A level Further Mathematics for AQA Student Book 1 (AS/Year 1)

Scheme of Work

This Scheme of Work has been compiled to help you teach the new AS/A Level Further Mathematics for AQA specifications.

Each page covers one chapter from Cambridge University Press’ brand new resource *A Level Further Mathematics for AQA Student Book 1 (AS/Year 1)*, for the specifications for first teaching in 2017, and has been divided into the following sections:

* **Specification references** and **chapter objectives** show you how the resources cover each point of the specifications, and what students should learn throughout the course of the chapter.
* A **teaching breakdown** provides a suggested number of teaching hours to cover the content of the chapter, based on 360 GLH for a full A Level, and 144 GLH for the AS content. You may need to adapt this guidance according to your timetable.
* An **introductory paragraph** gives you some context for the chapter, and explains its content in more detail.
* **Common misconceptions** provide guidance on areas where students typically make mistakes, and techniques you might use to address these.
* **Chapter links** provide reference to supplementary resources, including Underground Mathematics and NRICH. These also highlight synoptic links between different areas of the book, including pure and applied, as well as flags to show where AS content is re-visited in greater depth in the Year 2 book where relevant.

# 1 Complex numbers (p.1)

### **AQA specification references:** B1-B7, L1

## Chapter objectives

* Work with a new set of numbers called complex numbers.
* Perform arithmetic with complex numbers.
* Use the fact that complex numbers occur in conjugate pairs.
* Interpret complex numbers geometrically.
* Interpret arithmetic with complex numbers as geometric transformations.
* Represent equations and inequalities with complex numbers graphically.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Definition and basic arithmetic of i | 1 |
| 2. Division and complex conjugates | 1 |
| 3. Geometric representation | 1 |
| 4. Locus in the complex plane | 1 |
| 6. Operations in modulus–argument form | 1 |

This chapter introduces students to the basics of working with complex numbers, by introducing the imaginary number i. Once the new terminology of real and imaginary part, conjugate, modulus and argument of a complex number are introduced, students will see a geometric application of complex numbers in the plane, describing loci.

## Common misconceptions

Students need to be clear what the notation and terminology associated with complex numbers means, particularly that the Im(z) for *x* + *iy* only refers to *y* and not *iy*. Fluency with writing complex numbers in the form *a* + *bi* is important, and Exercise 1B question 2 will consolidate this skill.

Students can often become overwhelmed when working with complex numbers and should remind themselves what type of variable they are dealing with in a question. Worked Examples 1.6 and 1.7 emphasise when it is a good idea to define *z* = *x* + *iy*, with *x* and *y* real, and when it is sensible to work with *z* as a complex number. Exercise 1B questions 4, 12 and 13 can be used to help draw out any misconceptions.

A clear understanding of what form a question wants the complex number in (modulus-argument form for a particular argument domain, Cartesian form) is important and students should be encouraged to reread a question to make sure they are answering it. For example Exercise 1E questions 1, 2 and 3 ask for the complex numbers in specific forms.

One of the most useful forms of a complex number is *r*(cos θ + *i* sinθ ), but students sometimes forget that this form is defined only with the addition sign. Exercise 1E question 6 will consolidate their understanding of this.

Students often struggle using complex numbers to describe loci in the plane. Encourage them to draw diagrams and identify what geometric object they need by studying Worked

# 1 Complex numbers (cont…)

Examples 1.15 to 1.19 and use the drill questions from Exercise 1F to help build confidence before tackling questions that require more than one or two steps (Exercise 1F remaining questions).

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **NRICH** | Complex Countdown: <https://nrich.maths.org/6441> |
| **Links with other topics** | Section 2: Fast Forward to Chapter 2  Section 3: Rewind to A Level Mathematics Student Book 2, Chapter 7  Section 3: Rewind to A Level Mathematics Student Book 2, Chapter 8  Section 5: Rewind to A Level Mathematics Student Book 2, Chapter 8 |

# 2 Roots of polynomials (p.31)

### AQA specification references: D1-2

## Chapter objectives

## Factorise polynomials and solve equations which may have complex roots.

## Link the roots of a polynomial and its coefficients.

## Use substitutions to solve more complicated equations.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Factorising polynomials | 1 |
| 2. Complex solutions to polynomial equations |
| 3 Roots and coefficients | 2 |
| 4. Finding an equation with given roots | 1 |
| 5. Transforming equations | 1 |

This chapter builds on students’ knowledge of the factor theorem and the tools from complex numbers. Students will fully factorise polynomials, find roots of them and make connections between the structure of the roots and the coefficients of the polynomial. In addition to this, students will make connections between transformations of functions to change one polynomial into another and the effect this transformation has on the roots.

