Modelling of Friction Stir Welding Process of Aluminum Alloys

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Summary

When the friction stir welding (FSW) process is used in practice, defects appear under uncertain operation conditions. Is it possible to understand the basic mechanisms of the defect generation so that it could be avoided. During the workshop, simplified models were developed for heat generation and plastic deformation and attempts were made to find analytical and numerical solutions, which were continued until shortly after the workshop completed. We found that the unrealistic solution obtained during the workshop was due to the choice of a balance between inertial forces and plastic stress. In contrast, when inertia is negligible, physically reasonable solutions can be obtained for plastic deformation irrespective of the compressibility of the material. Investigating the dominant physical processes that drive this FSW process, the solutions from the simplified one-dimensional heat model and a two-dimensional non-Newtonian fluid model are also presented here. The temperature distribution matches the results using direct numerical simulation and the experimental measurements. However, we did not have sufficient time to solve the time-dependent one-dimensional thermal-plastic-elastic model. This latter problem has been left for future research.

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1 Introduction

In many industrial applications, joining aluminum alloys and other metal workpieces can be accomplished via various approaches, such as adhesive bonding and welding. FSW is a relatively new technology (patented in 1991) with the advantages of being more environmentally friendly and producing better joints when compared with traditional approaches. The technique has gained popularity in aerospace, automotive and marine industries.

FSW is a solid-state welding process (cf. Figure 1), where a hardened steel tool rotates and presses into the metal plates¹ that are to be welded [2]. The steel tool consists of a pin and a shoulder. Axial force is applied to the tool, to keep it in contact with the material, and the rotation of the pin and shoulder against the workpiece generates frictional heating which softens the aluminum. The material is not heated beyond the melting temperature as with traditional welding but rather reaches at temperature of about 80–90% of the melting temperature. While in this state, the heating and rotation of the tool induce a large plastic deformation and the metal in the plastic region behaves like a fluid, ultimately joining the plates.

Figure 1: Schematic illustration of the FSW process.

It has been observed experimentally that when the applied pressure or the rotational speed is too high, material may flow out of the tool to form uneven surface deposits. On the other hand, when the applied pressure or the rotational speed is too low, or the tool moves too fast, an incomplete joint may occur. The objective of this project is to

¹We focus on aluminum plates in this report.

understand the underlying mechanisms and provide guidance for improving the process. In particular, two elements play an important role in the FSW process: heat generation and large plastic deformation. Revealing the mechanisms or influencing factors on these two aspects can shed light on the size of plastic region and the possible generation of defects that are possible with the process.

During the workshop, we built a three-dimensional (3D) general plastic/elastic model coupled with heat transfer. With axisymmetry and the plane-strain assumption, the general model was reduced to a one-dimensional (1D) model. To keep the problem tractable, we studied heat generation and plastic deformation processes separately even though in practice they are interrelated. For the heat generation and temperature distribution problem we used a numerical method, while for the plastic models, analytical solutions were developed for two separate 1D scenarios. This was continued after the workshop and permitted the investigation of the role of several key physical parameters that induce plastic deformation such as tool pressure.

2 A general model

In this section, we first introduce a general 3D model for the FSW process and illustrate various mechanisms involved in the process and how they are coupled.

We assume that the aluminum workpiece can be divided into two regions: an elastic and a plastic region. In the elastic region, away from the steel tool, we assume that the deformation is governed by linear elasticity. By ignoring the body forces, the force balance reads

$$\nabla \cdot \boldsymbol{\sigma} = \vec{0}, \quad (1)$$

where $\boldsymbol{\sigma}$ is the Cauchy stress and the constitutive relation is given by

$$\boldsymbol{\sigma} = \lambda \text{tr}(\boldsymbol{\varepsilon}) \mathbf{I} + \mu \boldsymbol{\varepsilon} = \frac{E\nu}{(1+\nu)(1-2\nu)} \text{tr}(\boldsymbol{\varepsilon}) \mathbf{I} + \frac{E}{2(1+\nu)} \boldsymbol{\varepsilon}, \quad \boldsymbol{\varepsilon} = \frac{1}{2}(\nabla \vec{u} + \nabla \vec{u}^\top), \quad (2)$$

where $\boldsymbol{\varepsilon}$ is the strain tensor, $\vec{u}$ is the displacement vector, λ and μ are Lamé constants, E is Young's modulus and ν is Poisson's ratio (see Table 1 for parameter values). The temperature field T is governed by the heat equation

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T), \quad (3)$$

Parameter	Value	Units	Parameter	Value	Units
k	237	J/s.m.K	σ_{y0}	240	MPa
E	70	GPa	T_R	20	°C
ν	0.35		T_{melt}	605	°C
ρ	2700	kg/m ³	m	1.34	
C_p	897	J/kg.K			

Table 1: Thermal-physical properties of Aluminum.

where ρ is the density, C_p is the specific heat capacity and k is the thermal conductivity (see Table 1).

