Value of Domestic Tourists in the Travel and Tourism Industry

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Summary
The COVID-19 pandemic has affected South Africa’s tourism industry, which had a large dependence on international tourists, due to the restrictive travel measures put in place to curb the spread of the virus. The tourism industry contributes significantly to South Africa’s economy, and the lack of international tourists and subsequent revenue loss has been felt across the industry. One proposition to mitigate this is to promote domestic tourism. The report is concerned with the design of a mathematical model which would boost domestic tourism in the Mpumalanga Province, South Africa. The model investigates the formulation and optimisation of a customised (tourism) package cost function.

1 Introduction
The COVID-19 pandemic has affected the majority of business sectors around the globe. Most companies were closed and others suffered severe loss in revenue. The tourism industry was no exception as it experienced a great loss of revenue. Thus the focus of our Study Group was to determine a mathematical model that would assist the Mpumalanga tourism industry to have customised packages of tourism products and/or services which shall be utilised to mitigate the challenge of catering for the domestic market.

1.1 Definitions
The World Tourism Organization defines tourism [4, p. 9-10] as travelling to and staying in places outside the usual home environment for no less than 24 hours and not more than one consecutive year for leisure, business and other purposes.

A tourist is defined as [4, 6, 9]:

- a person who contributes an economic input with regard to any area other than in which they generally live or work;
- or a person who voluntarily visits a place, away from their normal residence for a period of at least 24 hours.

Furthermore, there are four subcategories of tourists [5, 9], namely:

- foreign (or international) tourists;
- domestic (or local/national) tourists;
- excursionists (see [6, p. 393] for a definition);
- inbound and/or outbound tourists.

There are roughly eight categories in the tourism sector:

- accommodation;
- conferences and events;
- food and beverage;
- other tourism services;
• recreation and travel adventures;
• transporation;
• tourism trade;
• tourist attractions.

Accommodation is one of the largest and fastest growing areas in the tourism industry and constitutes approximately 70 percent of tourist revenue. Conferences and events also include conventions, meetings, special conferences and events, trade shows and the catering for the aforementioned. Food and beverage include dining places like restaurants, bars, etc. Other tourism services include associations, organisations, government agencies, and companies that specialise in serving the needs of the tourism industry. Recreation and travel adventures include tourist activities in a tourism area (for example, World Heritage Sites, nature parks or reservations, etc) or with relation to the culture of indigenous communities in the tourism area. Transporation refers to the various modes of transport that tourists may make use of in a tourism area. Tourism trade refers to the economic trade in support of tourism. Some examples include booking accommodation and transporation via tourism operators and travel agencies.

1.2 Mpumalanga Province Tourism

Mpumalanga province (previously known as the Eastern Transvaal), see Figure 7 on page 15, is located in eastern South Africa.

The province shares its borders with four South Africa provinces, the Free State, Gauteng, KwaZulu-Natal and Limpopo, and with two countries, Eswatini (formerly Swaziland) and Mozambique. Its capital city is Mbombela, previously known as Nelspruit.

Mpumalanga means “east” or “the place where the sun rises”. It is for this meaning, that the province is also well known as “the place of the rising sun”.

It is one of South Africa’s top tourism destinations with a spectacular natural diversity, being rich in flora and fauna, and tourist attractions. Mpumalanga Province is home to the world famous Kruger National Park (Figure 8 page 16), the Bourke’s Luck Potholes (Figure 9a page 17), the Three Rondavels (Figure 9b page 17), and the Blyde River Canyon (Figure 9c page 17). In addition, Mpumalanga houses South Africa’s tenth World Heritage Site, the Barberton Makhonjwa Mountains World Heritage Site (Figure 9d page 17). It also hosts many other adventure and cultural attractions, which includes a living (human) international heritage icon in Dr Ester Mahlangu from the Ndebele Culture (Figure 9e page 17). This province may be regarded as a true South African showcase of natural beauty and cultural diversity.

1.3 Domestic Tourism

Prior to the COVID-19 pandemic, tourism had been seen as a fast-growing industry globally, with the economies of different countries benefitting from the increased revenues 10.

In South Africa, the tourism sector contributes significantly to the economy. The local tourist industry is heavily reliant on international tourists. However, during the COVID-
19 pandemic, with the increase of travel restrictions to curb the spread of the virus, the local tourism industry has suffered significantly.

Therefore, it has been proposed that the industry’s focus should be shifted to domestic tourists to save the tourism sector, specifically for the Mpumalanga province.