## Common misconceptions

Work It Out 2.1 draws out the relationship between conjugates and roots of polynomials with real coefficients and Exercise 2B questions 4, 5, 6 and 7 will help to consolidate this and draw out any misconceptions students may have.

Students often miss the alternating sign in the structure of the roots and the division by the coefficient of the highest power in the polynomial when working with roots and coefficients. The drill questions in Exercise 2C and 2D will help to consolidate this.

Students don’t always make the connections between transformations of functions and finding a new polynomial based on a linear transformation of the roots of another polynomial. Work It Out 2.2 and Exercise 2F questions 2, 3 and 7 work specifically on linear transformations. Once students make these connections, they can explore other compositions of functions to transform one root set into another. Exercise 2F question 1 explores these ideas and questions 8 and 9 apply them.

# 2 Roots of polynomials (cont…)

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **Underground Mathematics** | Can we show *x*4 −*px*3 −6*x*2 + *px* + 1 = 0 always has four real roots? <https://undergroundmathematics.org/polynomials/r9189>  Two values of *x* that differ by 5 satisfy *x*2 − 12*x* + *k* = 0. What is *k*? <https://undergroundmathematics.org/polynomials/r5949> |

# 3 The ellipse, hyperbola and parabola (p.54)

### AQA specification references: D9-15

## Chapter objectives

* Recognise and work with Cartesian equations of ellipses, hyperbolas and parabolas.
* Solve problems involving intersections of lines with those of curves, and find equations of tangents.
* Recognise the effects of transformations (translations, stretches and reflections) on the equations of those curves.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Introducing the ellipse, hyperbola and parabola | 2 |
| 2. Solving problems with ellipses, hyperbolas and parabolas | 2 |
| 3. Transformations of curves | 1 |

In this chapter, students will extend their geometric knowledge of the parabola and discover that this curve is just one of a family of curves that come from taking slices through a cone: the conic sections. The conic sections are also the possible graphs for a degree two polynomial in two variables, *ax*2+*by*2+*cxy*+*dx*+*ey*+*f*=0, where *a*, *b*, *c*, *d*, *e* and *f* are constants (*a* and *b* non-zero).

For an alternative introduction to conic sections and their use in real life, see Underground Mathematic <https://undergroundmathematics.org/circles/conic-sections-in-real-life>.

The chapter introduces each conic section and some special cases are introduced, with the equation that defines each of them. The terms focus and directrix are not used. The second section looks at intersections, tangents and normal for each of the conic sections and finally the third section looks at transforming each curve.

## Common misconceptions

Students often find it challenging to remember how each of the conic sections are defined and what features are created in their graphs given their equation. To help with this, students could use graphing software (such as Desmos) to explore the equations given in Exercise 3A, question 1, 3 and 4, before using it to reproduce the graphs given in question 2 and 5.

Exercise 3B allows students to solve at intersections, tangents and normal problems, without necessarily differentiating to find the gradient function. This exercise should help develop students’ geometry-to-algebra and vice versa tool box. This is where a geometric property can be proved using algebra or an algebraic property gives rise to a geometric consequence for the graphs.

# 3 The ellipse, hyperbola and parabola (cont…)

The final section extends the ideas of graph transformations from stage one mathematics and students must have a solid idea of what the different transformations to a curve will do geometrically and algebraically. The drill questions in Exercise 3C question 2 will help to fortify these relationships.

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **Underground Mathematics** | Elliptical crossings: <https://undergroundmathematics.org/circles/elliptical-crossings>  Review questions:  <https://undergroundmathematics.org/calculus-of-powers/r8286>  <https://undergroundmathematics.org/chain-rule/r5678>  <https://undergroundmathematics.org/chain-rule/r6710> |
| **Links with other topics** | Section 2: Rewind to A Level Mathematics Student Book 1, Chapter 3  Section 2: Rewind to A Level Mathematics Student Book 1, Chapter 6  Section 3: Rewind to A Level Mathematics Student Book 1, Chapter 5  Section 3: Rewind to A Level Mathematics Student Book 1, Chapter 3 |