In the plastic region, near the tool, the force balance and incompressibility condition are

$$\nabla \cdot \boldsymbol{\sigma} = \rho \frac{D\vec{v}}{Dt}, \quad \nabla \cdot \vec{v} = 0, \quad (4)$$

where $\vec{v} = \dot{\vec{u}}$ is the velocity, and $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla$ is the material time derivative. For the constitutive relations, we adopt a perfect plasticity model with von-Mises yield function [3, see chap 8], and more precisely

$$\dot{\boldsymbol{\epsilon}} = \frac{\Lambda}{\sigma_y} \left(\boldsymbol{\sigma} - \frac{1}{3} \text{Tr}(\boldsymbol{\sigma}) \mathbf{I} \right) \equiv \frac{\Lambda}{\sigma_y} \boldsymbol{\sigma}^{\text{dev}}, \quad \sigma_y^2(T) = \frac{3}{2} \text{tr}(\boldsymbol{\sigma}^{\text{dev}} \boldsymbol{\sigma}^{\text{dev}}) \quad (5)$$

where σ_y is the yield stress which depends on temperature T , and Λ is a Lagrange multiplier which in general can be a scalar function of position and time. We use a simple form of Johnson-Cook constitutive model for the yield stress [2, 4]

$$\sigma_y^2(T) = \sigma_{y0} \left(1 - \left(\frac{T - T_R}{T_{melt} - T_R} \right)^m \right), \quad (6)$$

where σ_{y0} is the yield stress at room temperature, T_R and T_{melt} are the room and melt temperatures, m is the thermal-softening exponent (see Table 1). The temperature field is governed by

$$\rho C_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + \text{tr}(\boldsymbol{\sigma} \dot{\boldsymbol{\epsilon}}), \quad (7)$$

where the last term represents heat generation due to plastic flow.

In summary, we have presented a model for both the plastic and elastic region of the material. To complete this model, we require suitable boundary conditions like convection

of heat for temperature and displacement or traction conditions for the mechanical field, and propose connection conditions on the free boundary between the elastic and plastic region. These conditions will be illustrated in later sections for the 1D case. In the governing equations above, the coupling of mechanical and thermal fields is mainly from two terms: $\sigma_y(T)$ and $\text{tr}(\boldsymbol{\sigma}\dot{\boldsymbol{\epsilon}})$.

3 Heat generation

In this section, we will present a 1D model for temperature field developed by our team. The simulation of temperature field is credited to Mohammad Samani.

In this 1D model (cf. Figure 2), we consider the case that the shoulder of the tool presses on the material with some axial force N_{axial} and the shoulder and the pin rotate at a constant frequency ω . The axisymmetric case is considered, and the temperature in the aluminum material is denoted by $T = T(r, t)$, where r measures the radial distance of material point from axis of pin and $r_p < r < L$. The temperature of pin and shoulder are denoted by $T_p(t)$ and $T_s(t)$ respectively. The effect of velocity $\vec{v}$ in plastic region is neglected in this model, and will be studied in the future.

Figure 2: Schematic illustration of 1D heat generation model.

The governing equation is

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r k \frac{\partial T}{\partial r} \right) + \dot{q}_s + \dot{q}_B, \quad r_p < r < L, \quad (8)$$

where $\dot{q}_s$ is the heat generation due to friction between the shoulder and the material, $\dot{q}_B$

Parameter	Value	Units	Parameter	Value	Units
N_{axial}	10	kN	$\sigma_{r\theta}$	100	MPa
r_p	5	mm	h_p, h_s, h_B	10^4	J/s · m ² · K
r_s	15	mm	h_L	5000	J/s · m ² · K
L	100	mm	ρ_s	8170	kg/m ³
H	10	mm	c_p, c_s	418	J/kg.K
ω	1400/60	1/s	h_{ps}	10^5	J/s · m ² · K
f	0.5		γ	0.9	

Table 2: Data used in the simulation of temperature field.

