# Errata and addenda for 2013-2017 printings of Horn & Johnson, *Matrix Analysis*, 2nd ed.*

July 16, 2021

All of the following edits are included in the final 2017 reprinting of *Matrix Analysis,* 2nd ed.

# Chapter 0

p. 2, line 14 of (0.1.3): Replace "; if $S$ is empty" with ". If $S$ is empty"
p. 2, line 15 of (0.1.3): Replace "of $V$. The" with "of $V$; since the"
p. 2, line 15 of (0.1.3): Replace "subspace $\{0\}$, so the" with "subspace $\{0\}$, the"
p. 4, line 20: *subspace intersection lemma* should be *subspace intersection theorem*
p. 7, lines 3-4: Replace "$(AB)^* = B^*A^*$ and $(AB)^T = B^TA^T$." with "$(AB)^T = B^TA^T$ and $(AB)^* = B^*A^*$."
p. 8, line 4: "$\sum_{k=1}^{n}$" should be "$\sum_{k=1}^{p}$"
p. 11, line $-11$: "the systems of linear equations" should be "consistent systems of linear equations".
p. 12, first line of **0.4.2**: "$b \in \mathbf{F}^n$" should be "$b \in \mathbf{F}^m$"
p. 15, line -6: "$v_1, \ldots, v_n$" should be "$x_1, \ldots, x_n$"
p. 16, line 1 of (0.6.6): Replace "Given any subset $S \subset \mathbf{C}^n$, the" with "Given any set $S \subset \mathbf{C}^n$, its".
p. 16, line 2 of (0.6.6): Replace "of $S$ is the set" with "is the set"
p. 16, line 2 of (0.6.6): Replace ". Even if" with "if $S$ is nonempty; if $S$ is empty, then $S^\perp = \mathbf{C}^n$. In either case, $S^\perp = (\operatorname{span} S)^\perp$. Even if"
p. 18, line -14:

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p. 20, line 7 should read "ably, the converse is true: If $A_{11} \in M_r$, rank $A = r$, and"
p. 20, in the display (0.7.6.1) replace "if rank $A =$" with "$r =$"
p. 20, line $-15$. "with $X, Y \in M_{m,r}(\mathbf{F})$; see (0.4.6c)" should be "with $X \in M_{m,r}(\mathbf{F})$ and $Y \in M_{n,r}(\mathbf{F})$; see (0.4.6(e))"
p. 21, third line of **0.8.1**. "$\alpha, \beta$, entry" should be "$\alpha, \beta$ entry"
p. 21, fourth line of **0.8.1**. "rows and column" should be "rows and columns"
p. 22, lines $-3$ and $-2$. "adj $A = \alpha x y^T$ for some nonzero $\alpha \in \mathbf{F}$ and nonzero $x, y \in \mathbf{F}$" should be "adj $A = xy^T$ for some nonzero $x, y \in \mathbf{F}^n$"
p. 24, line 4 of **0.8.3**. "(0.8.2.9)" should be "(0.8.2.10)"
p. 25. In (0.8.5.6), the $A_{11}$ in the $1, 1$ entry of the displayed matrix should be $A_{11}^{-1}$.
p. 27. Replace the last 8 lines of **0.8.5** with the following text:

Alternative assumptions about commuting blocks and consideration of the Schur complement of $A_{22}$ in $A$ lead to other identities:

$$\det \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{cases} \det(A_{11}A_{22} - A_{21}A_{12}) \text{ if } A_{11}A_{21} = A_{21}A_{11} \\ \det(A_{22}A_{11} - A_{21}A_{12}) \text{ if } A_{11}A_{12} = A_{12}A_{11} \\ \det(A_{11}A_{22} - A_{12}A_{21}) \text{ if } A_{22}A_{21} = A_{21}A_{22} \\ \det(A_{22}A_{11} - A_{12}A_{21}) \text{ if } A_{22}A_{12} = A_{12}A_{22} \end{cases} \tag{0.8.5.13}$$

A continuity argument shows that these identities are valid even if $A_{11}A_{22}$ is singular.

