

# Application of Lorentz transformations to the Schwarzschild metric

V. B. Belayev<sup>a)</sup>

<sup>a)</sup>Corresponding author: bvl2879@gmail.com

**Abstract.** Lorentz transformations are applied to the isotropic Schwarzschild metric. The change in the flow of time obtained from the geodesic equations is consistent with the annual fluctuations in the frequency of the signal from Pioneer 10.

## INTRODUCTION

The Schwarzschild metric describes the gravitational field of a spherical body. The special theory of relativity, based on Lorentz transformations, has a number of experimental confirmations. Application of Lorentz transformations to the linearized isotropic Schwarzschild metric was done in [1]. The resulting metric is used to determine the active gravitational mass of a rarefied gas cloud of relativistic particles.

In present paper the geodesic equations for the resulting metric are found and used to analyze the frequency shift data of the Pioneer 10 signal [3, 4]. It is concluded that the annual frequency variations are caused by a change in the velocity of the time flow on the apparatus from the point of view of an observer on Earth.

## APPLYING LORENTZ TRANSFORMATIONS TO SCHWARZSCHILD METRIC

Lorentz transformations should be performed for the Schwarzschild metric in rectangular coordinates. For weak gravity it has the form

$$ds^2 = c^2 \left(1 - \frac{\alpha}{r}\right) dt^2 - \left(1 + \frac{\alpha}{r}\right) (dx^2 + dy^2 + dz^2). \quad (1)$$

At low velocity, the accelerations of a material particle along the coordinates follow from the geodesic equations are

$$\frac{d^2 r}{ds^2} = -\frac{\alpha}{2r^2} \left(\frac{d(ct)}{ds}\right)^2, \quad (2)$$

$$\frac{d^2 t}{ds^2} = -\frac{\alpha}{r^2} \frac{d(ct)}{ds} \frac{dr}{ds} \quad (3)$$

without small quantities of higher orders. After Lorentz transformations, when the observer moves along the coordinate  $x$  with velocity  $v$ ,

$$t = \frac{t' + \frac{\beta}{c} x'}{\sqrt{1 - \beta^2}}, \quad x = \frac{x' + vt'}{\sqrt{1 - \beta^2}}, \quad y = y', \quad z = z' \quad (4)$$

with the notation  $\beta = v/c$  metric takes the form

$$ds^2 = c^2 \left(1 - \frac{1 + \beta^2}{1 - \beta^2} \frac{\alpha}{r'}\right) dt'^2 - \frac{4v}{1 - \beta^2} \frac{\alpha}{r'} dt' dx' - \left(1 + \frac{1 + \beta^2}{1 - \beta^2} \frac{\alpha}{r'}\right) dx'^2 - \left(1 + \frac{\alpha}{r'}\right) (dy'^2 + dz'^2), \quad (5)$$

with the designation

$$r' = \sqrt{\left(\frac{x' + vt'}{\sqrt{1 - \beta^2}}\right)^2 + y'^2 + z'^2}. \quad (6)$$

From geodesics equations for  $\beta \ll 1$ ,  $\alpha/r \ll 1$  it follows that accelerations of motion along the space coordinates and the flow of time will be

$$\frac{d^2x'}{ds^2} = -x' \frac{\alpha}{2r'^3} \left( \frac{d(ct')}{ds} \right)^2, \quad (7)$$

$$\frac{d^2y'}{ds^2} = -y' \frac{\alpha}{2r'^3} \left( \frac{d(ct')}{ds} \right)^2, \quad (8)$$

$$\frac{d^2z'}{ds^2} = -z' \frac{\alpha}{2r'^3} \left( \frac{d(ct')}{ds} \right)^2, \quad (9)$$

$$\frac{d^2t'}{ds^2} = -\frac{\alpha x}{r'^3} \frac{d(ct')}{ds} \frac{dx'}{ds} - \frac{\alpha y}{r'^3} \frac{d(ct')}{ds} \frac{dy'}{ds} - \frac{\alpha z}{r'^3} \frac{d(ct')}{ds} \frac{dz'}{ds} \quad (10)$$

without small quantities of higher orders [2]. This result can explain the annual variations in the expected additional acceleration of Pioneer 10, see [3, 4].

