CAMBRIDGE

Higher

MATHEMATICS

GCSE for OCR

Student Book Answers

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Contents

Answers

The answers to past paper questions are the work of the publisher and authors and have been neither seen nor verified by OCR.

1 Basic calculation skills

BEFORE YOU START ...

1 **a** > **b** > **c** = **d** <

2 **a** Mistake is just working left to right not doing operations in order. Correct answer is 23.
b Mistake is adding before multiplying. Correct answer is 31.
c Order of operations is correct, but calculation is wrong. Correct answer is 132.

3 **a** C **b** B **c** B

LAUNCHPAD

1 **a** 1433 **b** 1117 **c** 855 **d** 18

2 **a** C **b** D **c** D **d** B **e** D

3 $\frac{128}{4} = 32$

4 208

5 1350

EXERCISE 1A

1 **a** Total cost and price per pen.
b 5 packets. We need to know this so we can work out the total cost.
c £19.50 **d** £1.30

2 £36.50

3 64

4 **a** 4320 **b** 42 mins, or a good approximation.

5 56 539

6 **a** 222 km **b** 6094 km

7 4803

8 5678 + 67 = 5745

EXERCISE 1B

1 **a** $^{-}2$ **b** 2 **c** 6 **d** 9 **e** 29

2 **a** 15 **b** 11 **c** 7 **d** 4 **e** $^{-}16$

3 **a** $^{-}3$ **b** 25 **c** 9 **d** $^{-}126$ **e** 0

4 **a** $^{-}8$ **b** 8 **c** $^{-}32$ **d** 128 **e** 0.25

5 **a** $^{-}8$ and 1, $^{-}6$ and 3 **b** $^{-}3$, 3 and 1
c $^{-}3$ and 1 **d** $^{-}6$ and 1

6 $^{-}26$

7 This is an investigative task.
The only integers between 5 and 150 that cannot be made by combining multiples of 5 and 7 are 6, 8, 9, 11, 13, 16, 18 and 23.

WORK IT OUT 1.2

1 A: in B, addition is done before multiplication.

2 B: in A, the 4 and 9 have been squared separately.

3 B: in A, incorrect calculation.

4 B: in A, the calculation has been performed left to right.

5 B: in A, the 4's have been cancelled incorrectly in the fraction.

6 A: in B, addition is done before the division.

EXERCISE 1C

1 **a**, **c** and **d** are correct.
b 608 **e** 368 **f** 10

2 **a** 13 − 18 ÷ 9 = 11
b 8 ÷ (16 − 14) − 3 = 1
c (9 + 5) − (6 − 4) = 12 or (9 + 5) − (12 − 4) = 6

3 **a** 3 × (4 + 6) **b** (25 − 15) × 9 **c** (40 − 10) × 3
d (14 − 9) × 2 **e** (12 + 3) ÷ 5 **f** (19 − 9) × 15
g (10 + 10) ÷ (6 − 2) **h** (3 + 8) × (15 − 9)
i (9 − 4) × (7 + 2) **j** (10 − 4) × 5
k 6 ÷ (3 + 3) × 5 **l** no brackets needed
m (1 + 4) × (20 ÷ 5) **n** (8 + 5 − 3) × 2
o 36 ÷ (3 × 3 − 3) **p** 3 × (4 − 2) ÷ 6
q no brackets needed **r** no brackets needed

4 **a** 12 ÷ (28 − 24) **b** 88 − 10 × 8
c 40 ÷ 5 ÷ (7 − 5) **d** 9 + 15 ÷ (3 + 2)

5 **a** 0.5 **b** 2 **c** $\frac{11}{60} = 0.183$ **d** 0.5
e $\frac{1}{3} = 0.333$ **f** 1 **g** 2 **h** $\frac{2}{3} = 0.667$

6 This is an investigation.
Students should find their own methods and explain their thinking.
The formula $\frac{n(a+b)}{2}$ will work for any sequence, where n is the number of terms, a is the lowest number and b is the highest number in the set.
There are many other methods of finding the answer though, and students could research these online if they are interested.

EXERCISE 1D

1 **a** 312 **b** 102 **c** 400
d $^{-}5$ **e** $^{-}145$ **f** 216 000

2 All calculations are correct.

3 **a** $h = 12$ cm **b** $h = 8$ cm, $b = 16$ cm

4 **a** $^{-}83$
b Students' own answers.
Suggestion:
i (1 + 2) × 3 × 4 × 5 × 6 × 7 = 7560
ii (1 − 2 − 3) × 4 × 5 × 6 × 7 = $^{-}3360$
c Students' own reasoning; should suggest that addition and multiplication (of whole positive numbers) produces higher values.

CHAPTER REVIEW

1 Students' own answers.

2 Students' own answers. Some possible solutions are:
$^{-}19 + 2$; $^{-}7 + {}^{-}10$; $^{-}34 + 17$, $51 \div {}^{-}3$; $17 \times {}^{-}1$, and so on.

3 **a** **i** 13 **ii** 5 **iii** 4
b Students' own answer, for example
i 4 + 3 + 2 − 1 = 8 **ii** 4 × 3 + 2 + 1 = 15

4 4032

5 256

6 $^{-}20$ and 5

7 $^{-}4$ and 5

8 £568

9 **a** 18 °C **b** 34 °C

2 Whole number theory

BEFORE YOU START ...

1 **a** 25 **b** 6 **c** 11

2 **a** 2 **b** 3 **c** 5

3 **a** 2, 3, 5, 7, 11, 13, 17, 19
b 1, 4, 9, 16 **c** 1, 8

4 **a** D **b** B **c** C **d** A

LAUNCHPAD

1 **a** F **b** F **c** T **d** T **e** F **f** F

2 **a** Multiples of 2; 29 is incorrect
b Multiples of 11, 56 is incorrect
c Factors of 12; 8 is incorrect
d Multiples of 3, 41 is incorrect
e Factors of 36, 24 is incorrect
f Multiples of 12; 86 is incorrect
g Primes to 20, 9 is incorrect
h Square numbers, 39 is incorrect

3 **a** B **b** A

4 **a** C (24) **b** D(360)

EXERCISE 2A

1 **a** 1, 3, 5, 7, ... 29 **b** 2, 4, 6, 8, ...30
c 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
d 1, 4, 9, 16, 25 **e** 1, 8, 27
f 1, 2, 3, 4, 6, 8, 12, 24 **g** 3, 6, 9, 12 ... 30
h 1, 2, 4 **i** 12, 24

2 **a** 209, 211, 213, 215
b Various options, for example: 502, 504, 506, 508
c 25, 36, 49, 64
d 1, 23
e 17, 19, 23, 29, etc (selection is technically infinite)
f 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000
g 8, 16, 24, 32, 40
h 1, 2, 3, 4, 6, 9, 12, 18, 36

3 **a** Even **b** Even **c** Even
d Odd **e** Even **f** Odd

EXERCISE 2B

1 **a** 348, 432, 456, 654, 843 **b** 606, 607, 660, 670, 706
c 123, 231, 312, 1231, 1321 **d** 12 700, 21 007, 21 700, 71 200

2 **a** 5 **b** 500 **c** 50 **d** 5000
e 50 000 000 **f** 500 000 **g** 5 000 000 **h** 50 000

3 **a** 640, 406 **b** 7531, 1357 **c** 643 210, 102 346

EXERCISE 2C

1 **a** 2, 3, 5, 7 **b** 53, 59 **c** 97, 101, 103

2 **a** $2^2 \times 3^2$ **b** 5×13 **c** 2^6 **d** $2^2 \times 3 \times 7$
e $2^4 \times 5$ **f** $2^3 \times 5^3$ **g** $2 \times 5 \times 127$ **h** 13×151

3 **a** 1080
b No. The product of prime factors is unique for each whole number.