p. 27. In the first display in **0.8.6**, "$\det A[\{1, \ldots k, k+i\}$" should be "$\det A[\{1, \ldots, k, k+i\}$", that is, a comma must be inserted.
p. 28, line 9 of **0.8.8**. "$A(\{1,3\}; 4, 2)$" should be "$A[\{1,3\}; 4, 2]$"
p. 29. Replace the three lines in **0.8.11** with the following:
**0.8.11 Dodgson's identity.** Let $n \geq 3$ and let $A \in M_n(\mathbf{F})$. Define $a = \det A[\{n\}^c]$, $b = \det A[\{n\}^c, \{1\}^c]$, $c = \det A[\{1\}^c, \{n\}^c]$, $d = \det A[\{1\}^c]$, and $e = \det A[\{1, n\}^c]$. Then

$$e \det A = ad - bc$$

p. 29, first line of (0.8.12): $\alpha \subseteq \{1, \ldots, m\}$ should be $\alpha \subseteq \{1, \ldots, n\}$.
p. 29, line preceding (0.8.12.2): $rth$ adjunct should be $rth$ adjugate.
p. 29, line following (0.8.12.2): $\mathrm{adj}_1(A) = A$ should be $\mathrm{adj}_1(A) = \mathrm{adj}\, A$.

p. 29, the display (0.8.12.3): $s^k t^{n-k}$ should be $s^{n-k}t^k$ and $C_r(B)$ should be $C_k(B)$, that is, the display should be

$$\det(sA + tB) = \sum_{k=0}^{n} s^{n-k}t^k \operatorname{tr}(\operatorname{adj}_k(A)C_k(B)) \tag{0.8.12.3}$$

p. 29, last line: $\operatorname{tr} C_r(B)$ should be $\operatorname{tr} C_k(A)$.
p. 31, end of line 5. "$((\det B)(\det A)A^{-1}\oplus$" should be "$((\det B)(\det A)A^{-1})\oplus$"
p. 31, beginning of line 6. "$(\det A)(\det B)B^{-1})$" should be "$((\det A)(\det B)B^{-1})$"
p. 34, line $-2$. "$b_k = a_0^{-1}(\sum_{m=0}^{k-1} a_{k-m}b_m)$" should be "$b_k = -a_0^{-1}\sum_{m=0}^{k-1} a_{k-m}b_m$"
p. 35, line $-2$. Delete "a sequence of 2-by-2 matrix products"
p. 35, display in last line. Delete the two zero columns, so that the display becomes

$$\begin{bmatrix} \det A_{k+1} \\ \det A_k \end{bmatrix} = \begin{bmatrix} a_{k+1} & -b_k c_k \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \det A_k \\ \det A_{k-1} \end{bmatrix}, \quad k = 2, \ldots, n-1$$

p. 40, line 17. The first term in the display should be ${}_{\mathcal{B}_2}[T]_{\mathcal{B}_2}[x]_{\mathcal{B}_2}$ instead of ${}_{\mathcal{B}_1}[T]_{\mathcal{B}_2}[x]_{\mathcal{B}_2}$

# Chapter 1

p. 43, line 5. "index" should be "Index"
p. 43, line 7. "Bibliography" should be "References"
p. 45, line -3. Replace "$2, 0, 0$" with "$3, 0, 0$".
p. 50, line 4. At the end of the line, "and $(t - a_{ii})$" should be "and $(t - a_{jj})$"
p. 50, line -4. "(1.2.4)" should be "(1.2.4a)"
p. 55, line 3 of **1.2.P2**. "$A, B \in M_n$" at the beginning of the line should be "$A \in M_n$"
p. 57. In **1.2.P17**, "(0.8.5.13–14)" should be "(0.8.5.13)"
p. 58, line 17. Replace "(e.g., Jordan canonical form)" with "(for example, the Jordan or Weyr canonical forms)"
p. 59, third line of the proof of Theorem 1.3.7: $S_2 \in M_n$ should be $S_2 \in M_{n,n-k}$.
p. 61, line -6. "$B_1, \ldots, B_d$" should be "$B_1, \ldots, B_{d-1}$"
p. 61, line -5. "$M_{n,n+m}$" should be "$M_{m,n+m}$"
p. 62, last line. "$F$-invariant" should be "$\mathcal{F}$-invariant".
p. 63, lines 4 and 9: $1 < k < n$ should be $1 \leq k < n$.

