Optimal configuration for communication antenna along a railway line

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Summary

Solvit's main task is to provide and maintain stable signal coverage along railway tracks, ensuring quality communications. To do this, the company should design a suitable antenna network in order to guarantee good signal coverage while reducing the costs associated with the entire network. In this report we formulate the problem as 0/1 linear optimization models, and report computational results obtained using the models on real data provided by the company.

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1 Introduction

Solvit is an engineering company that develops and implements projects on Information and Communications Technology. The company's challenge for ESGI 161 is to select, from a number of possible sites, a subset of sites for the installation of antennas in order to ensure stable signal coverage along a railway track, while minimizing costs.

The antennas are located on the railway domain and should preferably coincide with existing geographical infrastructures, since the following requirements are needed to install antennas: energy, free area space, road accessibility. However, there are other requirements that must be taken into account, which are listed below.

The goal of this work is to select locations where to install communication antennas in order to minimize their installation cost and ensure a minimum signal level along the track. The installation cost depends on the site where the antenna is installed.

The company considers two issues in assessing the quality of solutions: the radio signal quality along the railway line, and the cost of installing new antennas, that depends on where the antenna is to be installed, that the company refers as priority site classification. In terms of radio signal quality, there are three scenarios to take into account: lack of coverage - defined as the % of the track where the signal is below $-95\,\mathrm{dBm}$; low signal coverage - defined as the % of the track where the signal is between $-95\,\mathrm{dBm}$ and $-80\,\mathrm{dBm}$; and low signal continuity - defined as the maximum distance, in km, with low signal. Lack of coverage is given a very high weight, while low coverage and low signal continuity is considered a medium one. In the optimization problem, these scenarios are addressed in the imposed constrains.

As mentioned above the site classification is related to the locations where the antennas are to be installed. The company classifies sites in terms of their position along the railway line, and assigns to sites within the same class a numerical value called "priority" (Table 1). We use these "priority" values as costs of installing antennas in the objective function. For example, the cost of installing an antenna placed at a level crossing site is 5.

Site classification	Priority
Anchor (a mandatory antenna)	0
Station	1
Halt	2
Level crossing	5
Sign	6
Other	10

Table 1: Selected sites classification (site priorities).

Wireless communication suffers from severe multi-path fading and path loss [11]. In fact, many factors can affect the system, such as, hand-off, transmission rate, train's velocity, wireless channel, among others. However, in the present challenge these issues will not be addressed since the company has mathematical models that are able to provide estimates of the measures of the signal along the railway track.

The choice of the locations for the installation of the antennas has a great impact on the performance of the system. In this study, we propose a 0/1 linear optimization formulation where each selected antenna should try to extend its coverage area while satisfying the company's requirements regarding coverage area, transmission length and number of antennas.

1.1 The case study

To tackle the proposed challenge, the company provided a case study for the Portuguese Cascais railway line (Figure 1). In this railway line there are 18 candidate sites for installing the antennas: nine stations (Cais do Sodré, Alcântara - Mar, Algés, Caxias, Oeiras, Carcavelos, São Pedro do Estoril, Estoril, Cascais); eight halts (Santos, Belém, Cruz Quebrada, Paço de Arcos, Santo Amaro, Parede, São João do Estoril, Monte Estoril); and one Anchor (PK16+8).

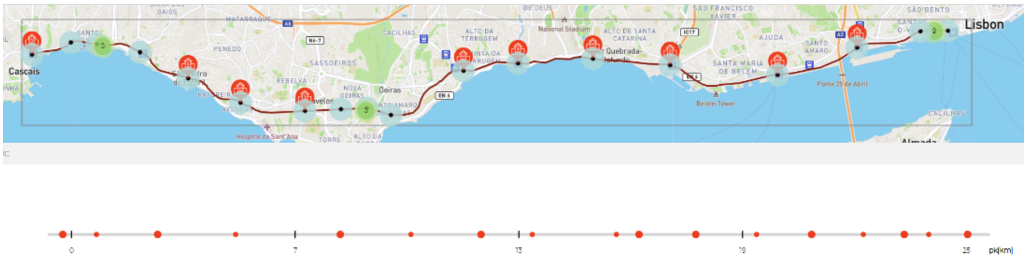


Figure 1. The case study: Cascais railway line.

Measures of the signal on the 18 antennas were given for a total of 5884 not evenly spaced positions along the railway line (Figure 2).

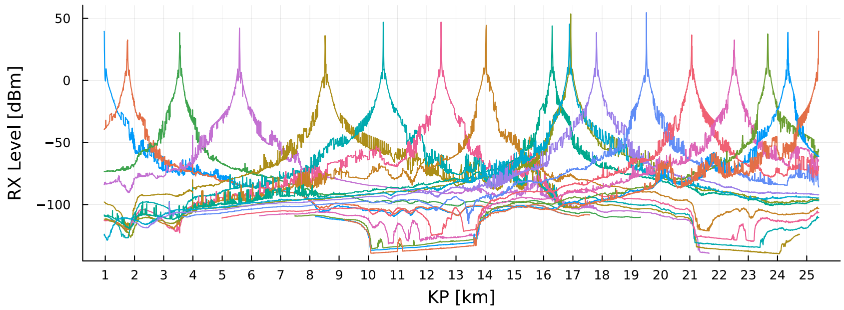


Figure 2. Signals of the 18 candidates antennas' sites.