4 This is an investigation.
The sieve of Eratosthenes is a grid of numbers on which multiples (of 2, 3, etc.) are crossed out systematically, leaving only primes uncrossed.

5 **a** They are named after the monk who first suggested them, although his initial thinking about them was proven to be wrong with later discoveries.
b GIMPS aims to use the spare processing power of linked computers to 'crunch' greater and greater numbers to see whether they are prime or not.

6 **a** 101, 103, 107 and 109, as well as 191, 193, 197 and 199.
b All even numbers are automatically excluded, as are numbers ending with 5 (multiples of 5); that leaves only 4 possible options, numbers ending in 1, 3, 7 and 9.
As these could be multiples of 3, 7 or 9, many of these are not prime.

EXERCISE 2D

1 **a** 18 **b** 36 **c** 90 **d** 24
e 36 **f** 24 **g** 72 **h** 96

2 **a** 6 **b** 18 **c** 9 **d** 3
e 10 **f** 1 **g** 12 **h** 50

3 **a** 18 m **b** 72 m: 4 pieces. 90 m: 5 pieces.

4 120 shoppers

5 20 students

6 20 minutes

7 **a** 60 days **b** 6 or 7 times (depending when the first ride fell compared to the start of the year)

8 6 cm

9 4.31 pm

10 After 420 seconds (7 minutes).
Francesca 21 laps, Ayuba 5 laps and Claire 4 laps.

11 **a** 900 cm^2 **b** 209 tiles

12 142 people

CHAPTER REVIEW

1

		a C									
	1 M	U	L	T	I	b P	L	E	S		
		B				R					
2 E	V	E	N		3 D	I	V	I	S	c O	R
				d F		M				D	
	4e S	Q	U	A	R	E				D	
	I			C							
	X			5 T	W	O					
		6 P	R	O	D	U	C	T			
				R							

2 Yes. You can decide by trial division of prime numbers up to the square root of the number in question.

3 **a** 20 **b** 6 **c** 17 **d** 34, 16 **e** 16

4 F_{20}: 1, 2, 4, 5, 10, 20
F_{35}: 1, 5, 7, 35
HCF = 5
M_{20}: 20, 40, 60, 80, 100, 120, 140
M_{35}: 35, 70, 105, 140
LCM = 140

5 $800 = 2^5 \times 5^2$

6 a $72 = 2 \times 2 \times 2 \times 3 \times 3$
$108 = 2 \times 2 \times 3 \times 3 \times 3$
HCF = 36
LCM = 216
b $84 = 2 \times 2 \times 3 \times 7$
$60 = 2 \times 2 \times 3 \times 5$
HCF = 12
LCM = 420

7 13th step

8 15th March and 27th March (or 14th and 26th)

9 8 boys and 13 girls per group

3 Algebraic expressions

BEFORE YOU START ...

1 a B **b** A **c** A

2 a i 19 and $^{-}11$ **ii** 27 and $^{-}3$
b 20

3 a $6y$ **b** $5x$

4 a C **b** B **c** D **d** A

LAUNCHPAD

1 a $3n + 4$ **b** $3(n - 4)$ **c** $\frac{4n^2 + 3}{2}$

2 a $5a + b$ **b** $6x + 4$ **c** $^{-}6a^2 + 3ab$

3 a $mn - mp$ **b** $7x + 23$ **c** $z^2 + 3z$

4 a $3x + 12 = [\,3\,](x + 4)$
b $5x + 10y = [\,5\,](x + 2y)$
c $x^2 - 3x = [\,x\,](x - 3)$
d $ab - ac = a([\,b\,] - [\,c\,])$
e $^{-}x + 7x^2 = {}^{-}x([\,1\,][\,-\,][\,7x\,])$

5 a $2(x + 2y)$ **b** $^{-}3(x + 3)$ **c** $5x(x + y)$

WORK IT OUT 3.1

B is correct.
A is incorrect: the order of subtraction is important. This is only true for $x = 5$.
C is incorrect: LHS is the reciprocal of RHS. This is only true for $y = 0.5$ and $y = {}^{-}2.5$

EXERCISE 3A

1 a $3x + 7y$ **b** $5(x^2 - 4)$ **c** $\frac{(x^3 + y^2)}{4}$
d $4(x + 6) - y$ **e** $\frac{x^2}{2}$ or $\frac{1}{2}x^2$ **f** $\frac{(3x - 25)}{5} = \frac{3x}{5} - 5$

2 a B **b** G **c** I **d** H **e** C
f D **g** F **h** A **i** No match

3 a $6a$ **b** $20b$ **c** $^{-}9d$ **d** $12ab$ **e** $10cd$
f $^{-}12mn$ **g** $6pq$ **h** a^2 **i** m^2 **j** $8a^2$
k $^{-}15a^2$ **l** $8m^2$ **m** $56a^2b$ **n** $12cd^2e$ **o** $8a^3$

4 a $3x$ **b** $9y$ **c** $3a^2$ **d** $5p$ **e** x
f $\frac{3}{2}y$ **g** $^{-}2x^2$ **h** $4a^2$ **i** $\frac{5}{n^2}$

5 a x^{13} **b** y^{13} **c** $15a^9$ **d** $10x^9$
e a^{-2} **f** $2b^5$ **g** $2p^{-1}$ **h** x^{12}
i $8a^{21}$ **j** $10x^{-2}y^4$ **k** 5 **l** $3x^4$

6 a Perimeter: $2x + 16$, area: $2x + 12$
b Perimeter: $4x + 16$, area: $x^2 + 8x + 16$
c Perimeter: $2x + 15$, area: $\frac{x}{2} + \frac{7}{2}$ or $\frac{(x+7)}{2}$
d Perimeter: $4x + 2y$, area: $x^2 + 2y$

7 a $x + 10$ **b** $x - 10$ **c** $\frac{x}{3}$

8 a £$(C - 5)$ **b** £$C/3$ **c** £15

EXERCISE 3B

1 a 24 **b** 21 **c** 54 **d** 15 **e** 24
f $^{-}15$ **g** 36 **h** 9 **i** $^{-}18$

2 a 60 **b** 24 **c** 2 **d** $^{-}200$
e $^{-}6$ **f** 9 **g** 22 **h** $^{-}8$
i $^{-}12$ **j** 3004 **k** 30 **l** $-\frac{12}{5}$

WORK IT OUT 3.2

Student B is right because $3x^2y$ and $2xy^2$ do not have the same combination of variables and so are not like terms.