2 Solution Approach

Our group decided to take a generalised approach: instead of focusing only on tourism in the Mpumalanga province, our solution should be applicable to any province in the country.

Our initial idea was to minimise the expense cost for a domestic tourist, which by our reasoning would be the foremost consideration for any tourist. We reasoned that this will have the indirect effect of increasing the number of domestic tourists, which would then lead to an increase of revenue for tourism companies.

We decided to make use of four of the eight tourism areas our solution approach, since these are the most general for any tourist destination:

- accommodation;
- transportation;
- food and beverage;
- attractions and recreation.

We designated each of these areas with a cost function variable as follows:

- $A : \mathbb{N} \rightarrow \mathbb{R}^+$, $n \mapsto A(n)$ (for accommodation);
- $T : \mathbb{N} \rightarrow \mathbb{R}^+$, $n \mapsto T(n)$ (for transportation);
- $F : \mathbb{N} \rightarrow \mathbb{R}^+$, $n \mapsto F(n)$ (for food and beverage);
- $R : \mathbb{N} \rightarrow \mathbb{R}^+$, $n \mapsto R(n)$ (for attractions and recreation);

where $\mathbb{N}$ is the set of natural numbers (excluding zero), $\mathbb{R}^+$ is the set of positive real numbers, and the independent variable $n \in \mathbb{N}$ represents the number of domestic tourists.

2.1 Package Cost Function

From the perspective of the tourist operators, it might be more cost effective to offer accommodation, transportation, food and beverage, attractions and recreation as a singular package to the domestic tourist. With this context in mind, we shall also refer to the functions $A$, $T$, $F$, and $R$ as the individual cost functions.

We define the package cost function as

$$ p(n) = \omega_A A(n) + \omega_T T(n) + \omega_F F(n) + \omega_R R(n) $$

(2.1)

where $\omega_j \in \mathbb{R}$ are weights subject to

$$ \omega_A + \omega_T + \omega_F + \omega_R = 1. $$

(2.2)

The weights determine how much emphasis is put on each individual cost function, that is, how much accommodation, transportation, food and beverage, attractions and
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recreation contribute to or make part of the package cost function. Our overarching goal was to determine the value of \( n \) which would minimise the function (2.1).

2.2 Modelling the individual cost functions

2.2.1 Quality and Season

The tourist experience is crucial, therefore we have to take the quality of the experience into consideration. Various factors may play a role in determining the quality of the tourist experience. For example, the standard of the accommodation and the standard of the transport provided to the domestic tourists.

Another factor which plays a role is the season during which the tourist experience takes place. High season is when the particular tourism experience is popular or in high demand; similarly, mid (respectively low) season is when the particular tourist experience is in moderate (respectively low) demand or popularity.

We decided on a very rudimentary classification for both quality and season: high, middle, low. We represent these by the discrete variables \( H, M, \) and \( L \), respectively. These variables may take values as we shall see later.

Let \( q \) be the variable which represents the quality of the tourist experience, and let \( s \) be the variable which represents the season of the tourist experience. Then

\[ q, s \in \{H, M, L\}. \]

These discrete variables \( q \) and \( s \) play a role in the individual cost functions. It stands to reason that, for example, a higher quality accommodation will be more expensive than a lower quality accommodation.

2.2.2 Individual Cost Functions Expressions

The individual cost functions were determined by reasoning, trial and error, with valuable input from the Industry Representative.

For most of the individual cost functions, we agreed that there should be some type of nonlinear relationship between the quality and season factors and each of the individual cost functions. The exception is the food cost function where the relationship is more direct. Furthermore, we agreed that there should be direct proportionality between the number of domestic tourists and the cost of accommodation and food and beverage—more tourists imply the costs are more expensive per person. However, for transportation and attractions and recreation, these costs tend to be inversely proportional to the number of domestic tourist paying for the activity—more tourists imply the costs are less expensive per person.

With this reasoning in mind, the following expressions were decided upon for accom-
Figure 1. Individual cost functions—accommodation $A(n)$, transportation $T(n)$, food and beverage $F(n)$, attractions and recreation $R(n)$—against number $n$ of domestic tourists. (Colour online.)