is the cooling due to convection of heat towards the base material, and they are given by

$$\begin{aligned}\dot{q}_s(r) &= f \frac{N_{axial}}{\pi(r_s^2 - r_p^2)} 2\pi\omega r \frac{1}{H} - h_p(T - T_p) \frac{1}{H}, \quad r_p < r < r_s, \\ \dot{q}_B(r) &= -h_B(T - T_B) \frac{1}{H}, \quad r_p < r < L,\end{aligned}\tag{9}$$

where f is the frictional coefficient between aluminum and the tool, H is the thickness of the material, h_B is the convection coefficient between the workpiece and supporting material and T_B is set to be room temperature (see Table 2 for parameter values). The temperatures of the pin and the shoulder are governed by

$$\begin{aligned}\rho_p c_p V_p \frac{dT_p}{dt} &= \dot{q}_p + h_{ps}(T_p - T_s) + h_p(T_p - T(r_p)), \\ \rho_s c_s V_s \frac{dT_s}{dt} &= -h_{ps}(T_p - T_s) + h_s(T_s - T|_{(r_p, r_s)})\end{aligned}\tag{10}$$

where $T|_{(r_p, r_s)}$ is the mean temperature of the material in the interval, $V_p = \pi r_p^2 H$ is the volume of the pin, V_s is the volume of the shoulder (we set $V_s = 100V_p$), c_p, c_s are heat capacities of the pin and shoulder, h_p, h_s, h_{ps} are some convection coefficients, and $\dot{q}_p$ is the heat generation caused by rotation of the pin

$$\dot{q}_p = \sigma_{r\theta} v_\theta A_p, \quad v_\theta = \gamma 2\pi\omega r_p, \quad A_p = \pi r_p^2 + 2\pi r_p H,\tag{11}$$

where $\sigma_{r\theta}, v_\theta$ are the shear stress and velocity and A_p is surface area of the pin (cf. Table 2).

The boundary conditions for equation (8) are

$$\begin{aligned}-k \frac{\partial T}{\partial r} &= h_p(T - T_p), \quad \text{at } r = r_p, \\ -k \frac{\partial T}{\partial r} &= h_L(T - T_R), \quad \text{at } r = L,\end{aligned}\tag{12}$$

and the initial values of temperatures are all set to be the room temperature. Figure 3 shows the temperature field inside the aluminum workpiece, with the data in Table 2. The temperature in the region under the shoulder is higher, suggesting that the shoulder is mostly responsible for heating. Since σ_y is a decreasing function of T , the region under the shoulder will first yield due to a combination of pressure and shear stresses. The time of reaching the desirable temperature in the workpiece matches experimental observations.

Figure 3: Evolution of temperature field.

4 A one-dimensional plastic-elastic model

In this section, we consider a 1D plastic-elastic problem where the yield stress σ_y is assumed to be a constant. For plastic deformation, we consider two different plastic models. Our first plastic model follows from the general framework of the previous section, which uses von-Mises yield criterion and assumes that the plastic flow is incompressible. The second plastic model assumes the material is compressible with a simpler yield criterion, which is used in a model for plastic deformation in a gun barrel problem [3]. The

work in this section was mainly carried out by the two faculty members (Sean Bohun and Huaxiong Huang), a student (Kate Powers) and a postdoc (Zilong Song) with valuable input from Kirk Fraser. Zilong Song re-examined the model and produced most of the results presented in this section after the workshop.

We consider a plane-strain problem, and use polar coordinates (r, θ) for the plane of aluminum plate and z for the thickness of plate in the formulation. By a 1D model we mean that all the functions are functions of r only. The displacement in the elastic region is $\vec{u} = \langle u_r(r), u_\theta(r), 0 \rangle$, and the velocity in the plastic region is $\vec{v} = \langle v_r(r), v_\theta(r), 0 \rangle$. In the following, we will consider an infinite domain $r_p < r < \infty$, which is divided into two regions: the plastic region $r_p < r < r_b$ and elastic region $r_b < r < \infty$, where r_b is the location of free boundary between them. We will study the elastic and plastic problems separately, and match the solutions by jump conditions on plastic-elastic interface (a free boundary).