EXERCISE 3C

1 Like terms: **b**, **c**, **d**, **g**, **h**, **k**, **l**
Unlike terms: **a**, **e**, **f**, **i**, **j**

2 a $6x + 5y$ **b** $2d - 3c$
c $^{-}3xy - y^2$ **d** $2a^2 + 2ab^2 + 2ab$
e $2g - f$ **f** $6a^2b$
g $+2mn^3$ **h** $9st^2 + s^2t$

3 a $2a + [\,5a\,] = 7a$ **b** $5b - [\,3b\,] = 2b$
c $8mn + [\,4mn\,] = 12mn$ **d** $11pq - [\,5pq\,] = 6pq$
e $4x^2 + [\,3x^2\,] = 7x^2$ **f** $6m^2 - [\,5m^2\,] = m^2$
g $8ab - [\,10ab\,] = {}^{-}2ab$ **h** $^{-}3st + [\,8st\,] = 5st$

4 a $8a \times [\,2\,] = 16a$ **b** $9b \times [\,2\,] = 18b$
c $8a \times [\,2b\,] = 16ab$ **d** $5m \times [\,3n\,] = 15mn$
e $3a \times [\,4a\,] = 12a^2$ **f** $6p \times [\,5p\,] = 30p^2$
g $^{-}5b \times [\,^{-}2b\,] = 10b^2$ **h** $4m \times [\,3mn\,] = 12m^2n$

5 a $^{-}28x$ **b** $16xyz$ **c** $10a^2$ **d** ab^2c^2d
e $24x^2y$ **f** $\frac{^{-}y^2}{x}$ **g** $^{-}3x$ **h** $\frac{^{-}12y}{x}$
i $3x^2$ **j** $^{-}5x$

6 a $\frac{x}{2}$ **b** $\frac{a}{3}$ **c** $\frac{^{-}2m}{3}$
d $\frac{5p}{7}$ **e** $\frac{2x^2}{5}$ **f** $\frac{3xy}{4}$
g $12b$ **h** $\frac{1}{3}$ **i** $\frac{10x}{y}$

EXERCISE 3D

1 a Incorrect; $4a + 4b$ **b** Incorrect; $5a + 5$
c Correct **d** Incorrect; $^{-}3p + 15$
e Incorrect; $a^2 + ab$ **f** Correct
g Incorrect; $^{-}6x + 30$ **h** Incorrect; $12a^2 - 21a$
i Correct **j** Incorrect; $6x^2 - 21xy$

2 a $2c + 5$ **b** $a + 9$ **c** $5b + 25$
d $2e + 5$ **e** $3f - 18$ **f** $8a^2 + 13a$
g $10b^2 - 9b$ **h** $15a^2 + 6a$ **i** $2b^2 - 15\,b$

3 a $5y + 14$ **b** $16b - 9$
c $a + 1$ **d** $b - 22$
e $x^2 + x - 6$ **f** $2p^2 - 3p - 5$
g $10z$ **h** $4y^2 - 16y$

4 a

	$15x - 3y$	
$6x - 2y$		$9x - y$
$2x - 3y$	$4x + y$	$5x - 2y$

b

	$11p + 2q$	
$5p + 5q$		$6p - 3q$
$4p - q$	$p + 6q$	$5p - 9q$

5 a Let $a = 1$: $1 + 1 = 2$ and $1 \times 1 = 1$ (Note if you use 2, it will be equal)
b Let $x = 1$: $3(1) + 4 - 1 + 2 = 8$ and $2(1) + 2 = 2 + 4 = 4$
c Let $m = 1$: $(1 + 2)^2 = 9$ and $1^2 + 4 = 5$
d Let $x = 1$: $\frac{1+3}{3} = \frac{4}{3}$ and $1 + 1 = 2$

EXERCISE 3E

1 a $2(x + 2)$ **b** $6(2m - 3n)$ **c** $3(a - b - 2)$ **d** $x(y - z)$
e $5xy(1 - 3z)$ **f** $7b(2a - 3c)$ **g** $p(q - r)$ **h** $x(x - 1)$
i $6ac(3b - 2)$ **j** $2x(x - 2y)$ **k** $2xy(x - 2y)$ **l** $-6(a + 2)$
m $^-3(a + 3)$ **n** $^-x(y + 5)$ **o** $^-x(x - 6)$

2 a $x(7 - y + x)$ **b** $2x(y + 2z + 5)$ **c** $5(2x - y + 3z)$
d $(x - 2)(x + 5)$ **e** $(a - 7)(a - 1)$ **f** $^-2(x - 3)$
g $3xy(x + 2y)$ **h** $\frac{1}{4}x^3(144 - x)$ **i** $^-a(x^2 + y^2)$

EXERCISE 3F

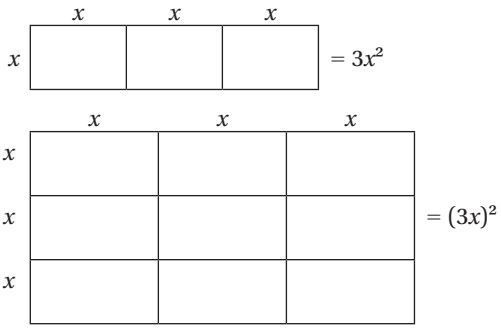
1 a False **b** True **c** True
d False **e** True

2 Yes. All total $3m$

3 a $3a + 1$ $2b + 1$
b $P = 10a + 6b + 10$
c 210.25

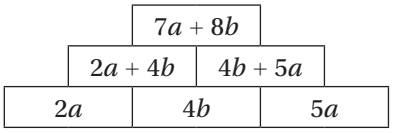
4 a Students' own answers.
b Students' own answers. Must sum to $x^2 + 2$ (two sides = half the perimeter)

5

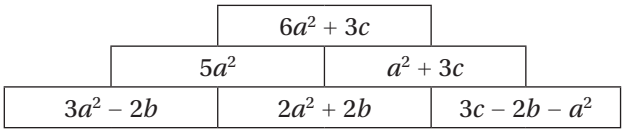


6 $A = 2xy + b(a - x) = 2xy + ab - bx$
$A = 2ay + (a - x)(b - 2y) = 2ay + ab - 2ay - bx + 2xy = ab - bx + 2xy$

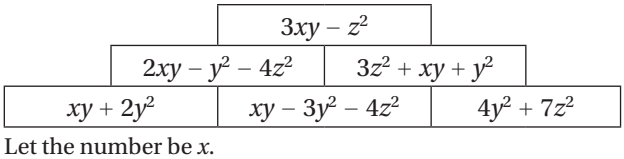
7 a



b



c



8 a Let the number be x.
Double is $2x$.
Add six gives $2x + 6$.
Halve it gives $x + 3$.
Subtract the original number (x) gives 3.
b Students' own ideas.

