Worked solutions

AS Level practice paper

1 Let X represent the number of beetles in a 2 m^2 area, $X \sim \text{Po}(2 \times 6.2 = 12.4)$.

$$\text{P}(X < 10) = \sum_{n=0}^{9} \frac{12.4^n \text{e}^{-12.4}}{n!} = 0.209 \text{ (3 s.f.) (Answer C)}$$

[1 mark]

2 Using the fact that the summed probabilities must be equal to 1:

$$k(1+4+9) = 1 \Rightarrow k = \frac{1}{14}$$

$$\text{E}(X) = \sum x\,\text{P}(X = x) = \sum kx^3 = \frac{18}{7} = 2.57 \text{ (3 s.f.) (Answer C)}$$

[1 mark]

3 **a** Using the fact that the summed probabilities must be equal to 1:

$$0.9 + k = 1 \Rightarrow k = 0.1$$

[1 mark]

b $\text{P}(1 \le Y < 3) = \text{P}(X = 1) + \text{P}(X = 2) = 0.3 + 0.5 = 0.8$

[1 mark]

c $\text{E}(Y) = 0.3 + 1 + 0.3 = 1.6$

$$\sigma(Y) = \sqrt{\text{E}(Y^2) - (\text{E}(Y))^2} = \sqrt{0.3 + 2 + 0.9 - 1.6^2} = 0.8$$

[4 marks]

4 **a** The total number of buses arriving in an hour. $T \sim \text{Po}(3 + 4.5 = 7.5)$

[2 marks]

b
$$\text{P}(T \le 3) = \text{e}^{-7.5} + 7.5\text{e}^{-7.5} + \frac{7.5^2\text{e}^{-7.5}}{2} + \frac{7.5^3\text{e}^{-7.5}}{6}$$
$$= 0.0591 \text{ (3 s.f.)}$$

[2 marks]

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c For example: Both are dependent on traffic.

[1 mark]

d Mean $= 7.45$ (3 s.f.)

Variance $= 11.5$ (3 s.f.)

Mean $\neq$ variance. The Poisson distribution is not feasible.

[4 marks]

5 **a** Using the fact that $\int_0^3 \mathrm{f}(x)\,\mathrm{d}x = 1$:

$$\int_0^3 kx^2\,\mathrm{d}x = \frac{27k}{3} = 1 \Rightarrow k = \frac{1}{9}$$

[3 marks]

b For the median, m:

$$\int_0^m kx^2\,\mathrm{d}x = \frac{m^3}{27} = 0.5$$

$$\Rightarrow m = 2.38 \text{ (3 s.f.)}$$

$$\mathrm{E}(X) = \int_0^3 x\,\mathrm{f}(x)\,\mathrm{d}x = \int_0^3 kx^3\,\mathrm{d}x = \frac{81}{36} = 2.25 < 2.38$$

[5 marks]

c $$\mathrm{Var}(1-X) = \mathrm{Var}(X) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2 = \int_0^3 kx^4\,\mathrm{d}x - (\mathrm{E}(X))^2$$

$$= \frac{243}{45} - 2.25^2 = 0.338 \text{ (3 s.f.)}$$

[3 marks]

6 H_0: school year and hours spent watching TV are independent.

H_1: school year and hours spent watching TV are dependent.

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Calculating the expected frequencies:

		School year		
		7–9	**10–11**	**12–13**
Hours	**$\leqslant 5$**	18.814	14.237	11.949
	6–10	17.977	13.605	11.418
	11–20	25.085	18.983	15.932
	>20	12.124	9.175	7.701

$v = (4-1)(3-1) = 6$

Calculating the χ^2 value and comparing this to the critical value from the table at the 5% significance level with $v = 6$:

$$\chi^2_{\text{calc}} = \sum \frac{(O_i - E_i)^2}{E_i} = 4.22 \text{ (3 s.f.)} < 12.592$$

Do not reject H_0. There is no significant evidence of an association.

[5 marks]

7 **a** Let the random variable X be the number of leaks in a 2 km stretch of pipe.

$X \sim \text{Po}(\lambda)$, where $\lambda = 9$.

H_0: $\lambda = 9$, H_1: $\lambda < 9$.

In a one-tailed test at the 5% significance level, the sample value $x = 3$ will lie in the critical region if $P(x \leqslant 3) < 5\%$.

$$P(X \leqslant 3) = \sum_{n=0}^{3} \frac{9^n e^{-9}}{n!} = 172 \times e^{-9} = 2.12\% \text{ (3 s.f.)}$$

$P(X \leqslant 3) < 5\%$, so reject H_0, as there is evidence to support the contractor's claim that the number of leaks has been reduced.

[5 marks]

b P(type 1 error) = significance level of test = 0.0212 (3 s.f.) as calculated in part **a**.

[2 marks]

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Worked solutions

A Level practice paper

1 Let the continuous random variable X be the average time in minutes between emissions of a beta particle, then $X \sim \exp(0.5)$, where $f(x) = 0.5e^{-0.5x}$.

$$P(1 < X < 2) = \int_1^2 f(x)\,dx = \int_1^2 0.5e^{-0.5x}\,dx = \left[-e^{-0.5x}\right]_1^2 = -e^{-1} + e^{-0.5}$$

$$= 0.239 \text{ (3 s.f.) (Answer B)}$$

[1 mark]

2 $P(Y \le 5) = 0.714$ (3 s.f.)

$P(Y \ge 5) = 0.524$ (3 s.f.)