p. 63, line -12. In the display, $B_1$ should be just $B$.
p. 63, line -5: $k \in \{2, \ldots, n-1\}$ should be $k \in \{1, \ldots, n-1\}$.
p. 71, last line of (1.3.P19): (4.6.P20) should be (4.6.P24).
p. 72, second line of (1.3.P21(f)): $\pm\mu_1 \ldots, \pm\mu_n$ should be $\pm\mu_1^{1/2} \ldots, \pm\mu_n^{1/2}$.
p. 74, end of second line of **1.3.P36**: "If $A$ and " should be "If $n = 2$ and if $A$ and $B$".
p. 74, third line of **1.3.P36**: $M_n$ should be $M_2$.
p. 76, lines 5 and 6. Replace the second sentence of the proof of (1.4.1) with the following sentence: Similarly, $p_{A^*}(t) = \det(tI - A^*) = \det(tI - \bar{A})^T = \det(tI - \bar{A}) = \overline{\det(\bar{t}I - A)} = \overline{p_A(\bar{t})}$.
p. 77, line -9. (0.2.5) should be (0.5).
p. 79, line 17. Replace "$= \lambda(S^{-*}y)^*$" with "$= \lambda(S^{-*}y)^*$ and $S^{-*}y \neq 0$ since $y \neq 0$."
p. 80, first line of (1.4.P1): $x, y \in M_n$ should be $x, y \in \mathbf{C}^n$.
p. 82, line -4: delete "is diagonalizable and"
p. 82, line -3: After "...and negative.", insert a new sentence: Why is $A$ diagonalizable?

# Chapter 2

p. 83, line -12. "$U \in M_m$ and $V \in M_n$" should be "$U \in M_n$ and $V \in M_m$"
p. 93, last line of (2.1.P23): Replace "(i.e., $A^*A =$" with "($A^*A =$"
p. 97. Delete lines 1-6 and replace with the following text:

Use (2.1.13) again to form a unitary matrix $(I_1 \oplus U_2)$ that takes the first column of $A_2$ into a vector whose entries below the second are all zero and whose second entry is nonnegative. Let $V_2 = I_2 \oplus U_2$ and let $\mathcal{A}_2 = V_2 \mathcal{A}_1 V_2^*$; the first column of $\mathcal{A}_1$ is undisturbed. After $n-2$ of these reductions, we obtain an upper Hessenberg matrix $\mathcal{A}_{n-2}$ that is unitarily similar to $A$ and has nonnegative subdiagonal entries except perhaps for the entry in position $(n, n-1)$; a final unitary similarity via $I_{n-1} \oplus [e^{i\theta}]$ may be necessary to rotate it to be nonnegative.

p. 99, first lines of (2.2.P3) and (2.2.P4): in both lines, $\operatorname{tr} W(A^T \bar{A})$ should be $\operatorname{tr} W(A^T, \bar{A})$.

p. 99, line 4 of (2.2.P4): "either (b) or (c)" should be "either (a) or (b)".

p. 112. Replace the proof of Theorem 2.4.4.1 with the following:

**Proof.** Consider the linear transformation $T : M_{n,m} \to M_{n,m}$ defined by $T(X) = AX - XB$. To ensure that the equation $T(X) = C$ has a unique solution $X$ for every given $C \in M_{n,m}$ it suffices to show that the only solution of $T(X) = 0$ is $X = 0$; see (0.5). If $AX - XB = 0$, we know from the preceding discussion that $p_B(A)X - Xp_B(B) = 0$. The Cayley–Hamilton theorem ensures that $p_B(B) = 0$, so $p_B(A)X = 0$. Let $\lambda_1, \ldots, \lambda_m$ be the eigenvalues of $B$, so $p_B(t) = (t - \lambda_1) \cdots (t - \lambda_m)$ and $p_B(A) = (A - \lambda_1 I) \cdots (A - \lambda_m I)$.

If $\sigma(A) \cap \sigma(B) = \varnothing$, then each factor $A - \lambda_j I$ is nonsingular, $p_B(A)$ is nonsingular, and the only solution of $p_B(A)X = 0$ is $X = 0$.

If $\lambda \in \sigma(A) \cap \sigma(B)$, let $y \in \mathbf{C}^m$ and $x \in \mathbf{C}^n$ be nonzero vectors such that $y^*B = \lambda y^*$ and $Ax = \lambda x$ (left and right eigenvectors). Then $\hat{X} = xy^* \neq 0$ and $A\hat{X} - \hat{X}B = \lambda\hat{X} - \lambda\hat{X} = 0$.

