University Press.* https://doi.org/10.33774/coe-2023-3ch85-v8.

[8] Annamalai, C. (2023) Computation of Algebraic Expressions and Geometric Series with Radicals. *TechRxiv, IEEE*. https://doi.org/10.36227/techrxiv.24311773.v2.

[9] Annamalai, C. (2024) Computing the Sum of Geometric Series based on Algebraic Expression. *Cambridge Open Engage, Cambridge University Press.* https://doi.org/10.33774/coe-2024-sb8lc-v3.

[10] Annamalai, C. (2024) New Method to compute the Sum of Geometric Series on Fractional Numbers. *Cambridge Open Engage, Cambridge University Press.* https://doi.org/10.33774/coe-2024-x63k8.