## ANALYSIS OF THE PIONEER 10 SIGNAL FREQUENCY SHIFT

Expected additional acceleration of Pioneer 10 is determined under assumption that signal frequency shift is caused by the Doppler effect [3, 4]. It has annual periodic component with an amplitude  $(2.9 - 2.4) * 10^{-8} \text{ cm}^2/\text{s}$  at a distance of 40 AU and  $(1.3 - 0.8) * 10^{-8} \text{ cm}^2/\text{s}$  for 60 AU. Distances and accelerations are determined approximately from the flight diagram Fig. 1 and graph Fig. 2.

The total additional acceleration was found from the formula relating the calculated frequency of the received signal to the observed one [4]:

$$v_{obs} = v_{model} * (1 - a_P * t/c) \quad (11)$$

assuming that this is caused by the acceleration of the spacecraft itself. However, the same effect will be produced by time dilation on it calculated by the formula

$$v_{obs} = v_{model} * (1 - c \int_s^{s_0} \frac{d^2t}{ds^2} ds) \quad (12)$$

for  $s \approx ct$ .

Let us establish that the  $x$ -axis is directed from the Sun to the Pioneer 10 apparatus and the  $x'$ -axis is parallel to it and begins on the Earth. The velocity  $u = \frac{dx'}{ds}$  is calculated as follows:

$$u = u_P + u_E \cos(\omega t' + \phi_0), \quad (13)$$

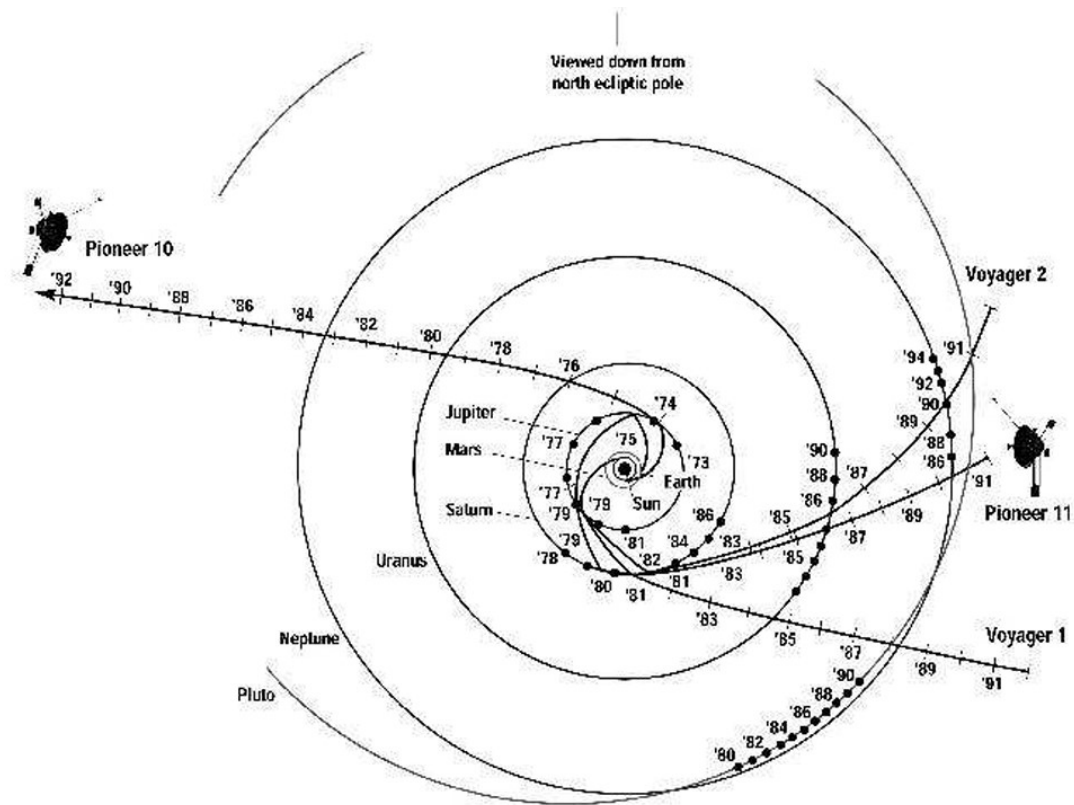
where  $u_P$  is the velocity of the Pioneer relative to the Sun,  $u_E$  is orbital velocity of the Earth,  $\omega$  is the period of the Earth's revolution around the Sun, and  $\phi_0$  is the initial angle. Calculations using formula (10), when converted to acceleration for the periodic component, yield  $3.7 * 10^{-8} \text{ cm}^2/\text{s}$  at a distance of 40 AE and  $1.6 * 10^{-8} \text{ cm}^2/\text{s}$  for 60 AE. These values are close to those observed.