4.1 1D elastic model

In the elastic region $r_b < r < \infty$, the force balance becomes

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}) = 0, \quad \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{2}{r} \sigma_{r\theta} = 0, \quad (13)$$

with the simplified constitutive relations

$$\begin{aligned} \sigma_{rr} &= (\lambda + 2\mu)\varepsilon_{rr} + \lambda\varepsilon_{\theta\theta}, & \sigma_{r\theta} &= 2\mu\varepsilon_{r\theta}, \\ \sigma_{\theta\theta} &= (\lambda + 2\mu)\varepsilon_{\theta\theta} + \lambda\varepsilon_{rr}, & \sigma_{zz} &= \lambda(\varepsilon_{\theta\theta} + \varepsilon_{rr}), \\ \varepsilon_{rr} &= \frac{\partial u_r}{\partial r}, & \varepsilon_{r\theta} &= \frac{1}{2} \left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right), & \varepsilon_{\theta\theta} &= \frac{u_r}{r}, \end{aligned} \quad (14)$$

where $\varepsilon_{rr}, \varepsilon_{r\theta}, \varepsilon_{\theta\theta}$ and $\sigma_{rr}, \sigma_{r\theta}, \sigma_{\theta\theta}$ are respectively the in-plane components of strain tensor $\boldsymbol{\varepsilon}$ and stress tensor $\boldsymbol{\sigma}$, and σ_{zz} is stress component along the thickness direction z . The boundary conditions are

$$\begin{aligned} u_r &= u_\theta = 0, & \text{as } r \rightarrow \infty \\ \sigma_{rr}(r) &= \sigma_{rr}^b, & \sigma_{r\theta}(r) &= \sigma_{r\theta}^b, & \text{at } r = r_b, \end{aligned} \quad (15)$$

where $\sigma_{rr}^b, \sigma_{r\theta}^b$ are unknown constants to be determined by matching the solutions in the plastic region.

The solution of above linear elastic problem can be readily obtained. The two displacements and non-zero stresses are given by

$$\begin{aligned} u_r &= -\sigma_{rr}^b \frac{1+\nu}{E} \frac{r_b^2}{r}, & u_\theta &= -\sigma_{r\theta}^b \frac{1+\nu}{E} \frac{r_b^2}{r}, \\ \sigma_{rr} &= \sigma_{rr}^b \frac{r_b^2}{r^2}, & \sigma_{\theta\theta} &= -\sigma_{rr}^b \frac{r_b^2}{r^2}, & \sigma_{r\theta} &= \sigma_{r\theta}^b \frac{r_b^2}{r^2}. \end{aligned} \quad (16)$$

One can see that the displacement decays like r^{-1} while stresses decay like r^{-2} .

4.2 1D incompressible plastic model

In this plastic model, we use the same formulation as in previous section but reduce it to 1D. We consider the steady state (quasi-static process) and the term $\partial \vec{v}/\partial t$ will be neglected. As this is a 1D problem, the partial derivatives are actually total derivatives but we retain the partial derivative notation for continuity.

In the plastic region $r_p < r < r_b$, the force balance and incompressibility condition read

$$\begin{aligned} \rho \left(v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} \right) &= \frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r} (\sigma_{rr} - \sigma_{\theta\theta}), \\ \rho \left(v_r \frac{\partial v_\theta}{\partial r} + \frac{v_r v_\theta}{r} \right) &= \frac{\partial \sigma_{r\theta}}{\partial r} + \frac{2}{r} \sigma_{r\theta}, \\ \frac{\partial v_r}{\partial r} + \frac{v_r}{r} &= 0. \end{aligned} \quad (17)$$

The constitutive relations and yield criterion are given by

$$\begin{aligned} \sigma_{rr} &= p + \frac{\sigma_y}{\Lambda} \frac{\partial v_r}{\partial r}, & \sigma_{\theta\theta} &= p + \frac{\sigma_y}{\Lambda} \frac{v_r}{r}, \\ \sigma_{r\theta} &= \frac{\sigma_y}{2\Lambda} \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right), & p &= \frac{1}{3} (\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz}), \\ (\sigma_{rr} - p)^2 + (\sigma_{\theta\theta} - p)^2 + (\sigma_{zz} - p)^2 + 2\sigma_{r\theta}^2 &= \frac{2\sigma_y^2}{3}, \end{aligned} \quad (18)$$

The boundary conditions are

$$\begin{aligned} v_\theta &= \gamma 2\pi\omega r_p, & \sigma_{r\theta} &= f\sigma_{rr}, & \text{at } r &= r_p, \\ v_\theta &= 0, & p &= p_b, & \text{at } r &= r_b, \end{aligned} \quad (19)$$

where p_b is assumed to be known for the time being, ω is the rotational frequency of the pin, $0 < \gamma < 1$ is a dimensionless scalar, and f is the frictional coefficient.