2 Methodologies

In this study, we propose a 0/1 linear optimization formulation to model the problem posed by the company. This formulation is described in Subsection 2.2. Before that, in Subsection 2.1, we present what was considered in a data pre-processing phase.

2.1 Pre-processing stage

The representation of the signal used was simplified at a pre-processing stage. Figure 3 presents a simplified representation of the signal of the 18 candidate antennas, indicating the positions on the railway track where the signal is above $-80\,\mathrm{dBm}$. The thickness of each dot is settled by the priority (site classification priority) of the respective antenna. It can be seen that there are overlaps of various antenna signals, and there are also different signal ranges for each antenna.

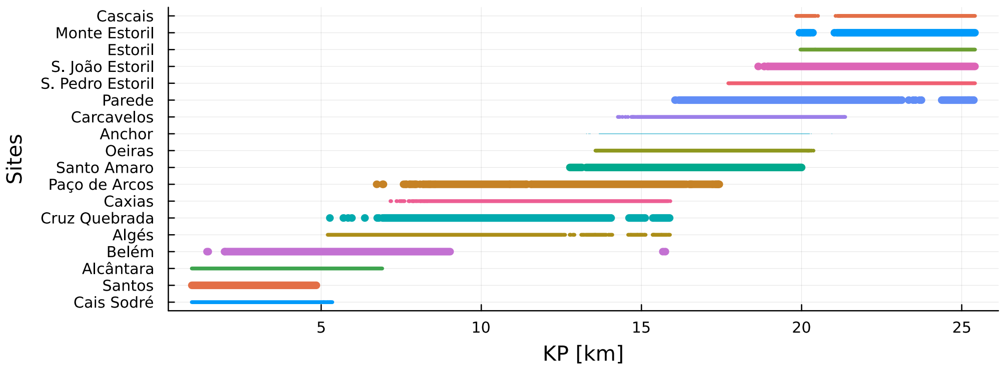
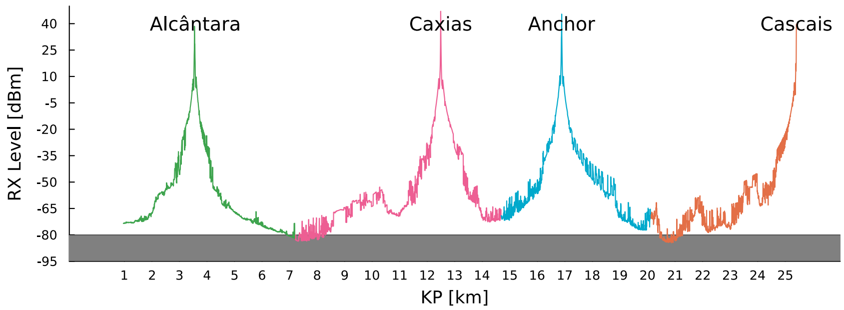


Figure 3. Signals above minimum per antenna. The thickness of each dot is settled by the priority (site classification) of the respective antenna.

Figure 4 presents a possible solution to the problem. For each $j \in \{1, \ldots, n\}$, where n is the number of sites where an antenna can be installed, the variable x_j takes the value one if it is decided to install an antenna in location j, and zero otherwise. In this case, $n = 18$ and the solution consists of installing antennas at the stations Alcântra-Mar, Caxias and Cascais, as well as at the Anchor (PK16+8). This solution is coded as 001000100010000001. It can be seen that for the entire length of the line the signal is above -95 dBm and only in two small portions of the line is the signal below -80 dBm.

We assume that each antenna built at a given location has a unique continuous interval with good signal coverage (see Figure 5). We can relax this condition and consider that the interval of good signal coverage for each antenna is given by a union of disjoint finite intervals. This improvement, although it could have been easily done, was not considered in this report due to lack of realistic data. If implemented, it could allow finding feasible solutions with fewer antennas.



Antennas Name	Cais do Sodré	Santos	Alcântara-Mar	Belém	Algés	Cruz-Quebrada	Caxias	Paço de Arcos	Santo Amaro	Oeiras	Anchor (PK16+8)	Carcavelos	Parede	S. Pedro do Estoril	S. João do Estoril	Estoril	Monte Estoril	Cascais
ID	1	2	**3**	4	5	6	**7**	8	9	10	**11**	12	13	14	15	16	17	**18**
Priorities	1	2	**1**	2	1	2	**1**	2	2	1	**0**	1	2	1	2	1	2	**1**
Combination	0	0	**1**	0	0	0	**1**	0	0	0	**1**	0	0	0	0	0	0	**1**

Figure 4. Interpretation of a possible solution.