# 4 Rational functions and inequalities (p.80)

### AQA specification references: D9-15

## Chapter objectives

* Solve cubic and quartic inequalities.
* Sketch graphs of the form
* Solve inequalities of the form
* Sketch graphs of the form

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Cubic and quartic inequalities | 1 |
| 2. Functions of the form | 1 |
| 3. Functions of the form |
| 4. Oblique asymptotes (**A Level only**) | 1 |

In this chapter students will extend their curve sketching skills to involve rational functions where the degrees of the polynomials involved are at most two. This will involve a careful analysis of the rational function, particularly looking at the roots of the numerator and denominator polynomials, and the long-term behaviour of the function.

A Level students can explore oblique asymptotes before examining reciprocal and modulus compositions of functions, or this can be left until stage 2 and used as a chance to revisit the concepts in this chapter.

## Common misconceptions

Sketching graphs is an important mathematical skill that can sometimes be ignored by students if they have access to graphing software. Students should be encouraged to sketch the rational functions they are looking for first as this will help to consolidate how the algebraic criteria give rise to certain geometric features.

Exercise 4A question 4 is a good question to draw out any misconceptions of sketching polynomial functions, and Exercise 4B questions 6 and 7 can help draw out key ideas for solving inequalities and understanding functions of the form . Students can sometimes get overwhelmed when looking at a rational function of the form but Exercise 4C, questions 1 and 2 has enough variety that a student can build some confidence.

Students can sometimes find it difficult to combine functions using the reciprocal and understanding where the features of the original graph go in the new graph. Exercise 4E question 3 gets the student to focus on the features of the graph, rather than the equation, to sketch the reciprocal function.

# 4 Rational functions and inequalities (cont…)

Working with the modulus functions causes problems for students when they do not sketch the original graph they are looking for. Worked Examples 4.9 to 4.11 start with sketching the graphs first; this will help minimise incorrect intersections found and sets of values for *x* which make inequalities using the modulus function true.

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **Underground Mathematics** | Function builder II: <https://undergroundmathematics.org/polynomials/function-builder-ii>  Can you find… asymptote edition: <https://undergroundmathematics.org/polynomials/can-you-find-asymptote-edition> |

# 5 Hyperbolic functions (p. 97)

### AQA specification references: H1, H4

## Chapter objectives

* Define the hyperbolic sinh *x*, cosh *x* and tanh *x*.
* Recognise and use the graphs, range and domain of the hyperbolic functions.
* Work with identities involving hyperbolic functions.
* Write the inverse hyperbolic functions in terms of logarithms.
* Solve equations involving hyperbolic functions.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Defining hyperbolic functions | 1 |
| 2. Hyperbolic identities | 1 |
| 3. Solving harder hyperbolic equations | 1 |

This chapter introduces the hyperbolic functions through the similarity of parameterising the circle using the circular functions sine and cosine, with parameterising a hyperbola. However, they have a more useful definition based on the average of the sum (and the difference) of the exponential functions ex and e-x. This creates an odd function (sinh *x*) and an even function (cosh *x*) based on a combination of the exponential function.

The hyperbolic functions have many uses in mathematics and the graph of cosh *x*, the catenary, has many applications in mechanics. This chapter focuses on the algebraic side of these functions by examining identities and solving equations involving them.

## Common misconceptions

Students sometimes find it difficult when solving hyperbolic equations to know when they need to leave their answers as a logarithm, in terms of arsinh/arcosh/artanh or as a calculated value. Exercise 5C gives the opportunity to read the question carefully and give their answers in the correct form.

Proof 4 outlines how to find the inverse function for cosh *x*, with the restricted domain to define arcosh *x* as a function. Students can struggle with remembering that for an inverse function we need the function to be one-to-one. However, when we are solving equations in hyperbolic functions we will need to find all solutions and Worked Example 5.2 outlines why we need to include the solution excluded from Proof 5. Mixed Practice 5 question 8 can help consolidate this idea.

It is also worthwhile students understanding why and making the connection to the symmetry of the cosh x graph.