9 $a = 2$, $b = 17$

10 a $13g + 10$ **d** $6x^2$ **e** x^8

11 a $\frac{13x}{3y^3}$ **b** $\frac{(6a+5)}{2b}$
c $^-20x + 80 = 20(4 - x)$ **d** $\frac{(2-a)}{5}$

12 a $10x^2y^2 - 5x^3$ **b** $2pq^2 + 3p^2q - 2p^4$

CHAPTER REVIEW

1 a $2b^2 - 15b$ **b** $^-8x^2 + 50x$ **c** $\frac{(5p+4)}{4}$ **d** $y^2 + 2y$

2 $\frac{8xy}{16} \neq \frac{xy}{4}, \frac{8xy}{16} = \frac{xy}{2}$
$\frac{15}{2x} \times \frac{2}{3x} \neq \frac{10}{x^2}, \frac{15}{2x} \times \frac{2}{3x} = \frac{5}{x^2}$

3 a Let $x = 1$.
$5(1 + 3) = 20$ and $5(1) + 3 = 8$, so this is not an identity.
b Let $m = 1$.
$^-3(1 - 2) = 3$ and $^-3(1) - 6 = ^-9$, so this is not an identity.
c Let $y = 1$.
$4(1 - 3) + 2(1 + 4) = 2$ and $6(1) - 4 = 2$, so this could be an identity. We need to use algebra to prove that they are:
LHS $= 4(y - 3) + 2(y + 4) = 4y - 12 + 2y + 8 = 6y - 4 =$ RHS

4 a **i** 10 **ii** 28
b $y^2 + 5y$ **c** $4p(p - 2)$

5 $\frac{1}{t} + \frac{1}{w} = \frac{(w+t)}{tw} \neq \frac{2}{(t+w)}$

6 $n + (n + 1) = 2n + 1$.
$2n + 1$ is always an odd number since $2n$ is always an even number.

7 $(n + 1)^2 - n^2 = n^2 + 2n + 1 - n^2$ (the difference between squares of two consecutive numbers)
$= 2n + 1$
$= n + (n + 1)$ (the sum of two consecutive numbers)
$= n + (n + 1)$ (the sum of two consecutive numbers
$= 2n + 1$
So the difference between the squares of two consecutive numbers is equal to the sum of those numbers.

4 Functions and sequences

BEFORE YOU START ...

1 a 7, 14, 21, 28, 35 **b** 66, 18, 54, 36

2 a 1, 16, 25, 4, 9, 49 **b** 9, 15

3 a The pattern is built up using square and triangle shapes with some common sides. Each new shape has three extra matches to make the sides of the next square and two extra matches to make the sides of the triangle.
b 31

LAUNCHPAD

1 a 59, 71, 83 **b** Add 12 to the previous term

2 a 29, 59, 299 **b** $3n - 4$ **c** 17

3 input → [× 2] →[− 4] → output

4 4, 5, 6, 7, 8 ...

5 $x \rightarrow \frac{2(x-1)}{3}$

6 The other function is $4x$. $4x$ is applied first.

7 $3n + 1$

EXERCISE 4A

1 **a** 16, 19, 22 ; add 3 **b** 58, 63, 68 ; add 5
c 15, 11, 7 ; subtract 4 **d** 45, 39, 33 ; subtract 6
e 16, 32, 64 ; multiply by 2 **f** 8, 4, 2 ; divide by 2
g 108, 324, 972 ; multiply by 3 **h** 27, 9, 3 ; divide by 3

2 **a** Add 7 **b** Subtract 4
c Multiply by 4 **d** Divide by 2

3 **a** Add 2; 9.5, 11.5, 13.5 **b** Multiply by 2; 9.6, 19.2, 38.4
c Add $1\frac{1}{2}$; 6, $7\frac{1}{2}$, 9 **d** Subtract 3; $^{-}1$, $^{-}4$, $^{-}7$
e Divide by 2; 9, 4.5, 2.25 **f** Add 6; 4, 10, 16

4 **a** 6cm **b** 7th bounce will be 0.75cm

5 **a** $\frac{1}{2}$ **b** $1\frac{1}{3}$ **c** Any negative value

WORK IT OUT 4.1

Option B is correct.
Option A is wrong because this sequence is defined as $3n - 5$.
Option C is wrong because this sequence is defined as $2n - 3$.

EXERCISE 4B

1 **a** 2, 5, 8, 11, 14, 17
b 59
c No. The 40th term is 119, which is not 2×59

2 **a** 5, 9, 13, 17, … , 41, … , 81, … , 401
b 1, 3, 7, 9, … , 35, … , 75, … , 395
c 10, 18, 26, 34, … , 82, … , 162, … , 802
d 4.5, 9.5, 14.5, 19.5, … , 49.5, … , 99.5, … , 499.5
e 1.5, 2, 2.5, 3, … , 6, … , 11, … , 51
f $^{-}1$, $^{-}3$, 13, 17, … , 41, … , 81, … , 401

3 $6n - 5$

4 **a** $u_n = 2n + 1$ **b** $u_n = 4n - 1$ **c** $u_n = 5n - 6$
d $u_n = 5n + 2$ **e** $u_n = 3n - 6$ **f** $u_n = 7n - 8$

5 $6n + 61$

6 **a** $2.2n + 2.3$
b 222.3 cm
c Sunflowers do not continue to grow at a constant rate. Over 2 m is unusual for a sunflower; 100 weeks is nearly two years and sunflowers are unlikely to live this long.

7a £308 **b** 75 weeks

8 **a** 14
b 22
c No, because any number in the sequence $2n + 2$ must be even

EXERCISE 4C

1 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

2 **a** $^{-}4$, $^{-}3$, $^{-}2$, $^{-}1$, 0, 1, 2, 3, 4, 5
b 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
c 8, 9, 10, 11, 12 ,13, 14, 15, 16, 17
d $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5

3 **a** 42, 44, 46, 48, 50, 52, 54, 56, 58, 60
b 13, 14, 15, 16, 17, 18, 19, 20, 21, 22
c 7, $7\frac{1}{3}$, $7\frac{2}{3}$, 8, $8\frac{1}{3}$, $8\frac{2}{3}$, 9, $9\frac{1}{3}$, $9\frac{2}{3}$, 10
d $21\frac{1}{2}$, $22\frac{1}{2}$, $23\frac{1}{2}$, $24\frac{1}{2}$, $25\frac{1}{2}$, $26\frac{1}{2}$, $27\frac{1}{2}$, $28\frac{1}{2}$, $29\frac{1}{2}$, $30\frac{1}{2}$