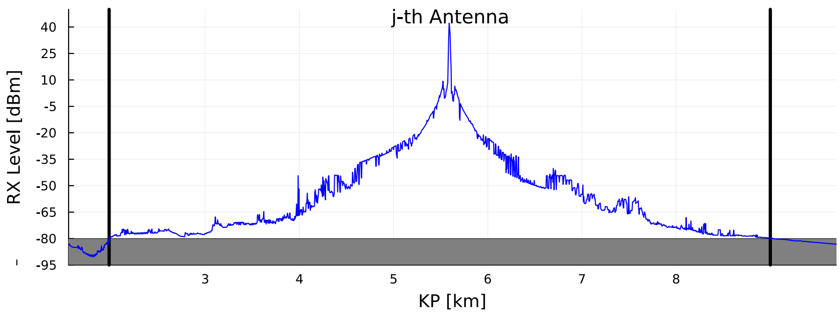


Figure 5. Interval of good signal coverage of the j-th antenna.

In the context of this industrial challenge, areas of overlapping signals from two antennas should be considered whenever they are in transition zones. In [1], a mixed-number optimization model for the coverage localization problem is presented, for which the existence of overlapping coverage areas is necessary while fulfilling all the requirements. Here we consider this situation in the definition of the interval with good signal coverage in a pre-processing stage.

The reduction of signal noise and the size of the data, can also be considered in the pre-processing phase. Here, we consider a smoothing algorithm where, for each antenna, the data is approximated by a cubic spline obtained from 200 equidistant points in the range in which the antenna signal is detected. In Figure 6 we can see that the noise is reduced in the interpolated signal while maintaining the main characteristics of the original signal. Apart from the issue of data smoothing, the number of points where the signal is considered is substantially reduced (from 5884 points to 200 points).

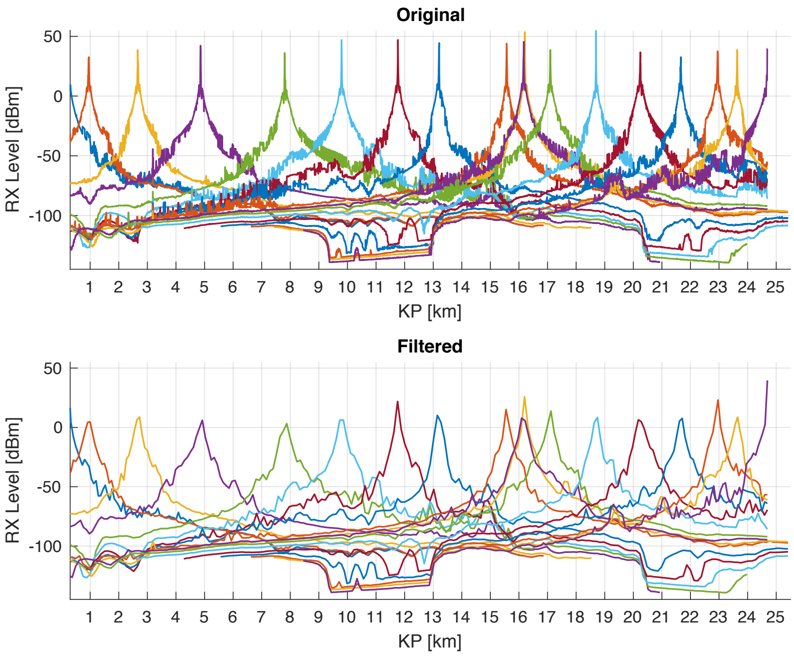


Figure 6. Original and filtered signal (200 equidistant points).

2.2 Optimizing the antennas selection

The proposed methodologies aim to optimize the selection of a subset of sites where to install the antennas, ensuring a minimum signal level along the track, according to the constraints imposed by the company Solvit.

We address the problem in the framework of set covering problems, and consider two different approaches: i) minimizing the cost of installing antennas, while ensuring strong signal on the entire railway line; and ii) installing antennas within a given budget, minimizing the length of the sections of the railway line that have weak or no signal.

Let $J = \{1, \ldots, n\}$ be the set of sites where antennas may be installed. Let $[S, E] \subseteq \mathbb{R}$ be the real segment corresponding to the railway line, and $\mathcal{P} = \{I_i = [s_i, e_i[, i = 1, \ldots, m\}$ be the interval partition of $[S, E]$ that satisfies the following conditions: (i) $s_1 = S$, $e_m = E$; (ii) $e_i = s_{i+1}$, $i = 1, \ldots, m-1$; (iii) every two points in $I_i = [s_i, e_i[$ are covered by exactly the same set of antennas; and (iv) $I_i = [s_i, e_i[$ is maximal with respect to (iii). We call $\mathcal{P}$ the signal partition of the railway line, and I_i the i-th interval of $\mathcal{P}$, $i = 1, \ldots, m$ (see Figure 7).