In this system, there are 4 unknowns v_r, v_θ, Λ and p (or equivalently σ_{rr}). One can easily solve v_r and Λ analytically with two integrating constants c_1, c_2 ,

$$v_r = \frac{c_1}{r}, \quad \Lambda = \left(\frac{3\sigma_y^2 c_1^2}{r^4 (\sigma_y^2 - 3\sigma_{r\theta}^2)} \right)^{\frac{1}{2}}, \quad \sigma_{r\theta} = \frac{\rho c_1 v_\theta}{r} + \frac{c_2}{r^2}, \quad (20)$$

then the system is reduced to two equations for v_θ and σ_{rr}

$$\frac{\partial v_\theta}{\partial r} = \left(\frac{1}{r} + \frac{2\Lambda\rho c_1}{\sigma_y r} \right) v_\theta + \frac{2\Lambda c_2}{\sigma_y r^2}, \quad (21)$$

$$\frac{\partial \sigma_{rr}}{\partial r} = \frac{2\sigma_y c_1}{\Lambda r^3} - \frac{\rho c_1^2}{r^3} - \frac{\rho v_\theta^2}{r}. \quad (22)$$

In principle, the two functions and integrating constants c_1, c_2 can be determined by the four boundary conditions. Note that σ_{rr} and p are related by $\sigma_{rr} = p - \frac{\sigma_y c_1}{\Lambda r^2}$.

This system of two ODEs was solved during the workshop by Kate Powers using the shooting method. To solve the system one chooses a value of c_1 and then solves (21), iterating on c_2 until the boundary condition $v_\theta(r_b) = 0$ is satisfied. Armed with values of c_1 , and c_2 , equation (22) is solved and c_1 allowed to update so that the condition $\sigma_{r\theta}(r_p) = f\sigma_{rr}(r_p)$. The solution is obtained when these nested processes converge simultaneously. Figure 4 shows the resulting MATLAB plots, with data $\omega = 1000/60$ 1/s, $\gamma = 1/2$, $f = 1/5$, $r_b = 3r_p = 0.015$ m, $p_b = 0$. We can see that these results are not realistic because the radial velocity is extremely high, implying that the material is moving away from the pin and creating a large hole.

To investigate the reason for the unrealistic solution, we carry out a dimensional analysis. We use a prime to denote the dimensionless quantities, and adopt the scales

$$\begin{aligned} r &= r_p r', & (\boldsymbol{\sigma}, p) &= \sigma_y (\boldsymbol{\sigma}', p'), & (v_r, v_\theta) &= V (v_r', v_\theta'), & \Lambda &= \bar{\Lambda} \Lambda', \\ c_1 &= C_1 c_1', & c_2 &= C_2 c_2' \end{aligned} \quad (23)$$

where $V, \bar{\Lambda}, C_1, C_2$ are some characteristic values. If we balance inertia and stress by choosing

$$\rho V^2 = \sigma_y, \quad C_1 = r_p V, \quad \bar{\Lambda} = V/r_p, \quad C_2 = \sigma_y r_p^2, \quad (24)$$

then all the terms in the two ODEs are in the same order. In this situation, from the data in Table 1, $\sigma_y \sim 10^8$ Pa, $\rho \sim 10^3$ kg/m³ the resulting speed is of the order of 10^2 – 10^3 m/s, explaining the large velocity of v_r in Figure 4. For the FSW technique, balancing the inertial forces with the characteristic yield stress requires the corresponding characteristic speed to be very large. The fact that these large velocities are not experienced indicates that inertial forces do not dominate. Other scales are also possible for this system.

A more reasonable scaling would be to choose the characteristic speed to match the

Figure 4: Numerical results for the 1D incompressible plastic model by shooting method. The red lines in the stress plot are the stresses in the elastic region, which we can see match up at the boundary.

axial velocity of the rotating pin radius resulting in

$$V = 2\pi\omega r_p, \quad C_1 = r_p V, \quad \bar{\Lambda} = V/r_p, \quad C_2 = \sigma_y r_p^2 \quad (25)$$

and with typical values of $\rho = 2.7 \times 10^3 \text{ kg/m}^3$, $\sigma_y = 10^8 \text{ Pa}$, $r_p = 0.5 \times 10^{-2} \text{ m}$, so that $V = 0.5236 \text{ m/s}$. Scaling in this fashion indicates that even with the significant rotation rates involved in the process, $\rho V^2 \ll \sigma_y$, indicating that the inertial terms are dominated by the yield stress. Practically, this means that the inertial terms with ρ in governing equations can be neglected.