So the median is 5. (Answer D)

[1 mark]

3 **a** $E(X) = \sum x P(X = x) = 0.5 + \frac{2}{3} + \frac{3}{6} = \frac{5}{3}$

[1 mark]

b $\text{Var}(X) = E(X^2) - (E(X))^2 = 0.5 + \frac{4}{3} + \frac{9}{6} - \left(\frac{5}{3}\right)^2 = \frac{5}{9}$

[2 marks]

c $\text{Var}(3X + 2) = 9\text{Var}(X) = 5$

[1 mark]

4 **a** H_0: Gender and lessons are independent,

H_1: Gender and lessons are not independent.

[2 marks]

b $\nu = (2-1)(2-1) = 1$

[1 mark]

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c Calculating the expected frequencies:

	Music lessons	No music lessons
Female	18.255	25.745
Male	20.745	29.255

Using Yates' correction and comparing the χ^2 value to the critical value from the table at the 5% significance level:

$$\chi^2_{\text{Yates}} = \sum \frac{\left(|O_i - E_i| - 0.5\right)^2}{E_i} = 4.84 \text{ (3 s.f.)} > 3.841$$

Reject H_0. There is significant evidence that gender and lessons are not independent.

[5 marks]

5 **a** Using the fact that $\int_1^2 \text{f}(x)\,\text{d}x = 1$:

$$\int_1^2 \frac{k}{x}\,\text{d}x = k\ln 2 = 1 \Rightarrow k = \frac{1}{\ln 2}$$

[3 marks]

b $$\text{E}(X) = \int_1^2 x\,\text{f}(x)\,\text{d}x = \int_1^2 k\,\text{d}x = \frac{1}{\ln 2} = 1.44 \text{ (3 s.f.)}$$

$$\text{Var}(X) = \int_1^2 x^2\,\text{f}(x)\,\text{d}x - (\text{E}(X))^2$$

$$= \int_1^2 xk\,\text{d}x - (\text{E}(X))^2 = \frac{3}{2\ln 2} - \frac{1}{(\ln 2)^2} = 0.0827 \text{ (3 s.f.)}$$

[4 marks]

c $$\text{Var}\left(\frac{2X+1}{X}\right) = \text{Var}(2) + \text{Var}\left(\frac{1}{X}\right) = \sigma^2\left(\frac{1}{X}\right)$$

$$= \int_1^2 \frac{1}{x^2}\text{f}(x)\,\text{d}x - \left(\int_1^2 \frac{1}{x}\text{f}(x)\,\text{d}x\right)^2 = \int_1^2 \frac{k}{x^3}\,\text{d}x - \left(\int_1^2 \frac{k}{x^2}\,\text{d}x\right)^2$$

$$= \frac{3}{8\ln 2} - \frac{1}{4(\ln 2)^2} = 0.0207 \text{ (3 s.f.)}$$

$$\Rightarrow \sigma\left(\frac{2X+1}{X}\right) = 0.144 \text{ (3 s.f.)}$$

[4 marks]

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6 **a** $s^2 = \frac{n}{n-1}\left(\frac{1}{n}\sum t^2 - \left(\frac{\sum t}{n}\right)^2\right) = 3.48$ (2 d.p.)

[2 marks]

b H_0: $\mu = 35$, H_1: $\mu < 35$

[2 marks]

c $\nu = 12 - 1 = 11$

[1 mark]

d $\overline{T} = 33.25$

$$T = \frac{\overline{T} - 35}{\frac{s}{\sqrt{12}}} = -3.25 \text{ (3 s.f.)}$$

Comparing your t-value to the critical value from the table at the 5% significance level:

$|T| = 3.25$ (3 s.f.) > 1.796

Reject H_0. There is significant evidence that the new technique improved the mean 50 m time.

[4 marks]

e Assume that the swimming times are drawn from a normal distribution. Any reasonable comment, for example: OK because swimming times will be mainly clustered around the average with few people at extremes; or not OK because the swimming club is likely to have people at the upper tail of the distribution.

[2 marks]

7 **a** Let X represent the number of phone calls received in a 3-hour period, $X \sim \text{Po}(4 \times 3 = 12)$.

$$P(X \le 6) = \sum_{n=0}^{6} \frac{12^n e^{-12}}{n!} = 0.0458 \text{ (3 s.f.)}$$

[3 marks]

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b The mean number of calls is 3.7 per hour, so $X \sim \text{Po}(3 \times 3.7 = 11.1)$.

$$P(X > 6) = 1 - P(X \le 6) = 1 - \sum_{n=0}^{6} \frac{11.1^n e^{-11.1}}{n!} = 0.925 \text{ (3 s.f.)}$$

[3 marks]

8 a Rectangular, between -0.5 and 0.5.

[2 marks]

b The PDF for E is given by $f(e) = \dfrac{1}{0.5-(-0.5)}$

i.e. $f(e) = 1$ for $-0.5 < e < 0.5$.

$P(|E| < 0.4) = P(-0.4 < E < 0.4)$, which is the area of a 0.8 by 1 rectangle,

so $P(|E| < 0.4) = 0.8$.

Tip

Alternatively, $P(|E| < 0.4) = \int_{-0.4}^{0.4} f(e)\,de = \int_{-0.4}^{0.4} 1\,de = [e]_{-0.4}^{0.4} = 0.8$

[1 mark]

c For each independent observation:

$P(|E| < x) = \int_{-x}^{x} 1\,de = [e]_{-x}^{x} = 2x$

So for two independent observations:

$P(|E| < x) \times P(|E| < x) = 4x^2$

[2 marks]

d $f(M) = \dfrac{d}{dx}(4x^2) = \begin{cases} 8x & 0 < x < 0.5 \\ 0 & \text{otherwise.} \end{cases}$

[3 marks]

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