The package cost function then becomes

$$p(n) = \omega A e^{q+s}\ln n + \frac{\omega T e^{q+s}}{n} + \frac{\omega F n q s}{n} + \frac{\omega R e^{q+s}}{n},$$

(2.7)

2.3 Minimisation of the package cost function

To determine the extreme values of the constructed package cost function, we first take the derivative of (2.7) with respect to $n$ to obtain

$$\frac{d}{dn} p(n) = e^{q+s} \left( \frac{\omega A}{n} - \frac{\omega T + \omega R}{n^2} \right) + \omega F q s,$$

(2.8)
Next, we solve the equation

\[ \frac{d}{dn} p(n) = 0, \]

that is,

\[ e^{q+s} \left( \frac{\omega_A}{n} - \frac{\omega_T + \omega_R}{n^2} \right) + \omega_F q s = 0. \]  

(2.9)

We used the \texttt{fsolve} function in the \texttt{optimize} submodule of SciPy (https://scipy.org) to solve equation (2.9) for various chosen values of \( \omega_A, \omega_T, \omega_F, \omega_R, q \) and \( s \).

3 Results

After constructing our model, we considered various scenarios which would affect the choice of weights in the package cost function (2.7).

For our simulations, we assigned the following numerical values to quality and season variables:

\[ L = 1, \quad M = \frac{3}{2}, \quad H = 2. \]

3.1 Preference for accommodation

In this scenario, the accommodation is the main attraction of the tourist operator’s package offering. We chose the weights as follows in this scenario: \( \omega_A = 0.4, \omega_F = 0.2, \omega_T = 0.2, \omega_R = 0.2 \). The quality and season variables were set at high-quality and high-season, that is, \( q = 2 \) and \( s = 2 \).

The corresponding package cost function is illustrated in Figure 2. The \texttt{fsolve} function failed to obtain a minimum for this scenario. However, we note from Figure 2 that the
minimum is obtained for one domestic tourist, and therefore
\[ p(1) = e^{4 \left( \frac{4}{5} \ln(1) + \frac{2}{5} \right)} + \frac{4}{5} (e^{4} + 2) \approx 22.639. \]

The minimal package cost a tourist operator may potentially charge is therefore 23 units.

We note that in the above choice of weights, we obtain \( \omega_F + \omega_R + \omega_T > \omega_A \)
which might imply that food and beverage, transportation, and attractions and recreation contribute more to the package function combined than the accommodation alone. Furthermore, we note \( A(1) = 0 \) so that the accommodation does not contribute at all to the package cost function for \( n = 1 \). This indicates that our choice of functions that constitute \( A(n) \) might not be ideal, which we only discovered after the initial study group. For other scenarios, the choice of functions which constitute the individual cost functions yielded plausible results.

### 3.2 Preference for attractions and recreation

In this scenario, the attractions and recreation are the main focus of the package. We again set the quality and season variables at high quality and high season, that is, \( q = 2 \) and \( s = 2 \). The weights were initially chosen as follows: \( \omega_A = 0.1, \omega_F = 0.2, \omega_T = 0.3, \omega_R = 0.4 \). The corresponding package cost function is illustrated in Figure 3. The local minimum is found at \( n_* \approx 3.4237 \) and the corresponding package cost value \( p(n_*) \approx 20.369 \).

The Industrial Representative suggested another similar scenario where attractions and recreation are preferred, but where food and beverage and transportation contribute
equally to the package cost. For this scenario, we set $\omega_A = 0.1$, $\omega_F = 0.25$, $\omega_R = 0.4$, $\omega_T = 0.25$, $q = 2$, and $s = 2$. The corresponding package cost function is illustrated in Figure 4. The local minimum is found at $n_\star \approx 3.823$ and the corresponding package cost value $p(n_\star) \approx 20.428$.

3.3 Preference for food (and beverage) and transportation

In this scenario, food (and beverage) and transportation are the main focus for the package. This scenario is feasible considering excursionists (day visitors) or tourists emphasising travelling to tourism areas. We set $\omega_A = 0.1$, $\omega_F = 0.4$, $\omega_R = 0.2$, and $\omega_T = 0.3$, $q = 2$, and $s = 2$.

The corresponding package cost function is illustrated in Figure 5. The local minimum is found at $n_\star \approx 3.4237$ and the corresponding package cost value $p(n_\star) \approx 20.396$.

3.4 Effect of quality and season variables on package cost

Next we considered the effect varying the quality and season variables would have on the package cost function.

Recall that the tourism sector generally experiences three seasons in a year, that is, high ($H$), mid ($M$) and low ($L$) seasons. These seasons may vary in length depending on various factors, but combined they should be roughly as long as a calendar year.

We consider the previously mentioned package where attraction and recreation is preferred. Recall that the weights for this scenario were chosen to be $\omega_A = 0.1$, $\omega_F = 0.25$, $\omega_T = 0.25$, $\omega_R = 0.4$. However, now we vary the quality $q$ and season $s$ variables.
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Figure 5. Package cost function for food (and beverage) and transporation preference scenario.

