$Collatz\ Sequence\ Proof\ (3rd\ Way)$

$$Abestract: Collatz\ Sequence\ rules: n\ (Even): \frac{n}{2}, or\ n\ (Odd): 3n+1.$$
$$With\ enough\ repetition, do\ all\ positive\ integers\ converge\ to\ 1?$$

$Proof: let\ Collatz\ Sequence\ of\ (n) = S(n), let\ loop\ of\ Collatz\ Sequence\ (n) = lS(n)$
$\&\ let\ n, a, b, c, w, r \in N_+$
$[S(n) = \{a, b, c, \dots, w\} \Leftrightarrow lS(n) = lS(a) = lS(b) = lS(c) = \cdots = lS(w)] \dots Taha's\ Fact1$

$i)$
$S(1) = \{4,2,1\} \Rightarrow lS(1) = \{4,2,1\}.$
$S(2) = \{1,4,2\} \Rightarrow lS(2) = \{4,2,1\}.$
$S(3) = \{10,5,16,8,4,2,1\} \Rightarrow lS(3) = \{4,2,1\}.$
$S(4) = \{2,1,4\} \Rightarrow lS(4) = \{4,2,1\}.$
$S(5) = \{16,8,4,2,1\} \Rightarrow lS(5) = \{4,2,1\}.$

$ii)$
$let\ S(r) = \left\{\frac{r}{2}\ or\ (3r+1), \dots, 4,2,1\right\} \Rightarrow lS(r) = \{4,2,1\}$
$\therefore\ lS(n) = \{4,2,1\}, \forall n \in\ Z = \left\{1,2,3,4,5,6,7,8,9, \dots, \frac{r}{2}\ or\ \frac{r+1}{2}, \dots, r\right\},$
$r \in N_{even}, or\ N_{odd}.$

$iii)$
$is\ lS(r+1) = \{4,2,1\}?$
$Part\ a)$
$If\ (r+1) \in N_{even}\ \Rightarrow S(r+1) = \left\{\left(\frac{r+1}{2}\right), \dots, ?\right\} by\ even, odd\ rules \Rightarrow$
$lS(r+1) =\ lS\ (\frac{r+1}{2}) \dots eq1]\ by\ Fact1$
$\because \frac{r+1}{2} \in\ Z \Rightarrow lS\ \left(\frac{r+1}{2}\right) = \{4,2,1\}.$
$\therefore\ lS(r+1) = \{4,2,1\} \dots (substitution\ in\ eq1)$
$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{even}.$

$Part\ b)$
$If\ (r+1) \in N_{odd} \Rightarrow 2(r+1) \in N_{even} \Rightarrow\ lS(2(r+1)) = \{4,2,1\} \dots by\ Part\ a$
$\therefore S(2(r+1)) = \{(r+1), \dots, 4,2,1\} \dots by\ Part\ a \Rightarrow$
$\because lS(2(r+1)) = lS(r+1) \dots eq2]\ by\ Fact1$
$\therefore \{4,2,1\} = lS(r+1) \dots (substitution\ in\ eq2)$
$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{odd}$
$\therefore lS(n) = \{4,2,1\}, \forall\ n \in N_{+} \dots by\ Part\ a\ \&\ Part\ b.$

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