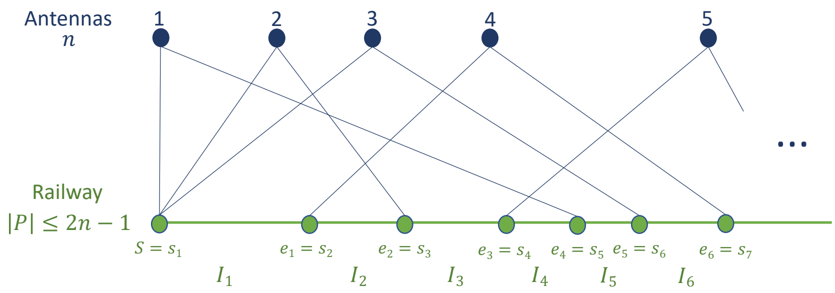


Figure 7. Railway partition.

Since we assume that each antenna has only one continuous signal interval and that the n antennas will fully signal cover the railway line $[S, E]$, it is easy to show that the number of intervals of $\mathcal{P}$, $m \leq 2n-1$ (with equality, in the likely case, that no two extreme points of different signal intervals of two antennas coincide). Clearly, the result holds for $n = 1$. To complete the proof, note that adding an extra antenna to a set of n antennas, will partition each of two of the $m \leq 2n-1$ intervals of $\mathcal{P}$ in two intervals, thus creating no more than $2n - 1 + 2 = 2(n + 1) - 1$ intervals.

Let $A = [a_{ij}]$ be an $m \times n$ 0/1 matrix, where $a_{ij} = 1$ if interval I_i will be signal covered by an antenna installed in location j. Let also c_j be the cost of installing an antenna at site j, $j = 1, \dots, n$.

To model i) and ii) above we use 0/1 decision variables $x_j, j = 1, \dots, n$, where $x_j = 1(0)$ if an antenna is installed in site j is (not) selected.

2.2.1 *Model i): Signal along the entire railway line*

We formulate the problem as a set cover problem (see, e.g., [3], [6], and [9]). Despite being NP-hard [8], there are solvers that produce optimal, or near optimal solutions, for median large instances (say about one thousand variables and five thousand constraints [5]).

Using the parameters and variables defined above, the model consists of:

$$\min \quad \sum_{j=1}^{n} c_j \, x_j \tag{2.1}$$

subject to

$$\sum_{j=1}^{n} a_{ij} \, x_j \geq 1, \quad i = 1, \dots, m, \tag{2.2}$$

$$x_j \in \{0, 1\}, \quad j = 1, \dots, n. \tag{2.3}$$

The objective function (2.1) expresses the total installation cost of the facilities we want to minimize. Constraints (2.2) guarantee that every interval I_i, $i = 1, \dots, m$, will be covered by at least an antenna. Finally, constraints (2.3) indicate the variables type.

The case where antennas have to be installed (or are already installed) in some given sites should be taken into account. The sites of these mandatory antennas are called anchor sites, and the set of these sites will be denoted by Ach. To incorporate this condition in the above model we add the following equations:

$$x_j = 1, \quad j \in Ach. \tag{2.4}$$

In case where different optimal solutions exist, instead of identifying a single one, it may be desired to provide a list of all the optimal solutions. To produce this list, we propose to solve the problem repeatedly, while the current optimal value equals the initial optimal value, adding in each iteration the inequality

$$\sum_{j \in sol} x_j \leq |sol| - 1,$$

where sol is the optimal "solution" obtained in the previous iteration, i.e., $sol = \{j \in J : x_j^* = 1\}$, where x^* is the optimal solution obtained in the previous iteration. This causes solution x^* to be non-feasible in subsequent iterations.

In Section 3, we present six optimal solutions given by the optimization model (2.1)-(2.4) for the case study given by the company (see Subsection 1.1). In all of them it is required that all points of the track have a good signal (they are the feasible solutions).

To assess sites combination in terms of the priorities given to the number of sites and to the selected sites combination, it is pertinent to control the number of antennas to be installed, and relate this to the total cost. To control the number of antennas we introduce the additional constraint

$$\sum_{j=1}^{n} x_j = b, \tag{2.5}$$

where b is number of antennas to consider.

The problem may now be solved for different values of $b \geq b_0$, where b_0 is the minimum b for which feasible solutions exist.

2.2.2 *Model ii): Relaxing having signal along the entire railway line*

Here, we will describe relaxed optimization models that allows feasible solutions not having signal cover along the entire railway line. The motivation is that the company wants to evaluate the costs of antenna networks that, although not guaranteeing full signal coverage on the railway line, can have acceptable coverage (with small failures) but with lower costs. It is possible that there are antenna configurations which do not guarantee full coverage of the rail network, but which are preferable to others which do guarantee full coverage, because they reduce the cost function, or because they have fewer antennas, or because the sum of the antenna priorities is lower.

In the following model we want to maximize the signal coverage of the railway line, assuming that there is a given budget B that limits the cost of the installation of antennas. This is the well-known maximal coverage problem (see, e.g., [7], [10],and [2]) that can be formulated as follows:

$$\max \quad \sum_{i=1}^{m} L_i y_i \tag{2.6}$$

subject to

$$y_i \leq \sum_{j=1}^{n} a_{ij} x_j \quad i = 1, \ldots, m, \tag{2.7}$$

$$\sum_{j=1}^{n} c_j x_j \leq B, \tag{2.8}$$

$$x_j \in \{0, 1\}, \quad j = 1, 2, \ldots, n, \tag{2.9}$$

$$y_i \leq 1, \quad i = 1, 2, \ldots, m, \tag{2.10}$$

where $L_i = e_i - s_i$ is the length of interval $I_i = [s_i, e_i[$.

