Pythagorean Theorem

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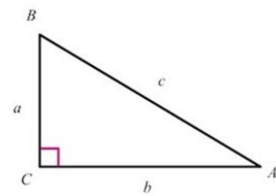
$$is\ a^2 + b^2 = c^2?$$
$$Proof$$

$ABC\ Right\ Triangle\ at\ C$
$c\ is\ hypotenuse$
$a < b, or\ a = b\ c < a + b, a < c + b, b < a + c$
(Triangle Property (T.P.))
$a, b, c, r, g, h, u, v, j, k, m, n \in R_+$

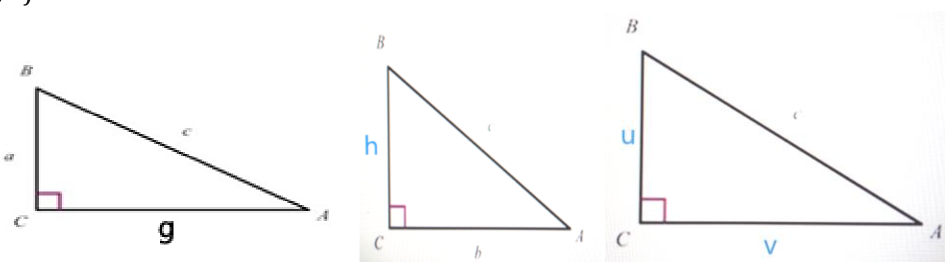


$let\ \ c^2 \neq a^2 + b^2 \Rightarrow$

$A)\ if\ c^2 > a^2 + b^2 \Rightarrow\ \ c^2 = a^2 + b^2 + r$

$i)\ if\ r = 2ab \Rightarrow\ \ c^2 = a^2 + b^2 + 2ab = (a+b)^2 \Rightarrow c = a + b \ldots Contradiction\ to$
$c < a + b \ldots (Tri.Pro.)$

$ii)\ if\ r \neq 2ab \Rightarrow\ c^2 = a^2 + b^2 + r \Rightarrow$

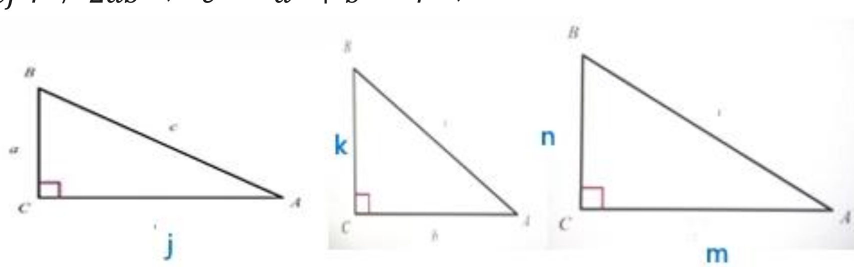


$[c^2 = a^2 + 2ag + g^2 \Rightarrow c^2 = (a+g)^2 \Rightarrow c = a + g \ldots Contradiction\ to\ c < a + g,$
$or\ c^2 = h^2 + 2hb + b^2 \Rightarrow\ c^2 = (h+b)^2 \Rightarrow c = h + b \ldots Contradiction\ to\ c < h + b,$
$or\ c^2 = u^2 + 2uv + v^2] \Rightarrow\ c^2 = (u+v)^2] \Rightarrow c = u + v \ldots Contradiction\ to\ c < u + v]\ \Rightarrow$
$r = 2ab \Rightarrow c = a + b \ldots Contradiction\ to\ c < a + b \ldots (Tri.Pro.)$
$\therefore by\ i\ \&\ ii{:}\ \ c^2 \ngtr a^2 + b^2 \ldots (A)$

$B)\ if\ c^2 < a^2 + b^2 \Rightarrow\ \ c^2 = a^2 + b^2 - r$

$i)\ if\ r = 2ab \Rightarrow\ \ c^2 = a^2 - 2ab + b^2 = (a-b)^2 \Rightarrow$
$c = a - b \Rightarrow c = 0\ or\ c \in R_{(-)} \ldots Contradiction\ to\ c \in R_+$

$ii)\ if\ r \neq 2ab \Rightarrow\ \ c^2 = a^2 + b^2 - r \Rightarrow$



$c^2 = a^2 - 2aj + j^2 = (a-j)^2 \Rightarrow c = a - j \Rightarrow c < a \ldots Contradiction\ to\ c > a$

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$or\ c^2 = k^2 - 2kb + b^2 = (k-b)^2 \Rightarrow c = k - b \Rightarrow c < k \Rightarrow \cdots Contradiction\ to\ c > k$

$or\ c^2 = m^2 - 2mn + n^2 = (m-n)^2 \Rightarrow c = m - n \Rightarrow c < m \ldots Contradiction\ to\ c > m] \Rightarrow$

$r = 2ab \Rightarrow c = a - b \Rightarrow c = 0\ or\ c \in R_{(-)} \ldots Contradiction\ to\ c \in R_+$

$\therefore by\ i\ \&\ ii{:}\ \ c^2 \nless a^2 + b^2 \ldots (B)$

$\therefore\ c^2 = a^2 + b^2 \ldots by\ A\ \&\ B$

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