INVERSE N-BODY PROBLEM

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Abstract. *Newton's law of gravity F = GMm/r^2 shows the force of gravitational interaction of two bodies. The solution of the inverse problem of two bodies (Bertrand problem) also leads to Newton's law of gravity. However, the solution of the direct and inverse problems of two bodies does not lead to a physical law that is capable of giving the full force of gravitational interaction of N-bodies. Newton's law of gravity does not take into account that all bodies of the Universe participate in gravitational interaction simultaneously. The fundamental law of gravity for N bodies has not been discovered. The obstacle was the unsolved gravitational problem of N bodies. The inverse problem of N-bodies, as a problem of obtaining a formula for the gravitational force, has not been studied in physics. Here we present a new method for finding the law of gravitational force for N bodies. The method is based on reducing the gravitational problem of N bodies to the two-body problem, where the central body is a system of N bodies. The problem of an N-body system is the inverse problem of the N-body problem. This is the problem of finding the law of gravitational force from the known integral characteristics of the N-body system. The solution of the inverse problem of N bodies gives a new law of gravitation F = (mc^2)√Λ. Instead of the gravitational constant G, the new law of gravity includes the cosmological constant Λ. The new law of gravity F = (mc^2)√Λ allows us to overcome the limitations inherent in Newton's law of gravity F = GMm/r^2 and leads to a new law of universal gravitation.*

Keywords: *Newton's law; N-body problem; law of universal gravitation; parameters of the observable universe; cosmological constant Λ.*

1. Introduction

For over 300 years, the force of gravity was represented by only one formula: Newton's law of gravity $F = GMm/r^2$. This formula gives only a part of the force of universal gravitation. The formula $F = GMm/r^2$ is accurate enough on the scale of the Solar System. But it is not applicable on the scale of the Universe. The formula of Newton's law shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula describes the attraction to only one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies. To describe the attraction of all N bodies in the Universe, Newton's law $F = GMm/r^2$ is not enough. Deriving a new law of gravity is a pressing problem. During the long history of the study of gravity, the law of gravitational force for N bodies has not been discovered.

2. The N-body gravitational problem and the problem of deriving the law of gravitation for all N bodies in the Universe

Newton's law $F = GMm/r^2$ does not provide a complete description of gravity. Newton's law does not work at large distances [1-3]. This law is applicable to the gravitational interaction of two bodies. The real picture of gravity is different. Bodies are simultaneously attracted to all other bodies, and not just to one local source of attraction. There are a huge number of bodies in the Universe. But their parameters are not included in the formula $F = GMm/r^2$. Even if it were possible to solve the problem of 3, 4, 5, etc. bodies, the description of gravity would still be incomplete until the gravitational action of the entire Universe is taken into account.

The problem of discovering the law of gravitation for all N bodies in the Universe is the inverse N-body problem. The inverse N-body problem, as a problem of obtaining a formula for the gravitational force, has not been studied in physics. It is known that the N-body problem has no analytical solution [4]. Direct application of Newton's law to N bodies requires an additive representation of the total force as a sum of forces. This leads to the insurmountable problem of the "bad infinity" (die Schlecht-Unendliche) [5].

To find the gravitational force that takes into account the action of N bodies, there is no need to solve the differential problem of N paired Newtonian forces. Instead of searching for a direct solution for N bodies, it is proposed to obtain the law of gravitational force for a system of N bodies. To do this, it is necessary to solve the inverse problem of N bodies.

3. The inverse two-body problem (Bertrand problem).

Newton's law of gravity F = GMm/r^2 shows the force of gravitational interaction between two bodies. The inverse two-body problem, namely, the problem of finding the law of gravitation based on the known trajectory of a body, had not yet been formulated at that time. Such a problem appeared in the late 1870s. It was formulated by Bertrand J. [6-8]. The first and second Bertrand gravitation problems are known [6-8]. The first Bertrand problem was formulated for trajectories that are conic sections. The second Bertrand problem was formulated for trajectories that are closed curves. In the general case, for trajectories represented by algebraic curves, this problem is known as the Koenigs problem [6-8]. The Bertrand and Koenigs problems are inverse two-body problems. The solution to these problems yields two known laws: Hooke's law and Newton's law of gravitation.