4 **a** 38, 41, 44, 47, 50, 53, 56, 59, 62, 65
b 15, 17, 19, 21, 23, 25, 27, 29, 31, 33
c $9\frac{1}{2}$, 10, $10\frac{1}{2}$, 11, $11\frac{1}{2}$, 12, $12\frac{1}{2}$, 13, $13\frac{1}{2}$, 14
d $44\frac{1}{2}$, $48\frac{1}{2}$, $52\frac{1}{2}$, $56\frac{1}{2}$, $60\frac{1}{2}$, $64\frac{1}{2}$, $68\frac{1}{2}$, $72\frac{1}{2}$, $76\frac{1}{2}$, $80\frac{1}{2}$
e $\frac{48}{11}, \frac{13}{3}, \frac{56}{13}, \frac{30}{7}, \frac{64}{15}, \frac{17}{4}, \frac{72}{17}, \frac{38}{9}, \frac{80}{19}, \frac{21}{5}$

5 **a** $x \to x + 7$ **b** $x \to \frac{x}{4}$ **c** $x \to x - 5$
d $x \to 3x$ **e** $x \to \frac{1}{2}(x - 4)$ **f** $x \to \frac{1}{4}(x + 5)$
g $x \to 5(x - 3)$ **h** $x \to \frac{4}{x + 2}$

6 **a** $y = 4x - 7$ **b** $y = 2x + 4$
c $y = 3(x - 2)$ **d** $y = (x + 1)^2$

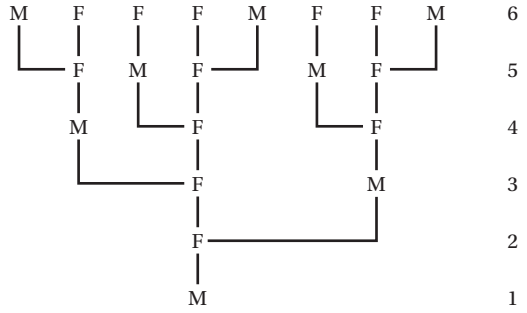
7 **a** $x \to \frac{x}{2}$
b function 1: $2x$, function 2: $\frac{x}{2}$, composite function $y = x$

EXERCISE 4D

1 **a** 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
b Adding 21 onto 100 (121), then 23 onto 121 (144)

2 **a** Triangular number. It can be arranged to form an equilateral triangle.
b 1, 3, 6, 10, 15, 21, 28, 36, 45, 55
c First difference 2, 3, 4, 5, 6, 7, 8, 9, 10, Second difference is 1
d A quadratic sequence

3 **a** 5 (3F 2M)
b **i**



ii (1,) 1, 2, 3, 5, 8
iii 34 (the ninth Fibonacci number)
iv These are the numbers from the Fibonacci sequence.

4 **a** 3, 4, 7, 11, 18, 29, 47, 76, 123, 199
b $^{-}2$, 3, 1, 4, 5, 9, 14, 23, 37, 60
c The terms in part **b** are multiplied by $^{-}1$ to make the terms in part **c**.

5 2, 5

6 Answer table as shown below:

Position-to-term rule	1st term	2nd term	3rd term	5th term	10th term	20th term	50th term
$n^2 + 5$	6	9	14	30	105	405	2505
$n^2 - 3$	$^{-}2$	1	6	22	97	397	2497
$2n^2 + 1$	3	9	19	51	201	81	5001
$2n^2 - 7$	$^{-}5$	1	11	43	193	793	4993

7 2, 9, 28, 65, 126, 217

8 2, 6, 12, 20, 30, 42, 56, 72, 90, 110

9 **a** $\frac{1}{2}(n^2 + n)$ **b** 55, 325

EXERCISE 4E

1 **a** $n^2 + 2n$ **b** $2n^2 + n$ **c** $4n^2 + 3n$
d $2n^2 + 5n - 1$ **e** $3n^2 - 2n$ **f** $4n^2 - 3n - 4$
g $^{-}2n^2$ **h** $^{-}2n^2 + 2n$

2 4, $4\sqrt{2}$, 8

3 **a** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$ **b** $\frac{10}{11}$

4 **a** 0, 5, 12, 21, 32 **b** 252

5 **a** 5, 7, 11, 17, 25 **b** 35 **c** $n^2 - n + 5$

CHAPTER REVIEW

1 a

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

b 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024
Powers of two (or doubles each time)

c Triangular numbers

d $\frac{1}{2}(n^2 + n)$

2 a $3n + 5$. **b** 7, 2, −3

3 a 28 **b** $5n + 3$ **c** 25 days

4 a 339 ($u_n = 2n^2 + 4n + 3$) Assuming the second difference is constant

b The assumption that no rabbits will die during the year is unrealistic.

5 4383 ($u_n = 12n^2 - 22n + 23$) Assuming the second difference is constant

6 a $n \rightarrow \boxed{\times 2} \rightarrow \boxed{+2}$

b 20th term = 42; 25th term = 52; nth term = $2n + 2$

7 a $b \rightarrow \boxed{\times 0.5} \rightarrow \boxed{+28}$

10 surveys = 32 days; 15 surveys = 35.5 days;
b surveys = $0.5b + 28$

b $d \rightarrow \boxed{-28} \rightarrow \boxed{\times 2}$ or $2(d - 28)$ where d = number of days holidays.

c 4

5 Properties of shapes and solids

BEFORE YOU START ...

1 a Point **b** Vertex **c** Edge
d Face **e** Right angle **f** Acute angle
g Base **h** Height

2 a Rectangle ABCE; triangle ADE; trapezium ABCD
b Cuboid

LAUNCHPAD

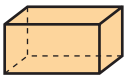
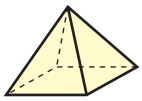
1 a Parallelogram **b** Lines are parallel
c PQ // SR **d** PQ̂R or RQ̂P

2 2 lines of reflective symmetry; 2 orders of rotational symmetry.

3 A Right-angled B Isosceles
C Equilateral D Obtuse-angled isosceles

4 a Parallelogram, rectangle, square, rhombus
b Rhombus, square
c Parallelogram, rectangle, square, rhombus
d Rectangle, square
e Trapezium
f Quadrilateral

5

Solid	Mathematical name	Number of faces	Number of edges	Number of vertices
	Cuboid	6	12	8
	Cube	6	12	8
	Square pyramid	5	8	5

EXERCISE 5A

1 a Equilateral triangle
b Pentagon (Regular only if the angles are also equal)
c Hexagon (Regular only if the angles are also equal)
d Regular octagon

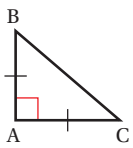
2 Many examples could be provided
a Stop sign
b Playing dice
c Squares on graph paper
d Shape of the side of a house