As in Model i), variables $x_j = 1(0)$ indicate that an antenna will (not) be installed in site j. The budget constraint (2.8) ensures that the total cost of antennas' installation will not exceed the given budget B. Inequalities (2.7) and (2.10) ensure that $y_i \leq 0$ if no antenna covering interval I_i is installed; and, if an antenna covering I_i is installed (i.e., $a_{ij} x_j = 1$, for some j), then y_i may assume values greater or equal to 1. Since $L_i > 0$, in the optimal solution $y_i = 0$, in the former case, and $y_i = 1$, in the latter case. Thus the objective function (2.6) seeks to maximize the sum of the lengths of the intervals that will be signal covered.

As we already notice, we did not consider in this report the situation where the range of good signal coverage for some antennas is the union of disjoint intervals. This can be easily handled creating as many copies of each of these antennas as the number of disjoint signal coverage intervals of that antenna. If the signal coverage of an antenna j consists of k disjoint intervals, then replace antenna j by the k fictitious "antennas" $j_1, \ldots, j_k$, with costs $c_{j_1} = c_{j_2} = \cdots = c_{j_k} = c_j/k$, and add to Models i) and ii) constraints $x_{j_1} = x_{j_2} = \cdots = x_{j_k}$, stating that either all fictitious "antennas" are selected, meaning that antenna j is to be installed, or no fictitious "antenna" is selected, meaning that antenna j is not installed.

3 Numerical results

In what follows we present the solutions obtained using the methods described in Section 2. These methods are implemented computationally using the **Julia** (v1.8.2) language, making use of some of its modules, such as **DataFrames**, **LinearAlgebra**, **XLSX** and **Plots**. Moreover, the **HiGHs** and **JuMP** modules (see [4]) are used to solve the constrained optimization problems described in Section 2.

Throughout this section, the costs c_j, $j = 1, \ldots, n$, (see (2.1)) are given by the priorities of antennas defined according to Table 1. So, the objective function take into account the number of selected sites and the sites classification. The parameters related with radio signal quality defined by the company (see Section 1) are considered in the constrains and are used to distinguish between feasibility of the solutions, where a feasible solution is defined as a solution that covers up to -80 dBm along all the railway, and non-feasible if not.

3.1 Full signal coverage

In this subsection we present the full signal coverage solutions given by the optimization model (2.1)-(2.4), for the case study given by the company in Subsection 1.1.

In Figures 8–10, we can see six optimal combinations of sites that ensure a good signal coverage (≥ -80 dBm) along the railway. We stress that, for all these six solutions, the minimum cost given by the objective function (see (2.1)) is 3. More specifically, we can see that it is possible to guarantee a full signal coverage along the railway with the Anchor antenna together with only three more antennas with priority classification of 1.

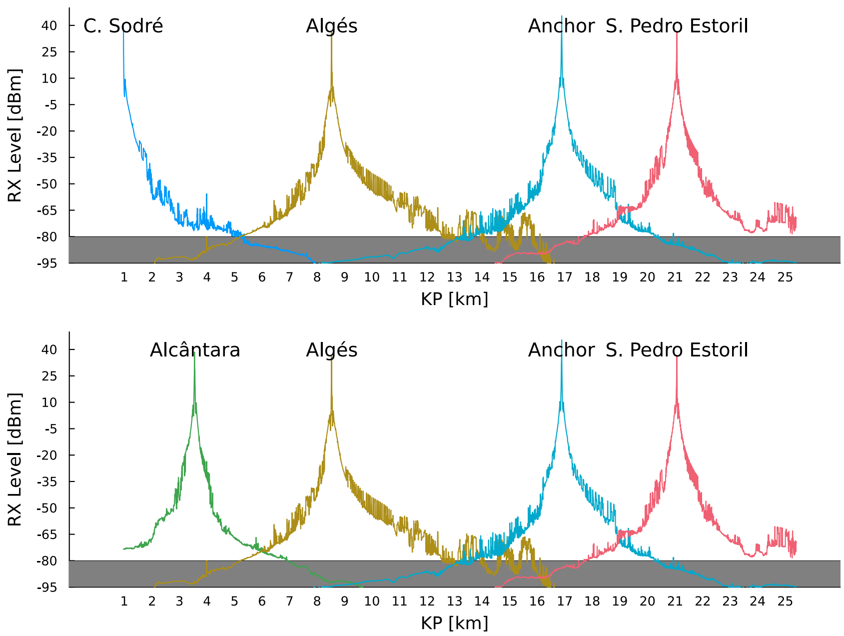


Figure 8. Solutions: (top) 100010000010010000; (bottom) 001010000010010000.