By neglecting all the inertial terms, the system can be solved analytically with four integrating constants. Now we present the solution in dimensionless quantities and for

convenience the prime is removed (e.g., in the following formula r in fact means r')

$$\begin{aligned} v_r &= \frac{c_1}{r}, \quad \sigma_{r\theta} = \frac{c_2}{r^2}, \quad \sigma_{rr} = p - \frac{c_1}{\Lambda r^2}, \quad \Lambda = \left(\frac{3c_1^2}{r^4 - 3c_2^2} \right)^{1/2}, \\ v_\theta &= \frac{\sqrt{3}c_1^2}{c_2 r} \left(\frac{r^4 - c_2^2}{c_1^2} \right)^{1/2} + c_3 r, \\ \sigma_{rr} &= \frac{c_1}{\sqrt{3}|c_1|r^2} \left(r^2 \log \left(r^2 + \sqrt{r^4 - c_2^2} \right) - \sqrt{r^4 - c_2^2} \right) + c_4. \end{aligned} \quad (26)$$

The four $O(1)$ integrating constants can be determined easily from four boundary conditions, for example, with the same data $\gamma = 1/2, f = 1/5, p_b = 0, r_b = 3$ as in the solution obtained earlier (but in dimensionless form), we get

$$c_1 = 1.5447, \quad c_2 = -0.3652, \quad c_3 = 7.3195, \quad c_4 = -1.6685. \quad (27)$$

This numerical example shows that the original quantities v_θ and v_r are in the same order of $V = 2\pi\omega r_p$, which is more realistic than previous calculation (cf. Figure 6(b) for stress distribution).

Finally in this subsection, we match the plastic solution with the elastic one. We adopt the jump conditions at free boundary r_b ,

$$v_\theta = 0, \quad [\sigma_{rr}] = [\sigma_{r\theta}] = [\sigma_{\theta\theta}] = 0, \quad r = r_b, \quad (28)$$

which implies $p_b = 0$. We need one extra condition to solve the system, since r_b is not known. One possibility is that we consider a quasi-static process, and the evolution of free boundary r_b follows

$$\frac{dr_b}{dt} = v_r(r_b), \quad r_b(0) = 2r_p. \quad (29)$$

where the initial position of the free interface is set to be the circumference of the shoulder $r_s = 2r_p$. Figure 5 shows the evolution of r_b with data $\gamma = 1/2, f = 0.2$ on the boundary. Figure 6 shows the stress distributions in both the elastic and plastic region for two cases $r_b = 2$ and $r_b = 3$.

Another possibility is to use approximate conditions from the pressure of the shoulder on the top surface of the material. Since there is singularity at the circumferential boundary of the shoulder, we can not use this approximation for the whole region. We only use such conditions at the two ends, at $r = r_p$ and $r = r_b$. Or we set $p(r_p) = \sigma_{zz}(r_p) = P_0$ where P_0 is the pressure by the axial force generated by the shoulder. The condition at

Figure 5: Evolution of r_b .

Figure 6: Distribution of stresses with $r_b = 2$ and $r_b = 3$.

$r = r_b$ is $p(r_b) = \sigma_{zz}(r_b) = 0$, consistent with the previous condition $p_b = 0$. With the extra condition at r_p , we can determine r_b , for example if $P_0 = -\sigma_y$ we get $r_b/r_p = 2.349$.

Numerical results show that the larger the pressure P_0 , the smaller the radial velocity v_r . Thus, the pressure has the effect of preventing the material from going away from the pin. Figure 7 shows the dependence of r_b/r_p on parameters $-P_0/\sigma_y$ and f . The plastic region becomes larger when pressure increases, which indicates pressure from the shoulder plays an important role in yielding.

4.3 1D compressible plastic model

The constitutive relation for the previous plastic model implies the incompressibility condition, which may be inconsistent with the plane-strain assumption (uniform in the thickness direction). In this subsection, we relax the incompressibility assumption and consider the force balance with a simpler yield criterion, which is used for a gun barrel problem in [3].

Figure 7: Dependence of r_b/r_p on P_0/σ_y and f

The inertial terms are assumed to be negligible, so the governing equations and yield criterion are

$$\frac{d\sigma_{rr}}{dr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0, \quad \frac{d\sigma_{r\theta}}{dr} + \frac{2}{r}\sigma_{r\theta} = 0, \quad \frac{1}{4}(\sigma_{rr} - \sigma_{\theta\theta})^2 + \sigma_{r\theta}^2 = \sigma_y^2(T), \quad (30)$$

where the yield stress $\sigma_y(T)$ can be a function of temperature T and consequently, a function of r . Consistent with the previous boundary conditions and matching conditions we take

$$\begin{aligned} \sigma_{rr} &= P_r, \quad \sigma_{r\theta} = f\sigma_{rr}, \quad \text{at } r = r_p, \\ [\sigma_{rr}] &= [\sigma_{r\theta}] = 0, \quad \text{at } r = r_b, \\ (\sigma_{rr}^b)^2 + (\sigma_{r\theta}^b)^2 &= \sigma_y^2, \quad \text{or } \sigma_{rr}^b + \sigma_{\theta\theta}^b = 0, \quad \text{at } r = r_b, \end{aligned} \quad (31)$$

where $P_r < 0$ and $|P_r|$ is the physical pressure on the surface of the pin. The continuity of $\sigma_{\theta\theta}$ is tacitly assumed in the last equation.