EXERCISE 5B

1

a	A shape with two fewer sides than an octagon has.	**ii**	Hexagon
b	A shape with two sides more than a triangle has.	**v**	Pentagon
c	A shape with four sides.	**vi**	Quadrilateral
d	A stop sign is an example of this shape.	**ix**	Octagon
e	A figure that has length and height.	**iv**	Two-dimensional
f	A closed plane shape with all sides x cm long.	**viii**	(Regular) polygon
g	A ten sided figure	**i**	Decagon
h	Another name for a regular 4-sided polygon.	**vii**	Square
i	The more common name for a regular 3-sided polygon.	**iii**	Equilateral triangle

2 a True **b** True **c** True **d** True
e False **f** False **g** False **h** True

3 a


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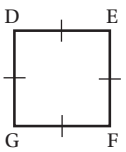
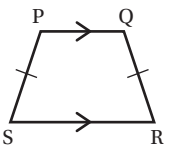


c


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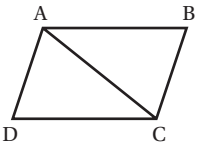


AB // DC and is intersected by CA.
DA // CB and is intersected by CA.
As alternate interior angles are equal ∠ACD = ∠CAB and ∠DAC = ∠ACB.
This means △DAC and △BCA have two angles equal and the side CA in common, so the triangles are congruent, and hence AB = DC and DA = CB.

EXERCISE 5C

1 **a** None **b** All lines (AB, CD, EF, GH)
c CD, GH **d** AB

2

Shape	Number of lines of symmetry	Order of rotational symmetry
Square	4	4
Rectangle	2	2
Isosceles triangle	1	1
Equilateral triangle	3	3
Parallelogram	0	2
Regular hexagon	6	6
Regular octagon	8	8

3 Students' own answer, example: propeller with three props

4 H

5 Lines of symmetry: vertical and horizontal axes through the centre of the image, rotational symmetry of order 4

6 Students' own answers

7 Students' own answers

EXERCISE 5D

1 Equilateral triangle, the balls are all the same size and each edge of the triangle has 5 balls along it.

2 D Obtuse-angled isosceles triangle

3 **a** (This is possible.)
b Not possible: the angles would sum to more than 180°.
c Not possible: angles do not sum to 180°.
d Not possible: angles of an equilateral triangle are all 60°.
e Not possible: isosceles triangle has two sides of equal length.

4 **a** 38° **b** Isosceles

5 $a = 54°$, $b = 66°$, $c = 60°$, $d = 115°$, $e = 115°$, $f = 16°$, $g = 104°$, $h = 76°$, $i = 70°$

6 **a** 17.5 mm **b** 23.2 mm **c** 12 mm

WORK IT OUT 5.1

Option B is correct.
Option A: 'angles on a straight line' only applies to angles that meet at a point to form a straight line.
Option C: The shape is not a rhombus as all sides are not equal lengths.

EXERCISE 5E

1 **a** Rectangle, square
b Rectangle, square, isosceles trapezium
c Parallelogram, rectangle, square, rhombus
d Quadrilateral
e Square
f Parallelogram, rectangle, square, rhombus

2

	Diagonals are...		
Shape	**Equal in length**	**Bisect each other**	**Perpendicular**
Rhombus		✓	✓
Parallelogram		✓	
Square	✓	✓	✓
Kite			✓
Rectangle	✓	✓	

3 Interior angles of a rhombus do not all equal 90°.

4 No, it could be a rhombus

5 Two possibilities: 90° and 90°, or 47° and 133° (in the second case it would also be a rhombus)

6 **a** Always true **b** Sometimes true **c** Sometimes true
d Always true **e** Sometimes true

7 **a** True **b** False assuming it isn't a square
c False **d** True

8 $x = 13.3°$

9 20 mm

10 AD = BC and AD//BC (as ABCD is a parallelogram).
AD = FE and AD//FE (as ADEF is a parallelogram).
Therefore, FE = BC and FE//BC.
So, FECB is a parallelogram (as one pair of opposite sides are equal and parallel).

11 No, it could be an isosceles trapezium, for example.

EXERCISE 5F

1 Answers here are for (**a**) a small cylinder mounted on the flat face of the larger cylinder, (**b**) pyramid fully on top of a cube, (**c**) pentagonal pyramids joined at base and (**d**) triangular prism fully on top of one of the flat surfaces of the cuboid. (Students may have other correct answers; check against their sketches.)

	3D Shape	Polyhedron	Faces	Vertices	Edges
a	Large and small cylinder		5	0	4
b	Cube and square pyramid	✓	9	9	16
c	Two identical pentagonal pyramids	✓	10	7	15
d	Triangular prism and cuboid	✓	7	10	15

2

3D Shape	Faces	Vertices	Edges
Cube	6	8	12
Cuboid	6	8	12
Triangular pyramid	4	4	6
Square pyramid	5	5	8
Triangular prism	5	6	9
Hexagonal prism	8	12	18

a $E = F + V - 2$
b 20 faces
c It works for both objects
Truncated pyramid: $F = 6$, $E = 12$, $V = 8$; $F + V - 2 = 12 = E$
Pyramid on top of cuboid: $F = 9$, $E = 16$, $V = 9$; $F + V - 2 = 16 = E$
d No. It doesn't fit the formula.

3 **a** 86 cm **b** 170.4 cm **c** 96 cm
Some assumptions have been made to reach these results
Other answers may be possible but assumptions must be clearly stated.

CHAPTER REVIEW

1 **a** False, assuming circular pizza. **b** True
c False **d** True **e** True

2 **a** **i** C and D **ii** B and D **iii** A and E
b **i** square **ii** rhombus **iii** kite

3 Hexagon has 6 lines of symmetry (lines connecting opposite vertices and lines connecting midpoints of opposite sides) and rotational symmetry of order 6.

4 $b = 117°,\ c = 117°,\ d = 63°$

5 No, the missing angle is 95° because angles in a quadrilateral total 360°.

6 a $x = 70°, y = 110°$ **b** $a = 97°,\ b = 83°,\ c = 97°$

7 Yes because both pairs of opposite sides are parallel. As equal length diagonals bisect we can also say that this shape is a rectangle.

8 No, the diagonals of both a rhombus and a kite intersect at right angles.

6 Construction and loci

BEFORE YOU START ...

1 a 120° **b** 40°

2 Check students' drawings.

3 a C **b** C

4 a Side **b** Vertex **c** Centre
d Radius **e** Diameter

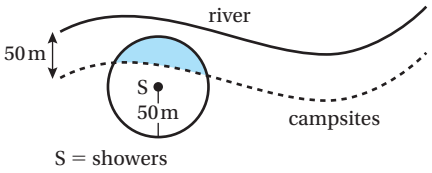
LAUNCHPAD

1 B

2 A, C

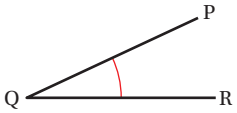
3 a Angle bisector
b Placed a compass at B to draw arcs on BA and BC.
c 12°

4

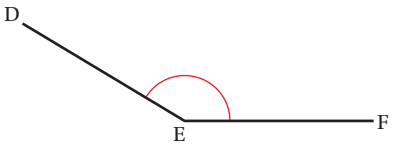


EXERCISE 6A

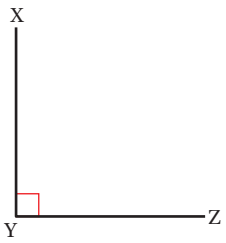
1 a



b



c



2 Use the protractor to measure the smaller (non reflex) angle at the point. In this case, this will measure 360 − 238 = 122°. Draw the non-reflex angle, but mark the angle arc on the reverse (reflex) side of the angle.