# 5 Hyperbolic functions (cont…)

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **Underground Mathematics** | From parabolas to catenaries: <https://undergroundmathematics.org/hyperbolic-functions/from-parabolas-to-catenaries>  Building catenaries: <https://undergroundmathematics.org/hyperbolic-functions/building-catenaries> |
| **Links with other topics** | Section 1: Rewind to A Level Mathematics Student Book 2, Chapter 2  Section 2: Rewind to A Level Mathematics Student Book 1, Chapter 10  Section 2: Fast Forward to Section 3 |

# 6 Polar coordinates (p.111)

### AQA specification references: G1-2

## Chapter objectives

* Use polar coordinates to represent curves.
* Establish various properties of those curves.
* Convert between polar and Cartesian equations of a curve.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Curves in polar coordinates | 1 |
| 2. Some features of polar curves | 1 |
| 3. Changing between polar and Cartesian coordinates | 1 |

This chapter introduces a new coordinate system based on distance from a pole and the angle above a fixed line containing the pole, the initial line. Students are introduced to the new way of defining curves based on this coordinate system and look at connections between the polar form and the Cartesian form for certain curves.

## Common misconceptions

Students may find it difficult initially to visualise what they need to do to plot a curve defined by a polar equation. As Worked Example 6.2 and Exercise 6A question 3 encourages, a table of values is very useful and should normally be the first thing a student does when trying to understand a certain curve from its polar equation.

In Section 2 links are made to differentiation to find the maximum and minimum values for *r*. Some students may confuse this polar coordinate analysis with the Cartesian one and assume that this corresponds with turning points. Exercise 6B questions 1, 2 and 3 should help draw out any of these misconceptions about differentiating *r* = *f*(θ) and understanding graphical features of the polar curve based on the polar equation.

Finally, students may need to be reminded that in the Cartesian coordinate system (*r*cos θ, *r*sin θ) are the Cartesian coordinates of a general point of a curve and they can use this to help find the Cartesian equation given a polar one and vice versa. Exercise 6C questions 3 and 4 give opportunities to consolidate this.

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **NRICH** | Polar Bearings: <https://nrich.maths.org/8055>  Polar Flower: <https://nrich.maths.org/2679> |
| **Links with other topics** | Section 3: Rewind to Chapter 1, Section 3 |

# 7 Matrices (p.136)

### AQA specification references: C1, C2, C4, C5, C6

## Chapter objectives

* Add, subtract and perform scalar multiplication with conformable matrices.
* Use and interpret zero and identity matrices.
* Calculate the determinant of a 2 x 2 matrix.
* Find and interpret the inverse of a 2 x 2 matrix.
* Use matrices to solve two simultaneous equations.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Addition, subtraction and scalar multiplication | 1 |
| 2. Matrix multiplication |
| 3. Determinants and inverses of 2 x 2 matrices | 2 |
| 4. Linear simultaneous equations | 1 |

This chapter introduces the topic of matrices and the basic tools needed to deal with 2 x 2 matrices. Connections to vectors from the A Level Mathematics course are extended to this new mathematical object.

## Common misconceptions

Key point 7.1 emphasises that an m x n rectangular matrix has m rows and n columns (some students confuse these). There are several new mathematical words and phrases that students need to become familiar with: square matrix, zero matrix (null matrix), identity matrix, conformable, dimension of a matrix, determinant, inverse of a matrix, singular and non-singular. Matrix algebra will be the first algebraic system students have come across where the order of multiplication matters. Exercise 7B questions 10–12 will help cement this property.

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **NRICH** | Matrix Countdown: <https://nrich.maths.org/7373> |
| **Links with other topics** | Section 1: Rewind to A Level Mathematics Student Book 2, Chapter 16 |

# 8 Matrix transformations (p.161)

### AQA specification references: C3

## Chapter objectives

* Interpret matrices as linear transformations in two and three dimensions.
* Find a matrix representing a combined transformation.
* Find invariant points and invariants lines of a linear transformation.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Matrices as linear transformations | 1 |
| 2. Further transformations in 2D | 1 |
| 3. Invariant points and invariant lines | 1 |
| 4. Transformations in 3D | 1 |

Matrices have many uses, from the arrangement of large data sets to describing transformations in two and three dimensions. In this chapter, students will apply the tools they learnt in Chapter 6 in two dimensions to describing transformations in the plane (including shears) and understanding that some transformations leave points invariant, whilst other transformations leave invariant lines (not necessarily the points). In three dimensions, transformations based on rotations and reflections, as well as stretches, are studied.

## Common misconceptions

Students often find it difficult to describe transformations that a matrix is performing. Exercise 8A and 8B, questions 1, 2 and 3 will help to students develop the tools they need to tackle these questions.