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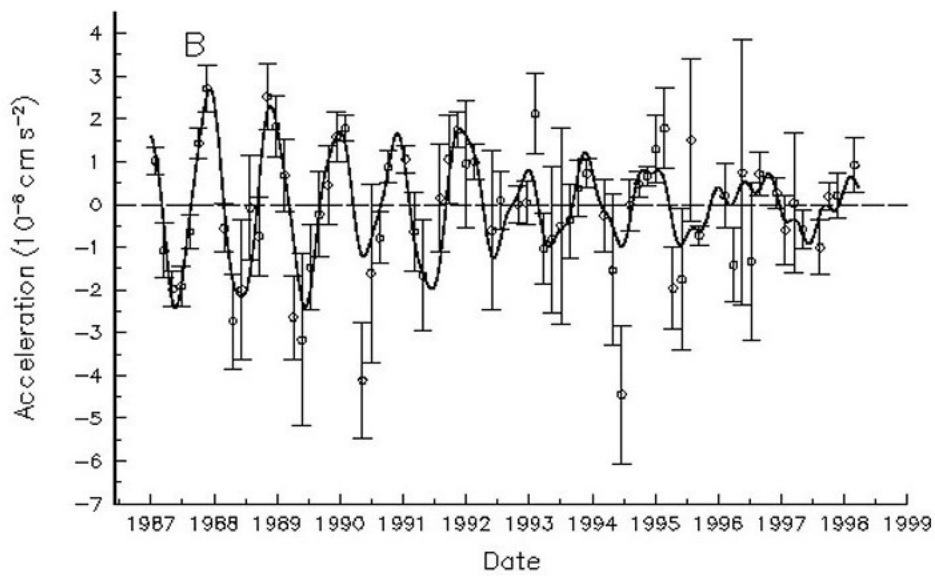
$$v_{obs} = v_{model} * (1 - a_P * t/c) \quad (14)$$

assuming that this is caused by the acceleration of the spacecraft itself. However, the same effect will be produced by time dilation on it calculated by the formula

$$v_{obs} = v_{model} * (1 - c \int_s^{s_0} \frac{d^2t}{ds^2} ds) \quad (15)$$



**FIGURE 1.** Ecliptic pole view of Pioneer 10 trajectory, [3].



**FIGURE 2.** The Pioneer 10 Doppler data: 50-day averages of anomalous acceleration – January 1987 to July 1998, [4].

for  $s \approx ct$ .

Let us establish that the  $x$ -axis is directed from the Sun to the Pioneer 10 apparatus and the  $x'$ -axis is parallel to it and begins on the Earth. The velocity  $u = \frac{dx'}{ds}$  is calculated as follows:

$$u = u_P + u_E \cos(\omega t' + \phi_0), \quad (16)$$

where  $u_P$  is the velocity of the Pioneer relative to the Sun,  $u_E$  is the velocity of the Earth's orbit,  $\omega$  is the period of the Earth's revolution around the Sun, and  $\phi_0$  is the initial angle. Calculations using formula (10), when converted to acceleration for the periodic component, yield  $3.7 * 10^{-8} cm^2/s$  at a distance of 40 AE and  $1.6 * 10^{-8} cm^2/s$  for 60 AE. These values are close to those observed.

## CONCLUSION

The annual frequency variations of Pioneer 10 signal are caused by a change in the velocity of the time flow on the apparatus from the point of view of an observer on Earth. This follows from the properties of the space obtained by applying Lorentz transformations to the linearized isotropic Schwarzschild metric.

## REFERENCES

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