If $A$ and $B$ are real, consider the linear transformation $T : M_{n,m}(\mathbf{R}) \to M_{n,m}(\mathbf{R})$ defined by $T(X) = AX - XB$. The same argument shows that: (a) If $\sigma(A) \cap \sigma(B) = \varnothing$ then $AX - XB = 0 \Rightarrow p_B(A)X = 0 \Rightarrow X = 0$, and (b) if $\sigma(A) \cap \sigma(B) \neq \varnothing$ then $\operatorname{Re}(A\hat{X} - \hat{X}B) = A(\operatorname{Re}\hat{X}) - (\operatorname{Re}\hat{X})B = 0$, $\operatorname{Im}(A\hat{X} - \hat{X}B) = A(\operatorname{Im}\hat{X}) - (\operatorname{Im}\hat{X})B = 0$, and either $\operatorname{Re}\hat{X}$ or $\operatorname{Im}\hat{X}$ is nonzero. ∎

p. 115, line 1. $T_{11}X - XS$ should be $T_{11}X - XS_2$
p. 121, line -12. "$U_k^* A U_k$ is ..." should be "$U_k^* A_k U_k$ is ...
p. 122, line 6. In the display labelled (2.4.9.3) delete the pair of curly brackets around the expression $|\lambda_{\pi(i)}(A_k) - \lambda_i(A)|$. The display will then read

$$\min_{\pi \in S_n} \max_{i=1,\ldots,n} |\lambda_{\pi(i)}(A_k) - \lambda_i(A)| \leq \varepsilon \text{ for all } k \geq N$$

p. 124, lines 3-4 of Problem 2.4.P2. The sentence "Write $A = \ldots$ upper triangular." should be "Write $A = UTU^*$, in which $U$ is unitary, $T = [t_{ij}]$ is upper triangular, and $|t_{11}| \geq \cdots \geq |t_{nn}|$."
p. 145, second line of (2.5.P51): "$\xi = [\xi_i] = Ux$" should be "$\xi = [\xi_i] = U^*x$".
p. 147, (2.5.P64): Delete "(a)".
p. 162, second line of (2.7.P4): "nullities of $U_{12}$ and $U_{21}$ are equal." should be "nullities of $U_{12}$ and $U_{21}^*$ are equal."

# Chapter 3

p. 166, display at the bottom of the page should be

$$\begin{bmatrix} 0 & 0 & a_2^T \\ 0 & J_{k_1} & 0 \\ 0 & 0 & J \end{bmatrix}$$

(no tilde over $J_{k_1}$ in the center of the matrix).
p. 171, line -8. "$D_{\epsilon,q}$, and compute" should be "$D_{\epsilon,k}$, and compute"
pp. 183-184. Replace all of Section 3.2.10 with the following:
**3.2.10 The index of an eigenvalue of a block upper triangular matrix.** If the indices of $\lambda$ as an eigenvalue of $A_{11} \in M_{n_1}$ and $A_{22} \in M_{n_2}$ are $\nu_1$ and $\nu_2$, respectively, then inspection of the Jordan canonical forms of $A_{11}$, $A_{22}$, and their direct sum reveals that the index of $\lambda$ as an eigenvalue of $A_{11} \oplus A_{22}$ is $\max\{\nu_1, \nu_2\}$.

The situation is different for block triangular matrices. If $A_{11}^{\nu_1} = 0$ and $A_{22}^{\nu_2} = 0$, then $A = \left[\begin{smallmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{smallmatrix}\right]$ is nilpotent. Compute

$$A^m = \begin{bmatrix} A_{11}^m & \sum_{k=0}^{m-1} A_{11}^k A_{12} A_{22}^{m-k-1} \\ 0 & A_{22}^m \end{bmatrix}, \quad m = 1, 2, \ldots$$

and let $m = \nu_1 + \nu_2$. Then $A_{11}^m = A_{11}^{\nu_1} A_{11}^{\nu_2} = 0$ and $A_{22}^m = A_{22}^{\nu_1} A_{22}^{\nu_2} = 0$. If $0 \le k \le \nu_1 - 1$, then $A_{22}^{m-k-1} = A_{22}^{\nu_1 - k - 1} A_{22}^{\nu_2} = 0$. If $\nu_1 \le k \le m-1$, then $A_{11}^k = A_{11}^{\nu_1} A_{11}^{k-\nu_1} = 0$. This shows that $A_{11}^k A_{12} A_{22}^{m-k-1} = 0$ for each $k = 0, 1, \ldots, m-1$. Therefore, $A^{\nu_1+\nu_2} = 0$, so the index of 0 as an eigenvalue of $A$ is at most $\nu_1 + \nu_2$. The example $A_{11} = J_2(0)$, $A_{22} = J_2(0)^T$, and $A_{12} = I_2$ shows that this upper bound on the index can be achieved: $A^4 = 0$ but $A^3 \ne 0$.