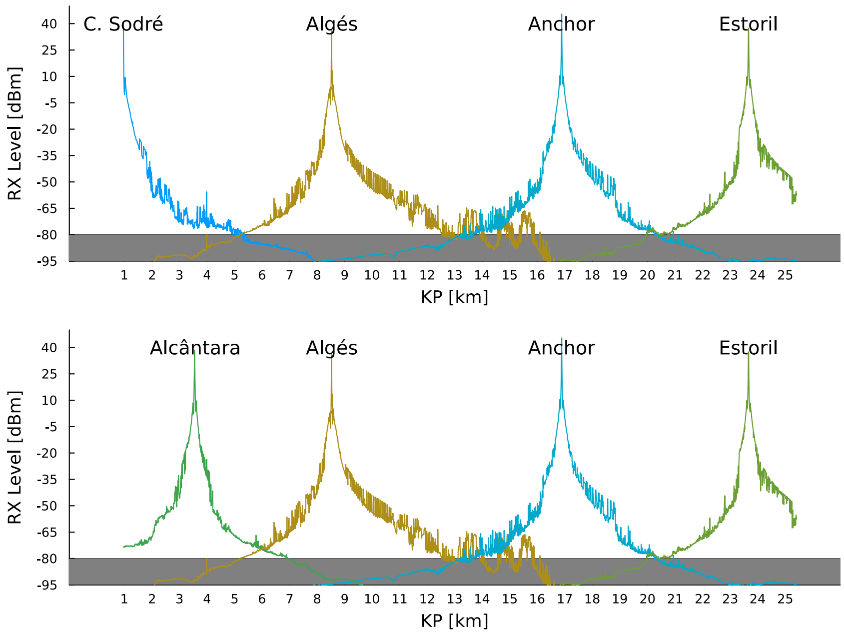


Figure 9. Solutions: (top) 100010000010000100; (bottom) 001010000010000100.

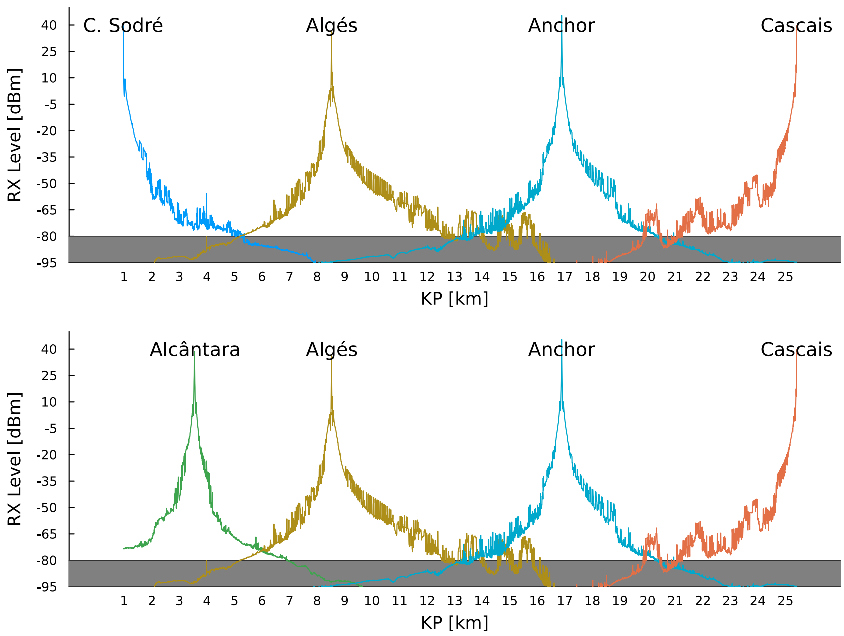


Figure 10. Solutions: (top) 100010000010000001; (bottom) 001010000010000001.

3.2 Relaxed signal coverage

Here, we show some results obtained using the model (2.6)–(2.10). More specifically, we present the solutions for which the installation of the antennas has a budget of less than 3. We stress that if one assumes the maximum budget of $B = 3$, i.e., the cost of the full signal coverage solutions presented in Subsection 3.1, the same six solutions are obtained (see Figures 8–10). Thus, in what follows, we present the maximal coverage solutions with allowed constructions budgets of $B = 2$ and $B = 1$, respectively.

If a maximum budget of 2 is imposed, three solutions that cover 82.6% of the railway are obtained (see Figure 11–12).