The two solutions to the Bertrand and Koenigs problems only confirmed the laws of force known at that time, but did not give new predictions. In fact, the Bertrand and Koenigs problems have not two, but three solutions. The third solution exists for closed elliptical trajectories of motion. The third solution to the Bertrand and Koenigs gravitation problems has the form: F = mR^3/T^2r^2 [9]. The third solution directly includes the parameters of the elliptical trajectory of motion. This unknown law of gravitational interaction of two bodies was pointed out by Robert Hooke in his correspondence with Newton [9]. The law of gravitation F = mR^3/T^2r^2 is a more accurate and perfect law of gravitational interaction of two bodies than Newton's law F = GMm/r^2.

4. Inverse N-body problem.

Unlike the inverse two-body problem, the inverse N-body problem, as a problem of obtaining the law of gravitational force, is not presented in physics. Perhaps because the direct N-body problem has no solution. Although the direct N-body problem has no solution, the inverse N-body problem has a solution and leads to a new law of gravitation. Here we formulate the inverse N-body problem and give its solution. This is a new problem of finding the law of gravitation based on the known integral characteristics of the N-body system. The N-body system is considered as a holistic object. This allows us to reduce the N-body problem to the two-body problem, where the central body is the N-body system.

We propose the following formulation of the inverse N-body problem: "*Knowing the integral characteristics of the N-body system, find the law of the gravitational force with which N bodies act on a body of mass m.*"

5. N bodies as a single system.

It is impossible to apply Newton's law to obtain N forces from all pairwise interactions. The complexity of the N-body differential problem is the need to take into account the parameters of each of the N bodies. The number of bodies in the Universe is enormous ($N \to \infty$). It is impossible to obtain a formula for the force for N bodies using this method. The impossibility of taking into account all the causes of motion and the impossibility of describing them using exact laws was pointed out by Newton in the text from the 1684 manuscript [10], known as the "Copernican Scholium": "*...the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind*."

The problem of obtaining a formula for the gravitational force that takes into account the gravitational interaction of N bodies requires abandoning the differential problem of N bodies and moving on to an integral problem for a system of N bodies. The integral inverse problem of N bodies has a solution. In the integral inverse problem of N bodies, it is envisaged to replace the set of forces from pairwise interactions of N bodies with one resultant force caused by the system of N bodies. In this case, there is no need to know the parameters of each body. It is enough to know the integral parameters that characterize the system of N bodies. For this, N bodies are considered as one system. The problem is reduced to the problem of two bodies, where the central body is a system containing N bodies. Such a possibility exists if the integral parameters of the N-body system are known. In relation to the Universe, integral parameters are known. These are the mass of the Universe Mu, the radius of the Universe Ru, the cosmological constant Λ, and the time of the Universe Tu.

6. Method for solving Inverse N-body problem.

The scheme for solving Inverse N-body problem is as follows:

1. N individual bodies are considered as a single object — as a system of N bodies.
2. The central body is a system of N bodies.
3. The N-body problem is reduced to the two-body problem.
4. The Universe with known parameters is considered as a single system of N bodies.
5. Selecting an integral parameter that is a characteristic for the system of N bodies.
6. Obtaining an acceleration formula for the central body, represented by a system of N bodies.
7. Obtaining an equation for a new law of gravitation, describing gravitation to all N bodies of the Universe.

7. Particular solutions of the Inverse N-body problem.

The gravitational problem of the N-body system as applied to the Universe has several solutions depending on the choice of the integral characteristic of the Universe (Mu, Ru, Λ, Tu):

F = (mc^2)/Ru (1)

F = mGMu/Ru^2 (2)

F = (mc^2)√Λ (3)

F = mc/Tu (4)

Where: m is the mass of the body; c is the speed of light in a vacuum; Mu is the mass of the Universe; G is the Newtonian constant of gravitation; Ru is the radius of the Universe; Λ is the cosmological constant; Tu is the time of the Universe.

The proof follows directly from Newton's second law F = ma. The acceleration caused by the action of the entire mass of the Universe has the form: a = GMu/Ru^2. Using the parameter Ru, the acceleration has the form: a = c^2/Ru. Using the parameter Tu, the acceleration has the form: a = c/Tu. Using the parameter Λ, the acceleration has the form: a = c^2√Λ.