If not both of $A_{11}$ and $A_{22}$ are nilpotent and the indices of 0 as an eigenvalue of $A_{11}$ and $A_{22}$ are $\nu_1$ and $\nu_2$, respectively, Theorem 3.1.11 ensures that $A_{11}$ is similar to $B \oplus N_1$ and $A_{22}$ is similar to $N_2 \oplus C$, in which $B$ and $C$ are nonsingular, $N_1^{\nu_1} = 0$, and $N_2^{\nu_2} = 0$. Then $A = \left[\begin{smallmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{smallmatrix}\right]$ is similar to $\left[\begin{smallmatrix} B \oplus N_1 & * \\ 0 & N_2 \oplus C \end{smallmatrix}\right]$, which is similar (see the proof of Theorem 2.4.6.1) to

$$\left[\begin{smallmatrix} B & 0 & 0 & * \\ 0 & N_1 & * & 0 \\ & & N_2 & 0 \\ & & 0 & C \end{smallmatrix}\right], \text{ which is permutation similar to } \left[\begin{smallmatrix} B & * & 0 & 0 \\ 0 & C & 0 & 0 \\ & & N_1 & * \\ & & 0 & N_2 \end{smallmatrix}\right]$$

We have shown that the index of 0 as an eigenvalue of $\left[\begin{smallmatrix} N_1 & * \\ 0 & N_2 \end{smallmatrix}\right]$ (and hence of $A$ itself) is at most $\nu_1 + \nu_2$. An induction permits us to extend this conclusion to any block triangular matrix and any of its eigenvalues.

**Theorem 1 (3.2.10.1)** *Let $A = [A_{ij}]_{i,j=1}^{p} \in M_n$ be block upper triangular. If the index of $\lambda$ as an eigenvalue of each $A_{ii}$ is $\nu_i$, then the index of $\lambda$ as an eigenvalue of $A$ is at most $\nu_1 + \cdots + \nu_p$.*

p. 185, lines -1, -2. Replace the text of the last Exercise on the page with the following: "If $A \in M_n$, show that $AA^D A = A$ if and only if $\operatorname{rank} A = \operatorname{rank} A^2$."
p. 189, line 3 of **3.2.P20**. "...similar to $BA$. (b) Explain why..." should be "...similar to $BA$. Explain why... "
p. 191, line 22. "(3.2.P31) and (3.2.P32)" should be "Problem 3.2.P30".
p. 191, line 25. "3.2.P34" should be "3.2.P35".
p. 191, line -3 of Section 3.2. "these two problems" should be "this problem"
p. 195, Problem 3.3.P2: Replace the first sentence with the following: Suppose that $\lambda_1, \ldots, \lambda_d$ are the distinct eigenvalues of $A \in M_n$.
p. 198, Problem 3.3.P20. Replace the last line of the problem with "(d) If $A$ commutes with $B$, show that $A = B$."
p. 200, penultimate line of (3.3.P32): "field $\mathbf{F}$, those ..." should be "field $\mathbf{F} \subset \mathbf{C}$, those ..."

# Chapter 4

p. 238, first line of (4.2.P7): "subspace intersection lemma" should be "subspace intersection theorem"
p. 258, first line of (4.3.P20): "$\lambda \in \mathbf{C}$" should be "$\lambda \in \mathbf{R}$"
p. 270, line -2: delete both instances of "$= [h_{ij}]$".
p. 271, second line of the proof of Theorem 4.4.24: "block is similar" should be "block is unitarily similar".
p. 274, lines 4 and 5 of (4.4.P16): both instances of "congruence normal" should be "conjugate normal".
p. 280, line 10. Replace "(e.g., the Laplacian" with "(for example, the Laplacian"
p. 290, line 1. "4.5.24" should be "4.5.24.1"
p. 294. The display defining $\Gamma_k$ in the center of the page should be labelled "4.5.24.1" instead of "4.5.24"
p. 298, first line of **4.5.P25**. "4.5.24" should be "4.5.24.1"
p. 298, line 12. Replace "$I_+(B)$" with "$i_+(B)$"
p. 298, line 15. Delete "that is, $I(A) = I(B) + I(S)$.