Working in 3D is often quite challenging for students as they cannot draw a ‘unit square’ to help them understand what a certain matrix is doing. Exercise 8D has a good variety of questions on working in 3D transformations, and students might find it helpful to revisit Worked Examples 8.14 to 8.17.

Understanding that an invariant line does not mean that the points are invariant can be difficult to come to terms with. Understanding Worked Example 8.11 and following this up with Exercise 8C question 5 will give good opportunities to draw out any misconceptions.

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **NRICH** | Square Pair: <https://nrich.maths.org/6875>  Matrix Meaning: <https://nrich.maths.org/6876> |

# 9 Further applications of vectors (p.191)

### AQA specification references: F1-6

## Chapter objectives

* Write an equation of a straight line in three dimensions, both using vectors and coordinates.
* Find the intersection point of two lines.
* Calculate an angle between two vectors or two straight lines (using the scalar product).
* Decide whether two lines are parallel or perpendicular.
* Solve problems involving distanced between points and lines.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Vector equation of the line | 1 |
| 2. Cartesian equation of the line | 1 |
| 3. Intersection of lines | 1 |
| 4. Angles and the scalar product | 1 |

This chapter extends students’ knowledge of vectors to include the vector equation of lines in two and three dimensions. Connections are made between Cartesian equations for lines in two and three dimensions, before looking at how lines can be arranged in three dimensions. Students will develop the skills to find angles between (position and direction) vectors and apply this to find the shortest distance between a line and a point.

## Common misconceptions

Students will need to draw diagrams marking the relative position of points and direction of vectors. This will help reduce errors in choosing direction vectors to work with and understanding which angle is calculated in a scalar or vector product. Exercise 9C question 6 is a good example of a question that benefits from a sketch of what the lines look like to provide a visual aid to how to solve the problem, as well as Mixed practice 9 question 11 where the perpendicular distance needs to be calculated.

Students sometimes forget what is required to find the angle between two lines and Work It Out 9.2 will help consolidate this using the scalar product. Furthermore, students occasionally forget that the output from the vector product is a vector (particularly if the output is the zero vector).

Students should be reminded often that there is an additional way to arrange lines in 3D, (skew lines) The Underground Mathematics resource may help address any issues with this (using the 3D view of GeoGebra to help).

# 9 Further vectors (cont…)

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **Underground Mathematics** | Three Lines: <https://undergroundmathematics.org/vector-geometry/three-lines>  Lots of vector lines!**:** <https://undergroundmathematics.org/vector-geometry/lots-of-vector-lines> |
| **Links with other topics** | Section 1: Rewind to A Level Mathematics Student Book 1, Chapter 15  Section 2: Rewind to A Level Mathematics Student Book 2, Chapter 12 |

# 10 Further calculus (p.222)

### AQA specification references: E2-3

## Chapter objectives

* Find the volume of a shape formed by rotating a curve around the *x*-axis or the *y*-axis.
* Find the mean value of a function.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Volumes of revolution | 1 |
| 2. Mean value of a function | 1 |

This chapter extends the work on calculus covered in AS/A Level Mathematics Student Book 1 in two directions. The first is calculating the volume of revolution generated by rotating a curve *y* = f(*x*) between *x* = *a* and *x* = *b* 360 degrees around the x-axis. The second looks at finding the average value of a function over a given domain.

## Common misconceptions

Students need to make sure they are confident at using the formula to find the volume and with integration in general. Typically, students can forget to square *y* or multiply by π.

It is a good idea to get students to draw a sketch of what they are looking for, particularly if the volume they are looking for is formed using two curves, such as in Worked Example 10.2. Exercise 10A question 8 will help consolidate this idea.

When calculating the average value for a function, students must be clear what domain they are working over to calculate this value. Exercise 10B questions 1, 3, 5 and 8 should draw out any misconceptions students have when calculating the average value for a function in each domain.

