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$Collatz\ Sequence\ Proof\ (2nd\ Way)$

$Abestract: Collatz\ Sequence\ rules: n\ (Even): \dfrac{n}{2}, or\ n\ (Odd): 3n+1.$
$With\ enough\ repetition, do\ all\ positive\ integers\ converge\ to\ 1?$

$Proof: let\ Collatz\ Sequence\ of\ (n) = S(n), loop\ of\ Collatz\ Sequence\ (n) = lS(n),\ \&\ n, a, b, c, w, r \in N_+$
$if\ S(n) = \{a, b, c, \dots, w\} \Rightarrow lS(n) = lS(a) = lS(b) = lS(c) =..= lS(w) \dots Fact1$

$i)\ S(1) = \{4,2,1\} \Rightarrow lS(1) = \{4,2,1\}.$
$S(2) = \{1, 4,2\} \Rightarrow lS(2) = \{4,2,1\}.$
$S(3) = \{10, 5,16,8,4,2,1\} \Rightarrow lS(3) = \{4,2,1\}.$
$S(4) = \{2,1,4\} \Rightarrow lS(4) = \{4,2,1\}.$
$S(5) = \{16,8,4,2,1\} \Rightarrow lS(5) = \{4,2,1\}.$

$ii)\ if\ S(r) = \left\{\dfrac{r}{2}\ or\ (3r+1), \dots, 4,2,1\right\} \Rightarrow lS(r) = \{4,2,1\}$
$\therefore\ lS(n) = \{4,2,1\}, \forall n \in Set\ Z = \{1,2,3,4,5,6,7,8,9, \dots, \frac{r+2}{2}, \dots, r\}.\ \ r \in N_{even} > 2$

$iii)\ is\ lS(r+2) = \{4,2,1\}, when\ (r+2) \in N_{even}?$
$\because S(r+2) = \left\{\left(\frac{r+2}{2}\right), \dots, ?\right\} \Rightarrow lS(r+2) =\ lS\ (\frac{r+2}{2}) \dots eq1]\ by\ Fact1$
$\because \frac{r+2}{2} \in Set\ Z \Rightarrow lS\ \left(\frac{r+2}{2}\right) = \{4,2,1\}.$
$\therefore\ lS(r+2) = \{4,2,1\} \dots (substitution\ in\ eq1)$
$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{even}.$

$iv)\ If\ n \in N_{odd} \Rightarrow S(n) = \{3n+1, \dots, ?\} \Rightarrow$
$lS(n) = lS(3n+1) \dots eq2]\ by\ Fact1$
$\because (3n+1) \in N_{even}$
$\therefore lS(3n+1) = \{4,2,1\} \dots by\ iii$
$\therefore lS(n) = \{4,2,1\}\ \forall\ n \in N_{odd} \dots (substitution\ in\ eq2)$
$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{+} \dots by\ \ iii\ \&\ iv$