# Chapter 5

p. 318, line 2 should read: is not derived from an inner product. You might consider $x = e_1$ and $y = e_2$.
p. 318, line 8. Replace "the polarization identity (5.1.10)." with "the parallelogram identity (5.1.9)."
p. 318, line 13 should read: preceding problem, show that (5.1.11) is not satisfied with $x = e_1$ and $y = e_2$.
p. 318. Replace line 16 with "$\alpha_0 = \langle x, y \rangle / \|y\|^2$, (b) $\|x - \alpha_0 y\|^2 = \|x\|^2 - |\langle x, y \rangle|^2 / \|y\|^2$, and (c) $x - \alpha_0 y$ is orthogonal to $y$."
p. 319. Delete the last three lines of **5.1.P13** and replace with "and show that the first line equals zero. (c) Is (5.1.13) correct for the norm $\|\cdot\|_\infty$ on $\mathbf{R}^3$, $x = [1\ 1\ -1]^T$, $y = [1\ -1\ 1]^T$, and $z = [-1\ 1\ 1]^T$?
p. 320, line 4. Replace "5.1.P10" with "5.1.P12"
p. 320, line 18. Replace "5.2.P7" with "5.1.P7"
p. 320, line 19. Replace "polarization" with "parallelogram"
p. 320, line 20. Delete "Show by example that it does not satisfy the parallelogram identity."
p. 386, line 3 of **5.8.P9**. $\kappa(H_n) = \rho(H_n)\rho(H_n)$ should be $\kappa(H_n) = \rho(H_n)\rho(H_n^{-1})$.
p. 386, line 24. Delete "a given matrix".

# Chapter 6

p. 400, display at top of page. The matrix $A_4$ should be

$$A_4 = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

# Chapter 7

p. 427. In lines 8-9, replace "(i.e., the elements" with "(the elements"
p. 427. Two lines below (7.0.4.2), replace "(i.e., the" with "(the"
p. 433. In line 12, replace "$x \in M_n$" with "$x \in \mathbf{C}^n$"
p. 441. At the very end of the first line in the proof of Corollary 7.2.9 insert "$(A^{1/2})^* A^{1/2} =$".
p. 441. In the last line of the proof of Corollary 7.2.9 replace "in (2.1.14)" with "in (b) and (e) of (2.1.14)".

p. 442. Replace all of part (c) of **7.2.P3** with: "(c) Show that $A^{-1} = R^{-1}R^{-T} = [\alpha_{ij}]$ is the symmetric tridiagonal matrix with $\alpha_{nn} = 1$, and with $\alpha_{ii} = 2$ and $\alpha_{i,i+1} = -1$ for $i = 1, \ldots, n-1$."

p. 451. In the last line of the proof of Corollary 7.3.6, replace "(7.6.7)" with "(7.3.7)".

p. 453. In line 9, replace "$Av_k = \sigma_k w_k$" with "$Aw_k = \sigma_k v_k$".

p. 453. In line 11, replace "$\begin{bmatrix} v \\ w \end{bmatrix}$" with "$\begin{bmatrix} v_k \\ w_k \end{bmatrix}$".

p. 454. In line 4 of **7.3.P12** replace "How can" with "If $A \in M_{m,n}$ and $B \in M_{m,p}$, how can".

p. 457. In line 1 of **7.3.P38** replace "$V = SBS^*$" with "$V = SWS^*$"

p. 457. In lines 20-21, "If $A, B \in M_n$, $A$ is normal and $B$ is Hermitian," should be "If $A, B \in M_n$ are normal,"

p. 484. In the second line of **7.5.P23**, $f(z) = (1 - z^3)^{-1}$ should be $f(z) = (1 - z)^{-1}$.

p. 503, **7.7.P30**. The three instances of $A^{-1} \circ A$ should all be $A^{-T} \circ A$, and "(7.7.17(c))" should be "(7.7.18)".

p. 514. In line 3 of **7.8.P12** replace "$(a_{nn} - x^* A_{11}^{-1} x) \det A \leq$" with "$(a_{nn} - x^* A_{11}^{-1} x) \det A_{11} \leq$".

p. 515. Insert the following after (7.8.P20):

**7.8.P21** Let $A = [a_{ij}], B = [b_{ij}] \in M_n$ be positive definite. When do the Oppenheim–Schur inequalities in (7.8.16) become equalities? The equality case in Hadamard's inequality (7.8.2) plays a key role in answering this question. (a) Explain why

$$\begin{aligned} \det(A \circ B) &\geq a_{11} \cdots a_{nn} \det B + (b_{11} \cdots b_{nn} - \det B) \det A \\ &\geq a_{11} \cdots a_{nn} \det B \geq \det(AB) \end{aligned}$$

(b) Show that $\det(A \circ B) = a_{11} \cdots a_{nn} \det B$ if and only if $B$ is diagonal, and $\det(A \circ B) = \det(AB)$ if and only if both $A$ and $B$ are diagonal. (c) If $n = 2$, show that the inequality (7.8.18) is always an equality. (d) If $n \geq 3$, it is known that (7.8.18) is an equality if and only if there is a permutation matrix $P \in M_n$ such that $PAP^T = A_1 \oplus A_2$ and $PBP^T = B_1 \oplus B_2$, in which $A_1, B_1 \in M_2$ and both $A_2, B_2 \in M_{n-2}$ are diagonal. Explain why this condition is sufficient for equality in (7.8.18).