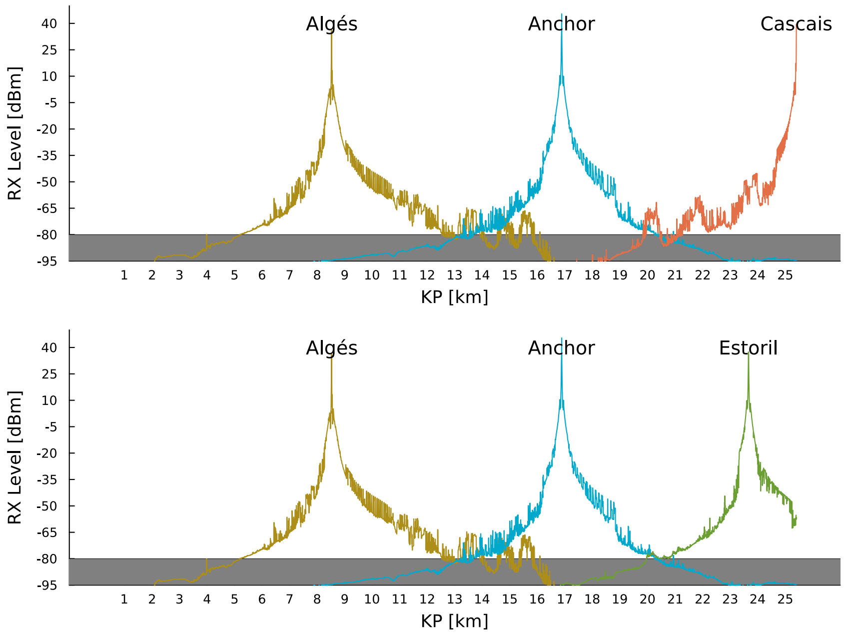


Figure 11. Solutions: (top) 000010000010000001; (bottom) 000010000010000100.

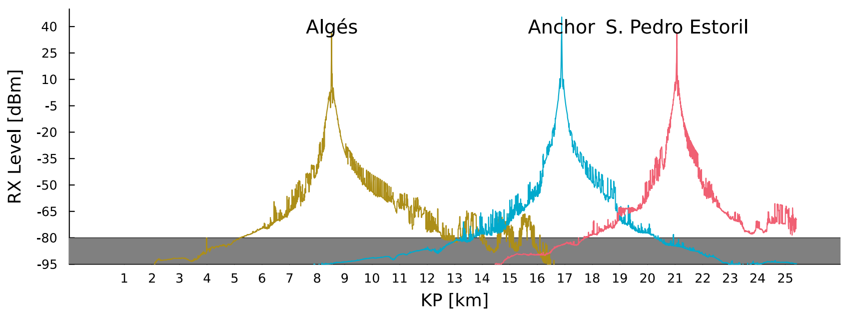


Figure 12. Solution: 000010000010010000

Finally, for an allowed budget of $B = 1$, only 64.4% of the railway track is covered and only one solution is obtained (see Figure 13).

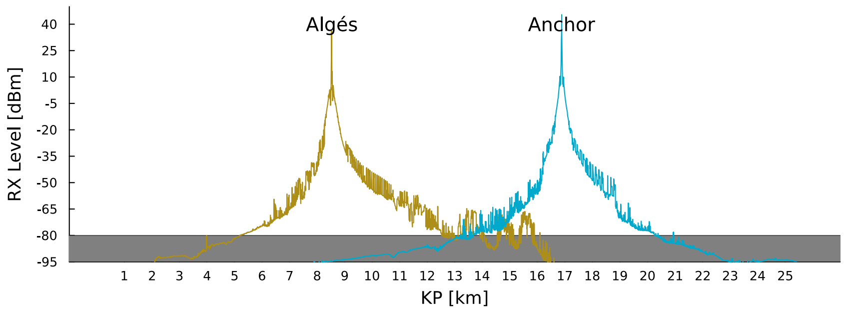


Figure 13. Solution: 000010000010000000

Note that the antenna located at Algés is common in all the four combinations presented here, as in the six solutions in the previous subsection.

It should be noted that this algorithm allows us to find these combinations with computational execution times in the order of seconds, using a standard laptop computer.

3.3 Ranking the optimal solutions

We will now describe a strategy to rank the optimal solutions given by optimization problems. We propose to consider a non-linear cost function, obtained by a more detailed description of the constraints given by the company, and which is used, a posteriori, to rank the solutions obtained by the algorithms described in Section 2.

Let us start by defining the following positive dimensionless quantities that, for feasible solutions $x = (x_j)_{j=1,\dots,n}$, are not expected to be much higher than 1:

$$0 \leq LC(x), LSC(x), C(x), N(x), P(x) \lesssim 1. \tag{3.1}$$

More specifically, functions LC, LSC, C, N and P stand for the lack of coverage, low signal coverage, signal continuity, number of sites and sites priorities, respectively. These notations are motivated by the concepts presented in Section 1, which are used empirically by the company to select and rank the possible combinations of sites.

In order to define these dimensionless functions we introduce the following scaling constants and coefficients: $-80\,\text{dBm}$ and $-95\,\text{dBm}$ are the low signal and lack of signal threshold values; S and E are the the starting and ending points of the railway, respectively [km]; $L = E - S$ is the railway length [km]; $\bar{D}$ is the average distance of coverage for the antennas (with signal $\geq -80\,\text{dBm}$)) [km]; $\bar{N} = \lceil \frac{L}{\bar{D}} \rceil$ is the minimum number of antennas required to cover the railway; $\bar{A} := \bar{D} \times 15$ is a reference value for the low signal coverage [km $\times$ dBm]. In addition, given the signal coverage associated with a solution x which can be represented by a function $f_x : [S, E] \longrightarrow \mathbb{R}\,(\text{dBm})$, in the sense of Figure 4, we define the following auxiliary functions:

$$f_x^h(t) = -80 - \min_{t\in[S,E]}\{-80, \max\{-95, f_x(t)\}\},$$
$$f_x^l(t) = -95 - \min_{t\in[S,E]}\{-95, f_x(t)\}.$$

We use f_x^h to evaluate the area of the graph of f_x between the low signal coverage region, i.e., where $-95 < f_x \leq -80$. On the other hand, with f_x^l we evaluate the area of the graph of f_x in the lack of coverage region, i.e., where $f_x \leq -95$.