Solutions to this system are readily obtained

$$\begin{aligned} \sigma_{r\theta}(r) &= fP_r \frac{r_p^2}{r^2}, \quad \sigma_{rr}(r) = \pm \int_{r_p}^r \frac{2}{s} \sqrt{\sigma_y^2(s) - \sigma_{r\theta}^2(s)} ds + P_r, \\ \sigma_{\theta\theta}(r) &= \sigma_{rr}(r) \pm 2\sqrt{\sigma_y^2(r) - \sigma_{r\theta}^2(r)}. \end{aligned} \quad (32)$$

Later calculation shows that the solution with positive sign is the correct one. As a special case, with constant σ_y , the explicit solution is

$$\begin{aligned} \sigma_{rr}(r) &= f(r) - f(r_p) + P_r, \\ f(r) &= \sigma_y \log \left(\sigma_y \left(\sqrt{r^4 \sigma_y^2 - f^2 P_r^2 r_p^4} + r^2 \sigma_y \right) \right) - \frac{1}{r^2} \sqrt{r^4 \sigma_y^2 - f^2 P_r^2 r_p^4}. \end{aligned} \quad (33)$$

The three matching conditions uniquely determine the free boundary r_b and the two matching parameters $\sigma_{rr}^b, \sigma_{r\theta}^b$ of the elastic solution.

In fact, the above solution is a generalization of the gun barrel problem. For the special case $f = 0$, i.e., when plasticity is induced by pressure only, we recover the solution in the gun barrel problem [3, pp. 351-352]. In this case, the plastic solution reads

$$\sigma_{r\theta} = 0, \quad \sigma_{rr} = 2\sigma_y \ln\left(\frac{r}{r_p}\right) + P_r, \quad \sigma_{\theta\theta} = 2\sigma_y \ln\left(\frac{r}{r_p}\right) + P_r + 2\sigma_y, \quad (34)$$

and yielding position r_b is explicitly given by

$$r_b = r_p \exp\left(\frac{-P_r}{2\sigma_y} - \frac{1}{2}\right) \quad (35)$$

which indicates plastic deformation when $|P_r| > \sigma_y$.

For the data $\{f \rightarrow 1/2, P_r \rightarrow -1.5 \times 10^8 \text{ Pa}, \sigma_y \rightarrow 10^8 \text{ Pa}, r_p \rightarrow 0.005 \text{ m}\}$, we get a numerical value $r_b \approx 0.007062 \text{ m}$. Figure 8 shows the stress distribution for both elastic and plastic regions. Figure 9 shows the dependence of r_b/r_p on the parameters $-P_r/\sigma_y$ and f . As the pressure P_r or frictional coefficient f increases, the plastic region becomes larger.

Figure 8: Distribution of stresses predicted by plastic model II.

5 Direct numerical solutions

During the workshop, direct numerical simulations were carried out. One was based on a 2D non-Newtonian fluid model for material flow in the plastic region [1], and the

Figure 9: Dependence of r_b/r_p on the parameters $-P_r/\sigma_y$ and f .

other was based on the preceding 1D dynamic system. We achieved partial success, but also encountered some problems in obtaining desirable results.

We consider the 2D non-Newtonian fluid model for the velocity $\vec{v} = \langle v_r(r), v_\theta(r), 0 \rangle$ coupled with a heat equation,

$$\begin{aligned} \nabla \cdot (2\mu \dot{\epsilon}) - \nabla p &= 0, \quad \nabla \cdot \vec{v} = 0, \\ \rho C_p \frac{DT}{Dt} &= \nabla \cdot (k \nabla T) + \alpha \text{tr}(2\mu \dot{\epsilon}), \end{aligned} \quad (36)$$

where α is a coefficient ranging from 0.9 to 1 when the mechanical power does not entirely dissipate in the form of heat (considering possible hardening), and the effective viscosity μ depends on the strain rate through the Norton-Hoff law

$$\mu = K(T)(2\text{tr}(\dot{\epsilon}\dot{\epsilon}) + 3\gamma^2)^{\frac{m(T)-1}{2}}, \quad (37)$$

where the two coefficients K and m are functions of temperature, which play a similar role as that of yield stress depending on temperature in preceding model. The finite element method is used to solve this system with some typical initial and boundary conditions. Figure 10 shows the distribution of temperature and two velocities. But we encountered a problem related to the stability of stress components.