3 a Student drawings as specified in question
b The lengths of the lines are irrelevant because the length of the line does not change the size of the angle.

4 Student drawings as specified in question.

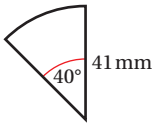
5 Student drawings as specified in question.

6 Student drawings as specified in question. Angles measured according to their diagram.

7 Student drawings as specified in question.

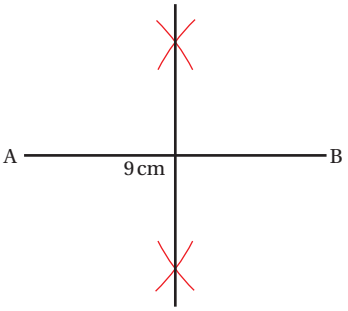
8 a Draw a line longer than 6.4 cm and mark a point A at one end of this line.
Set the compass to 6.4 cm and placing the compass at A draw an arc that crosses this line.
Label this point B.
Placing the compass at A draw an arc above the line that crosses above the midpoint of AB.
Repeat this step from point B.
Mark the point where the arcs intersect as C.
Use the ruler to join A to C and B to C.
ABC is an equilateral triangle with sides of 6.4 cm.
b Draw a line longer than 60 mm and mark a point O towards the middle of this line
Set the compass to 30 mm.
Place the compass at O draw a semi-circle by drawing the arc above the line.

9

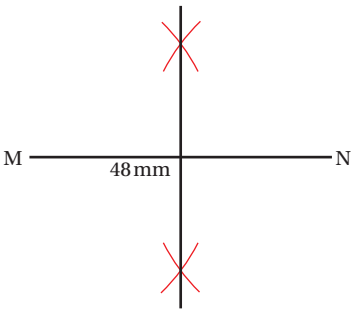


EXERCISE 6B

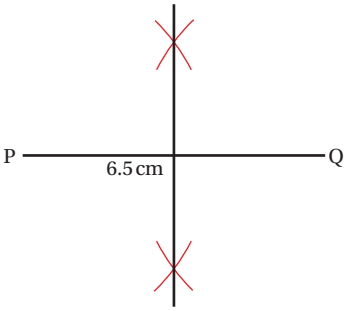
1 a



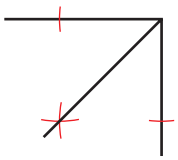
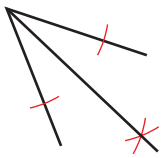
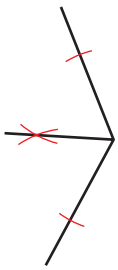
b

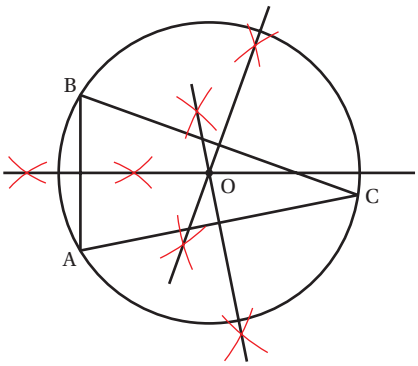


c

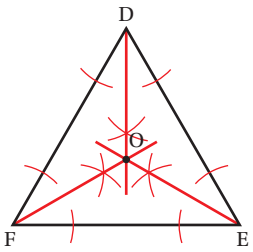


2 a Students' drawings of angle bisectors.
b Constructions can be checked for accuracy with a protractor.

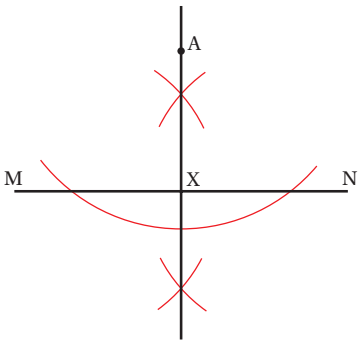
3 a  **b**  **c** 

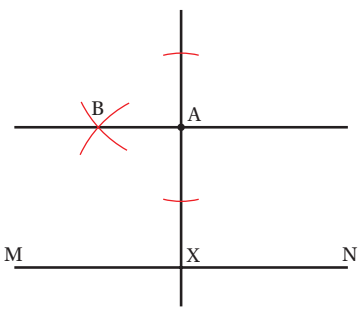
4 a, b 

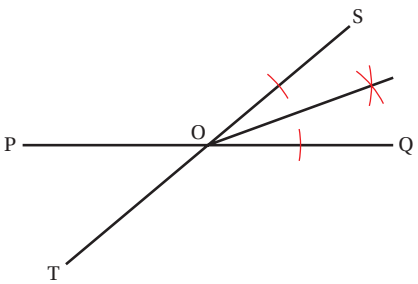
c For any triangle ABC, a circle with a centre at the intersection of the perpendicular bisectors and passing through point A will also pass through points B and C.

5 a 

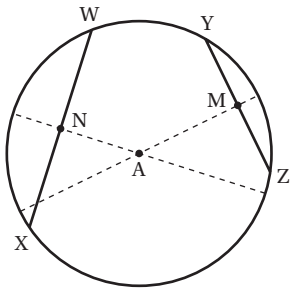
b DO = EO = FO ≈ 4 cm

6 a 

b 

7 a 

b Yes. Angles SOQ and POT are opposite angles and therefore equal.

8 a 

b Centre of the circle.

c The perpendicular bisector of any chord of a circle passes through the centre of the circle.

EXERCISE 6C

1 Discussion question.

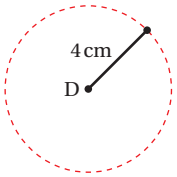
2 a Points on a circle centre at X, radius 200 km.

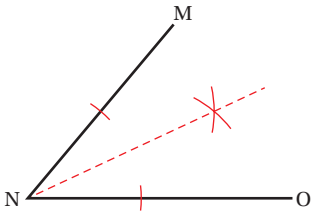
b Points within the area defined by lines A and B: line A is an oval defined by parallel lines 1 km long and at 2 km either side of the straight fence joined by semi-circles of radius 2 km at each end; line B is an oval defined by parallel lines 1 km long and at 3 km either side of the straight fence joined by semi-circles of radius 3 km at each end.