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **Underground Mathematics**  **NRICH** | What volume is generated when *y* = *ax* − *x*2 is rotated about the *x*-axis?: <https://undergroundmathematics.org/calculus-of-powers/r5007>  Average speed: <https://undergroundmathematics.org/introducing-calculus/average-speed>  Weekly Challenges: <https://nrich.maths.org/6497> |

# 11 Series (p.234)

### AQA specification references: D3, D4, D5

## Chapter objectives

* Use given results for the sums of integers, squares and cubes to find expressions for the sum of other series.
* Use a technique called the method of differences to find expressions for the sum of *n* terms of a series.
* Use given results for infinite series expansions of functions such as sin *x* and cos *x*, to find series for more complicated functions.
* Understand for which values of *x* these infinite series are valid.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. Sigma notation | 1 |
| 2. Using standard formulae |
| 3. Method of differences | 1 |
| 4. Maclaurin series | 1 |

This chapter builds on the idea of summing a sequence and finding a closed form for the summation. Formal summation notation is introduced and the standard formulae for the sums of integers, squares and cubes are given. Students use these series throughout the chapter to find new sums based on these series.

The method of difference is a powerful tool to find closed formulas for summations and is introduced in Section 3.

In the final section students are introduced to the idea of an infinite polynomial that can be used for some common functions. The series defined here are used to combine with each other to form new series.

## Common misconceptions

It can take students some time to be comfortable with summation notation if they have not been introduced to it before. Exercise 11A will draw out any misconceptions about this notation before proceeding with using it in the chapter.

The method of differences can sometimes cause students problems if they try to rush and find the terms that remain without showing enough of the series. Exercise 11C has enough variety that students will need to make sure they have written out enough terms to understand what is going on for each summation.

Since there is no motivation for why you can define an infinite polynomial for a function, students will find it difficult to remember to check the domain that each Maclaurin series is valid for, especially the trigonometric functions that are only defined for radians. Using some graphing software to plot different truncated versions of each series could be used to give some justification as to why these infinite series can be used for each function.

# 11 Series (cont…)

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **External resources:**  **Underground Mathematics**  **NRICH** | Power Series: Designing a polynomial: <https://undergroundmathematics.org/power-series/designing-a-polynomial>  Telescoping Series: <https://nrich.maths.org/267> |
| **Chapter links** | Section 2: Fast Forward to Chapter 12, Section 2  Section 4: Rewind to A Level Mathematics Student Book 1, Chapter 6 |

# 12 Proof by induction (p.253)

### AQA specification references: D3, D4, D5

## Chapter objectives

* Use the principle of induction to prove that patterns continue forever.
* Apply proof by induction to series.
* Apply proof by induction to matrices.
* Apply proof by induction to divisibility problems.
* Apply proof by induction to inequalities.
* Adapt the method to solve problems in a range of other contexts.

## Teaching breakdown

|  |  |
| --- | --- |
| **Chapter section** | **Number of lessons** |
| 1. The principle of induction  2. Induction and series | 1 |
| 3. Induction and matrices | 1 |
| 4. Induction and divisibility | 1 |
| 5. Induction and inequalities | 1 |

Students have met the idea of algebraic proof at GCSE, where they would have looked at simple examples involving stringing together a series of logical statements. They will also be able to disprove a statement by finding a counter example. Some students may also be familiar (from A Level Mathematics) with proving using a contradiction. However, some simple mathematical statements are easy to understand, but attempting to prove them by stringing together a series of logical statements requires demonstrating that a relationship holds for all positive integer values. This is where mathematical induction comes in.

This chapter explores the concept of mathematical induction through four examples: series, powers of matrices, number theoretic results relating to divisibility and proving that inequalities hold for a range of positive integers. Students will need to be very familiar with the contents of Chapters 7 and 11 of this book before starting this chapter.

## Common misconceptions

Students often find the formal layout and language associated with mathematical induction challenging. Key Point 12.1 outlines the process, but seeing this formally set out in Worked Examples 12.1 to 12.6 is very valuable for a student. The Common error box after Worked Example 12.1 is something that students do all too often and should be highlighted at every opportunity. Students can then apply their formal method to the green and blue questions in the Exercises in the chapter. The summary grid given in the Checklist of learning and understand can be used to go back over the questions students have answered and identify where and how they have used the inductive step in each question.

# 12 Proof by induction (cont…)

## Chapter links

|  |  |
| --- | --- |
| **Source** | **Description** |
| **Underground Mathematics**  **NRICH** | Triominoes: <https://undergroundmathematics.org/divisibility-and-induction/triominoes/>  Review question: <https://undergroundmathematics.org/divisibility-and-induction/r5634>  OK! Now Prove It: <http://nrich.maths.org/297> |
| **Chapter links** | Section 2: Rewind to Chapter 11, Section 1  Section 3: Rewind to Chapter 7, Section 2 |