*Further Reading.* For a proof that the condition stated in (7.8.P21(d)) is necessary if (7.8.18) is an equality, see A. Oppenheim, Inequalities connected with definite Hermitian forms, *J. London Math. Soc.* 5 (1930) 114-119. The equality cases for positive semidefinite $A$ and $B$ are discussed in X. D. Zhang and C. X. Ding, The equality case for the inequalities of Oppenheim and Schur for positive semi-definite matrices, *Czech. Math. J.* 59 (2009) 197-206.

# Chapter 8

p. 530, line 4. "$\rho(A) \geq a$" should be "$\rho(A) \geq \alpha$"
p. 534, line $-11$. Replace "*of* $A$. the vector $y$ in (8.4.4(d)) is its the *left*" by "*of* $A$; the vector $y$ in (8.4.4(d)) is its *left*"
p. 535, last line of (8.4.6(c)): The last character should be 1, not *1* (math mode, not italics).
p. 541, line 12."Corollary 8.4.8" should be "Corollary 8.4.7"

# Appendix B

p. 557, line -12: Replace "(e.g., a" with "(for example, a"
p. 557, line -11: Replace "(e.g., a closed" with "(for example, a closed"
p. 557, line -10: Replace "(e.g., the closed" with "(for example, the closed"

# References

p. 571, line -7: Replace "*Numerical Ranges of Normed Operators...*" with "*Numerical Ranges of Operators...*"
p. 572, line 1: "Campell, S. L." should be "Campbell, S. L."
p. 572, line 22: Replace "Fieldler, M. 1975" with "Fiedler, M. 1975"
p. 573, line -7 and -8: Delete both lines of the listing for *Advanced Topics in Linear Algebra*. This book is also listed in lines 7-8 of page 572.

# Notation

p. 575, line $-10$: "adjunct" should be "adjugate".

# Hints for Problems

p. 582, line 5: “**2.4.P11**(b) (2.4.P2)” should be “**2.4.P11**(c) (2.4.P2)”.
p. 582, line $-8$: delete the line containing **Section 2.4** and adjust the vertical spacing.
p. 583, line $-7$: In 2.5.P64, delete “(a)”.
p. 587. Insert the following between **3.3.P19** and **3.3.P22**: “**3.3.P20**. (d) What is column $n-1$ of $AB$?
p. 591, line 3: delete “(c)” at the beginning of the line.
p. 599, line 2 of **7.3.P7**: Replace “$= X^*Y^*Y =$” with “$= A^*Y^*Y =$”
p. 600, line 9 should be: “**7.3.P43** (7.3.P35); $\|AB\| = \|PUQV\| = \|PQ\| = \|QP\|$.”
p. 606, line 10 should be: “**8.7.P9** If $A$ has $n+1$ positive entries...”

# Index

p. 607. Left-hand column, middle of page. Replace the entry

$$\begin{bmatrix} 0 & A \\ \bar{A} & 0 \end{bmatrix}, 72, 307$$

with

$$\begin{bmatrix} 0 & A \\ \bar{A} & 0 \end{bmatrix}, 72, 277, 307$$

p. 607. Top entry in right-hand column. Replace “is real, *see* barAA” with “is real, *see* $A\bar{A} = \bar{A}A$”
p. 607. Delete line 14, right-hand column, “$AB$ *vs.* $BA$” and re-alphabetize its two sub-entries (eigenvalues and singular values) with the two sub-entries (Jordan blocks and similarity) under “$AB$ versus $BA$”. After reorganization we will have

$AB$ versus $BA$
eigenvalues, 65
Jordan blocks, 184
similarity,189
singular values, 455

p. 608. Line 24, left-hand column. Entry for “adjugate, rank”. Replace “1, 23” with “22, 80, 155”
p. 608. Line 6, right-hand column. Entry for “Ando, T.”. Replace “xvi” with “xiv”

p. 609. Line 23, left-hand column. Entry for "Barrett, W.". Replace "xii,xvi" with "xiv, xviii"
p. 610. Line -17, right-hand column should read

positive semidefinite matrix, 90, 441, 442

p. 615. In lines 20-22, right-hand column, delete two of the three instances of "*see* *congruence, diagonalizable".
p. 615. Right-hand column. Insert the following new line after the last line:

Ding, C.X., 515

p. 616. Line 19, left-hand column. Entry for Domanov, I. Replace "xvi" with "xiv"
p. 617. Line -3, right-hand column. Entry for Fill, J. Replace "xvi" with "xiv"
p. 618. Line 11, left-hand column. Entry for Fonseca, C. M. Replace "xvi" with "xiv"
p. 618. Line -14, right-hand column. Entry for Gerasimova, T. G.. Replace "xvi" with "xiv"
p. 619, left-hand column. In the index entry for "graph", insert a new secondary entry "directed, 399"
p. 619. Line -11, left-hand column. Entry for Guralnik, R. Replace "xvi" with "xiv"
p. 619. Line -15, right-hand column. Entry for Hawkins, T. Replace "xvi" with "xiv"
p. 619. Line -8, right-hand column. Entry for Herman, E. Replace "xvi" with "xiv"
p. 620. Line -22, right-hand column. Entry for Ikramov, Kh. Replace "xvi" with "xiv"
p. 621, left-hand column. After line 15, insert a new entry "indicator matrix, 399"
p. 621. Line -1, right-hand column. Entry for Ipsen, I. Replace "xvi" with "xiv"
p. 622. Line -20, left-hand column. Entry for Jespersen, D. C. Replace "xvi" with "xiv"
p. 622. Line -14, left-hand column. The entry should be "unitarily similar to a symmetric matrix, 217, 272"

p. 622. Line -14, right-hand column. Entry for Kosaki, H. Replace "xvi" with "xiv"

p. 623. Line -17, left-hand column. Entry for Li, C. K. Replace "xvi" with "xiv"

p. 623. Line -15, left-hand column. Entry for Li, Z. Replace "xvi" with "xiv"

p. 623. Line -10, left-hand column. Entry for Lim, T. C. Replace "xvi" with "xiv"

p. 623. Line 17, right-hand column. Entry for Lippert, R. A. Replace "xvi" with "xiv"

p. 624. Line 3, right-hand column. Entry for Mathias, R. Replace "xiv, xvi" with "xii, xiv"

p. 626. Line 25, right-hand column. Delete the entry for Merino, D.

p. 626. Line 26, right-hand column. Entry for Merino, D. I. Replace "279,311" with "xiv, 279, 311"

p. 626. Line -22, right-hand column should read

positive semidefinite, 435, 436, 442

p. 627. Line 17, right-hand column. Entry for Newmaier, A.. Replace "xvi" with "xiv"

p. 629. Left-hand column. Between the entries "principal submatrix" and "$QR$ factorization", insert a new entry:

product of two, 455

p. 629. Line 28, right-hand column. Entry for O'Meara, K. C. Replace "xiv, xvi" with "xii, xiv"

p. 629. Right-hand column, delete lines 37-38 (the entries for Oppenheim inequality and Oppenheim–Schur inequality) and replace them with

Oppenheim, A., 515
Oppenheim inequality
  case of equality, 515
  inequality, 509
Oppenheim–Schur inequality
  case of equality, 515
  inequality, 509

p. 632. line -1, left-hand column. Insert an entry "principal submatrix, definition, 17".

p. 634. Line -10, right-hand column. Entry for Rosenthal, P. Replace “xvi” with “xiv”

p. 636. Line 1, left-hand column. Entry for Sergeichuk, V. V. Replace “xvi” with “xiv”

p. 637. Line -20, right-hand column. Entry for So, W. Replace “xvi” with “xiv”

p. 639. Line 2, left-hand column. Entry for intersection lemma. “235, 238” should be “4, 235, 238”

p. 639. Delete line 3 (the entry for “intersection theorem”), left-hand column.

p. 639. Between lines -14 (normal) and -13 (involution) insert a new secondary item: “Jordan block unitarily similar to a, 271”

p. 642. Line 9, left-hand column. Entry for “to a complex symmetric matrix”. Replace “273” by “271, 273”

p. 642. Line -26, right-hand column. Entry for Vinsonhaler, C. Replace “xvi” with “xiv”

p. 643. Line 8, right-hand column. Entry for Woerdeman, H. Replace “xvi” with “xiv”

p. 643. Line -2, right-hand column. Entry for Zhang, F. Replace “xvi” with “xiv”

p. 643. Right-hand column. Insert a new, last entry “Zhang, X. D., 515”

# Errata contributors: Thank you!

Wayne Barrett, Harm Bart, Nelson Beebe, Dennis Bernstein, Rajendra Bhatia, Frank Hall, Sylvia Hobart, Zhongshan Li and his students, Momo Matsuda, Keiichi Morikuni and his students, and the University of Utah students in Math 5910.