The second simulation is based on the previous dynamic 1D plastic model. After some

Figure 10: Distribution of temperature and velocity predicted by the Non-Newtonian fluid model.

manipulation, the system is simplified to four equations for four unknowns $(v_r, v_\theta, \Lambda, p)$

$$\begin{aligned}
 \rho \frac{\partial v_r}{\partial t} + \left(\rho v_r - \frac{\sigma_y}{\Lambda r} \right) \frac{\partial v_r}{\partial r} - \frac{\partial}{\partial r} \left(\frac{\sigma_y}{\Lambda} \frac{\partial v_r}{\partial r} \right) + \frac{\sigma_y}{\Lambda r^2} v_r &= \frac{\rho}{r} v_\theta^2 + \frac{\partial p}{\partial r}, \\
 \rho \frac{\partial v_\theta}{\partial t} + \left(\rho v_r - \frac{\sigma_y}{\Lambda r} \right) \frac{\partial v_\theta}{\partial r} + \frac{\partial}{\partial r} \left(\frac{\sigma_y}{2\Lambda r} v_\theta - \frac{\sigma_y}{2\Lambda} \frac{\partial v_\theta}{\partial r} \right) + \left(\frac{\rho v_r}{r} + \frac{\sigma_y}{\Lambda r^3} \right) v_\theta &= 0, \\
 \Lambda &= \sqrt{\frac{3}{2} \left(2 \left(\frac{v_r}{r} \right)^2 + \frac{1}{2} \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \right)^2 \right)}, \\
 -\frac{\partial}{\partial r} \left(r \frac{\partial p}{\partial r} \right) &= -\frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \left(\frac{\sigma_y}{\Lambda r} v_r \right) + \frac{2\sigma_y}{\Lambda r} v_r + \rho r v_r \frac{\partial v_r}{\partial r} - \rho v_\theta^2 \right),
 \end{aligned} \tag{38}$$

with the boundary conditions

$$\begin{aligned}
 v_\theta &= \gamma 2\pi\omega r_p, \quad \sigma_{rr} = P_r, \quad \text{at } r = r_p, \\
 v_\theta &= 0, \quad v_r = 0, \quad \text{at } r = r_b.
 \end{aligned} \tag{39}$$

The finite volume method is used to solve this system, but we encountered a problem regarding the stability of solving the pressure equation.

6 Future work and discussion

In this report, we have investigated separately the heat generation and plastic deformation. The simulation of the temperature field and 1D analytical results provide some insights into the mechanism of the FSW process. Future work will be directed to the coupling of the two aspects. For example, the effect of yielding material is not considered in the 1D temperature model, and we need to estimate the stress power $\text{tr}(\boldsymbol{\sigma}\dot{\boldsymbol{\epsilon}})$ based on velocity in the plastic deformation. On the other hand, the dependence of yield stress on

temperature is a crucial factor in the plastic-elastic deformation, and this should be taken into account to determine the location of free boundary or the size of yielding region.

In the plastic-elastic model, we have used a plane strain formulation, by assuming everything is uniform along the thickness direction. This causes some inconsistency between yielding condition and incompressibility condition. It suggests that the component v_z or $\partial v_\theta / \partial z$ is important. A plate model will be needed to reduce the 3D problem to a 2D one, by keeping some information of these components.

References

- [1] Eric Feulvarch, J-C Roux, and J-M Bergheau. A simple and robust moving mesh technique for the finite element simulation of friction stir welding. *Journal of Computational and Applied Mathematics*, 246:269–277, 2013.
- [2] Kirk Fraser, Lyne St-Georges, and Laszlo I Kiss. A mesh-free solid-mechanics approach for simulating the friction stir-welding process. In *Joining Technologies*. InTech, 2016.
- [3] Peter Howell, Gregory Kozyreff, and John Ockendon. *Applied solid mechanics*, volume 43. Cambridge University Press, 2009.
- [4] Rafal Smerd. Constitutive behavior of aluminum alloy sheet at high strain rates. Master’s thesis, University of Waterloo, 2005.