Table 1. Minimal values $n_\ast$ of the package cost function for attraction and recreation preferred package for varying quality and season.

<table>
<thead>
<tr>
<th>Season</th>
<th>Quality</th>
<th>$n_\ast$</th>
<th>$p(n_\ast)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>Low</td>
<td>3.4819</td>
<td>7.9963</td>
</tr>
<tr>
<td></td>
<td>Mid</td>
<td>3.5866</td>
<td>12.910</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.8230</td>
<td>20.428</td>
</tr>
<tr>
<td>Mid</td>
<td>Low</td>
<td>3.2496</td>
<td>5.0912</td>
</tr>
<tr>
<td></td>
<td>Mid</td>
<td>3.3525</td>
<td>8.2098</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.5866</td>
<td>12.921</td>
</tr>
<tr>
<td>Low</td>
<td>Low</td>
<td>3.1477</td>
<td>3.1600</td>
</tr>
<tr>
<td></td>
<td>Mid</td>
<td>3.2496</td>
<td>5.0911</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>3.4819</td>
<td>7.9963</td>
</tr>
</tbody>
</table>

In Figure 6, we plot the package cost function with the aforementioned weights for a fixed season and varying the quality in the season.

From our numerical simulations we obtained the data listed in Table 1 on page 10. We note that there is not a remarkable difference in the minimal values $n_\ast$ of the package cost function, except past the decimal point which would be rounded away. Thus, packages should be designed to cater for three to four people to minimise the package cost. However, we do note the variation in the values of the package cost function as the quality of the offering is varied per season. From Table 1, we note that the similarity of package cost values between different seasons and quality offerings. For example, the same value was obtained for a high quality package in low season as compared to a low quality package.
Figure 6. Quality variation per season for attraction and recreation preferred package.

in high season. This gives some leeway for domestic tourists in terms of what they would prefer to choose.

Our choice of functions and variables and the numerical values assigned to the quality and season variables would yield a similar result as illustrated in Figure 6 in the scenario where we fix the quality of the offering and vary the season. This suggests that we should reconsider our modelling of the quality and season as discrete variables. Although these two concepts, logically, should parallel each other realistically, they might not be equal as we have set in our model.

4 Conclusions

We investigated the problem of boosting domestic tourism in lieu of the COVID-19 pandemic. Tourism operators generally offer their services in a combined tourism package. We reasoned that domestic tourism will increase if tourist operators reduced the cost of these tourism packages, since a lot of people were affected financially by the pandemic.

We proposed a rudimentary mathematical model in which we constructed a package cost function as a linear combination of individual cost functions. The individual costs we considered were accommodation, attraction and recreation, food and beverage, and transportation, as these are the most common services found in tourism. We constructed mathematical formulae for each individual cost function based on recommendations from the Industrial Representative. We also took into account the roles that the quality of the
tourist package and tourist seasons play on the individual cost functions and, hence, also on the package cost function.

We then considered various scenarios; each of the proposed scenarios determined different weights assigned to the individual cost functions in the package cost function. With the weights assigned we determined the critical value \( n^* \), representing the minimal number of domestic tourists, to minimise the package cost function. In most of our scenarios, this was found to be between three to four people.

### 4.1 Future work

The individual cost functions and package cost function could be improved to better represent what happens realistically. For example, we note in the accommodation preferred scenario that \( A(n^*) \) becomes identically zero for \( n^* = 1 \), which means that the accommodation cost function does not contribute at all to the package cost function for the minimal value. This is counter-intuitive to the proposed scenario.

Also, the obtained minimal values of the package cost function do not reflect realistic monetary costs that domestic tourists would have to pay for the tourism package. The design of the package cost function, and individual cost functions, could be bettered by using real-world data.

Furthermore, we may also introduce the quality and season of the tourism experience as independent variables for the individual cost functions and package cost function, instead of discrete numerical values which were arbitrarily chosen.

A better understanding of all of the economical processes involved will assist in the construction of better cost functions.

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### References


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Figure 7. Mpumalanga Province Map

(Figures continue on next page...)
Figure 8. Map of the Kruger National Park.
(a) Bourke’s Luck Potholes [1]
(b) Three Rondavels [2]
(c) Blyde River Canyon [3]
(d) Barberton Makhonjwa Mountains World Heritage Site [5]

(e) Dr Ester Mahlangu from the Ndebele Culture. Source: [5]

Figure 9. Tourist attractions of Mpumalanga province.