We are now able to define the functions given in (3.1) as follows:

$$LC(x) = \frac{1}{\bar{A}}\int_S^E f_x^l(s)\,\mathrm{d}s,$$
$$LSC(x) = \frac{1}{\bar{A}}\int_S^E f_x^h(s)\,\mathrm{d}s,$$
$$C(x) = \frac{1}{\bar{D}}\max_{t_1<t_2\in[S,E]}\{|t_2-t_1| : \forall t\in[t_1,t_2],\ f_x(t) < S_s\},$$
$$N(x) = \frac{1}{\bar{N}}\sum_{j=1}^n x_j,$$
$$P(x) = \frac{1}{\bar{N}\max\limits_{j=1,\ldots,n}\{c_j\}}\sum_{j=1}^n x_j c_j,$$

where c_j, $j = 1,\ldots,n$, are the site priorities defined in Table 1.

From the previous definitions we define the following cost function:

$$F(x) = 10^{LC(x)} + e^{N(x)} + LSC(x) + C(x) + (P(x))^2. \tag{3.2}$$

This cost function should be seen as a mere example; it can easily be changed by another one that the company considers more convenient without having to change the optimization algorithm. The choice of this function took into account the parameters related with radio signal quality defined by the company (see Section 1).

In what follows, we evaluate $F(x)$ in order to rank the solutions presented in the previous subsections, along with some others. We also test some feasible and non-feasible combinations, in order to illustrate the use of the cost function proposed here. Recall that we assume that a solution is non-feasible if the signal ($\geq -80\,\mathrm{dBm}$) along the railway is not guaranteed.

In Table 2 one can see the ordered solutions using the function defined in (3.2). We note that in the last column of this table, whenever they are shown, we refer to the figures in this work. Moreover, the column labeled with N shows the number of antennas in the corresponding solution and the column labeled LP refers to the cost function (2.1) of the LP model (2.1)-(2.4).

F / ID	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	LP	N	Figures
3.8790462	0	0	1	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	3	4	Figure 8(bottom)
3.8790468	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	3	4	Figure 9(bottom)
3.8790480	1	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	3	4	Figure 8(top)
3.8790486	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	3	4	Figure 9(top)
3.8843242	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	2	2	Figure 13
3.8949483	0	0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	3	4	Figure 10(bottom)
3.8949501	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	3	4	Figure 10(top)
3.9298649	0	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	3	3	
3.9384414	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	3	3	
4.0444962	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	2	
4.1074113	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	2	3	Figure 12
4.1074118	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	1	0	0	2	3	Figure 11(bottom)
4.1159878	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	1	2	3	Figure 11(top)
4.1445329	1	0	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	3	4	
4.1531094	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	3	4	
4.3537200	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	2	
4.8450397	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	2	
6.2108113	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	
10.801664	0	0	1	0	1	0	1	0	0	0	1	0	0	0	0	1	0	0	5	5	Figure 14(bottom)

Table 2: Ordered combination of sites.

Finally, in Figure 14 a non-optimal feasible solution of five antennas is shown.

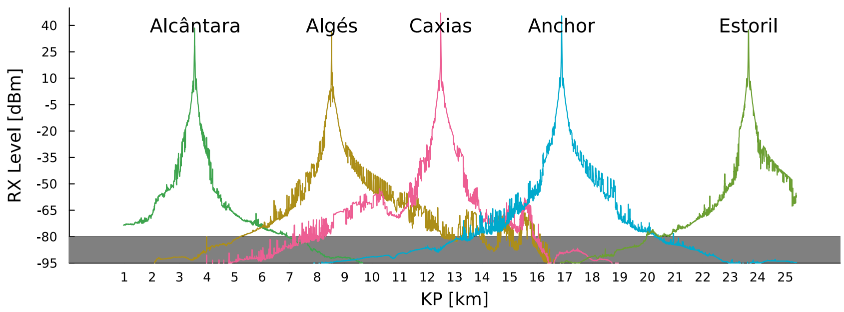


Figure 14. A non optimal feasible combination of five antennas.

4 Conclusions and recommendations

In this report, we present an efficient strategy to solve the signal coverage problem in a railway line. We considered a 0/1 Linear Programming algorithm that took into account the priorities presented by the company. The work was carried out based on a case study that showed the effectiveness of the considered approach.

The proposed approach now needs to be validated with real data, in particular in the definition of the weights/costs associated to each antenna, and also for more problematic railway lines. We recommend that the company stays in contact with the team of mathematicians who developed this proposal to make the necessary improvements to overcome the new challenges that will arise.

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