Not all solutions to the N-body inverse problem are useful. Of all the parameters of the Universe, the cosmological constant Λ is measurable. A practically applicable solution to the gravitational problem of a system of N bodies is the law of gravitation of the form: F = (mc^2)√Λ. This is a new law of gravitation - the law of cosmological force. The unknown gravitational force does not obey the law of inverse capdrats and has a linear dependence on the mass of the body. The value of acceleration associated with the cosmological force (Ao = 10.4922... • 10^(-10) m/s^2) turned out to be very close to the MOND prediction [11]. The law of cosmological force leads to a new law of universal gravitation, which takes into account not only the gravity of two bodies, but also the gravity of all N bodies in the Universe.

8. The Law of Cosmological Force F = (mc^2)√Λ.

The cosmological force, which Newton's law of gravitation does not detect, arises from the gravitational effect of all bodies in the universe on a body of mass "m". Since all bodies in the universe are distributed in space, the formula for the law of cosmological force does not include the inverse square law:

$$F = mc^2\sqrt{\Lambda} \qquad (5)$$

This new law of gravitation reveals an unknown cosmological force that acts on any body in the universe. It is the gravitational force with which bodies simultaneously gravitate toward all bodies in the universe. It is the gravitational force that makes the law of universal gravitation complete.

Instead of the gravitational constant G, the law of cosmological force contains the cosmological constant Λ. The cosmological force has a linear dependence on the mass of the body and does not obey the inverse square law.

On small scales, the additional cosmological force is much smaller than the Newtonian force. At small distances, the main part of the universal gravitational force is the Newtonian force. On the scale of the Universe, the cosmological force is enormous. At large distances, it exceeds the

Newtonian force. At large distances, the main part of the universal gravitational force is the cosmological force F = (mc^2)√Λ.

Studying the equation of the new law of cosmological force shows that the magnitude of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2\sqrt{\Lambda} = 1.21027 \bullet 10^{44} N = \frac{c^4}{G} \qquad (6)$$

The theoretical limit of the cosmological force at m → M_U reaches a huge value: c^4/G = 1.21027 x 10^{44} N.

9. Conclusion

For over 300 years, the force of gravity was represented by only one formula: the two-body law of gravity F = GMm/r^2. The two-body law of gravity was insufficient to explain numerous gravitational anomalies. A new law of gravity was needed to describe the gravity of all N bodies in the Universe. The obstacle to deriving the N-body law of gravity was the unsolved N-body gravitation problem. Here, another approach to deriving the N-body law of gravity is proposed. For this purpose, the inverse N-body problem, similar to the Bertrand two-body problem, is formulated and solved for the first time. The solution of the N-body inverse problem leads to a new law of gravity F = $mc^2\sqrt{\Lambda}$, which is not present in either standard Newtonian dynamics or MOND. This is the fundamental law of gravity for N bodies. The new law of gravitation F = (mc^2)√Λ allows us to overcome the limitations of Newton's law of gravitation F = GMm/r^2 and leads to a new law of universal gravitation.

10. Conclusions

1. A new method for finding the law of gravitational force for N bodies is proposed. The method is based on the inverse N-body problem. In this case, the N-body gravitational problem is reduced to the two-body problem, where the central body is the N-body system.

2. The problem of finding the law of gravitational force based on the known integral characteristics of the N-body system is formulated for the first time. This is the inverse N-body problem, similar to the inverse Bertrand problem for two bodies.

3. A new law of gravitation F=$mc^2\sqrt{\Lambda}$ is discovered. The new law of gravitation is obtained as a result of solving the inverse N-body problem. The new law of gravitation shows the gravitational force that Newton's law "does not see". This is the gravitational force with which bodies simultaneously gravitate to all other bodies in the Universe.

4. Instead of the gravitational constant G, the new law of gravitation includes the cosmological constant Λ.

5. The law of the cosmological force F=$mc^2\sqrt{\Lambda}$ shows that any body of mass m is acted upon by a cosmological force proportional to the mass of the body. The cosmological force is linearly dependent on the mass of the body and does not obey the inverse square law.

6. On small scales, the additional cosmological force is much smaller than the Newtonian force. On small scales, the Newtonian force is the main part of the force of universal gravitation.

7. On large scales, the cosmological force exceeds the Newtonian force. On large scales, the main part of the force of universal gravitation is the cosmological force.

8. The new law of gravitation $F = (mc^2)\sqrt{\Lambda}$ allows us to overcome the limitations inherent in Newton's law of gravitation $F = GMm/r^2$ and leads to a new law of universal gravitation.

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