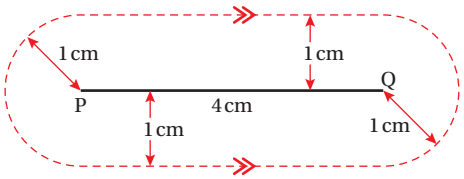
c Points on the straight line across the centre of the court.

d The centre spot.

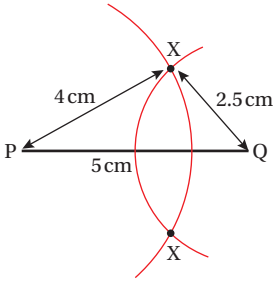
e Points within the area defined by two parallel lines that are 1 km either side of the railway line.

3 

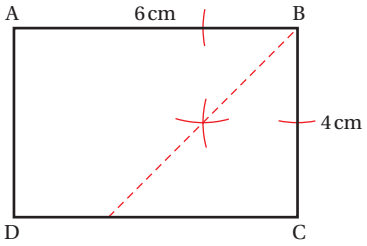
4 

5 

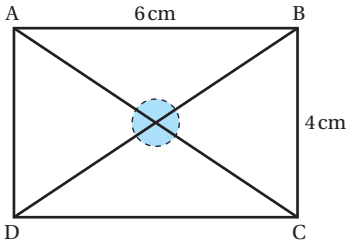
6 There are two points that are 4 cm from P and 2.5 cm from Q.



7 a



b



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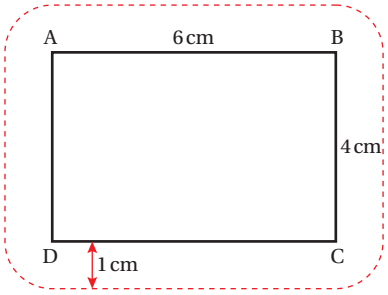

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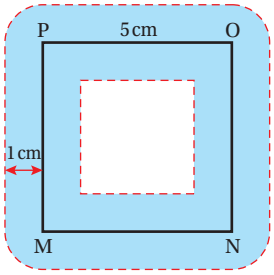
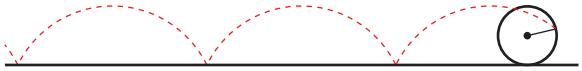


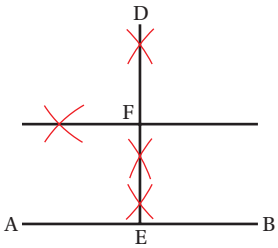
9 a



b

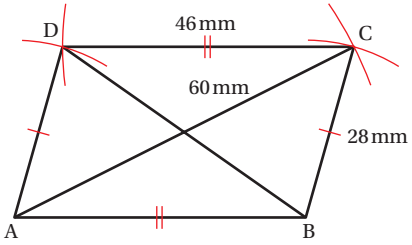
EXERCISE 6D

1

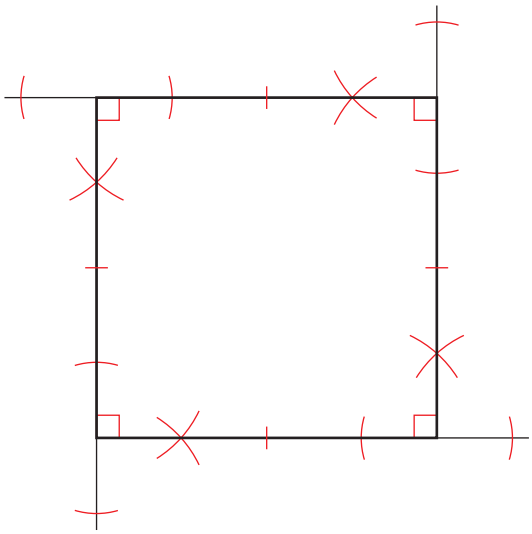


Drawings may vary as E is not specified as being on AB. General form and measurements should be accurate.

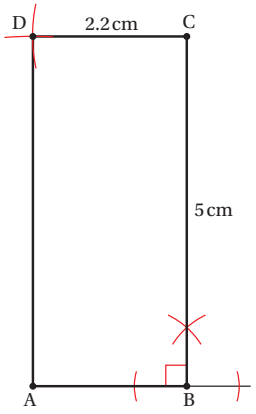
2 Length of diagonal DB is 47 mm.



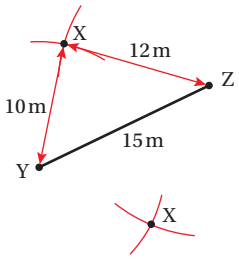
3



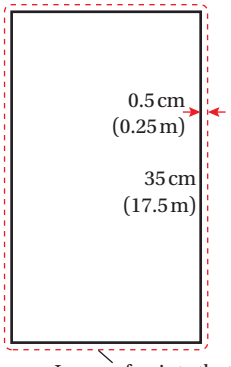
4 A rectangle.



5 Drawings should be to scale 1 cm to 2 m, so line YZ measures 7.5 cm.

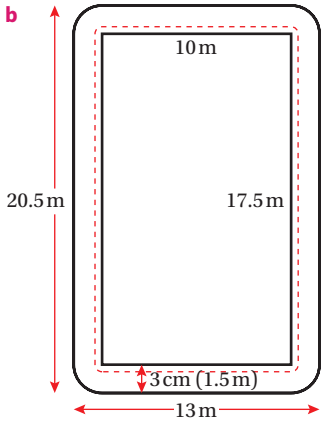


6 a



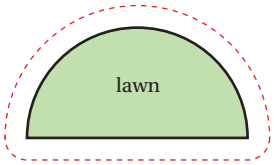
Locus of points that the monkey can reach.

b



b The barrier should be 1.5 m away from enclosure because people can reach out about 1 m. This means that visitors will not be able to touch the monkeys when both are stretching out (leaving a small margin for error).

7



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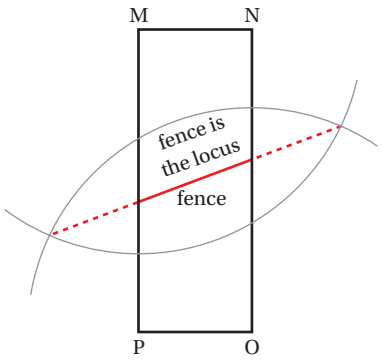
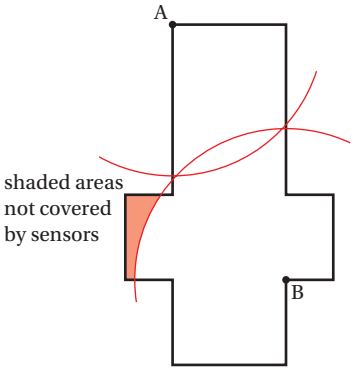
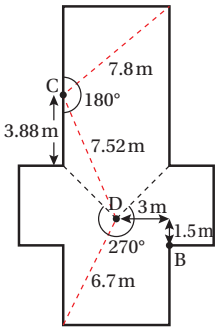


9 a



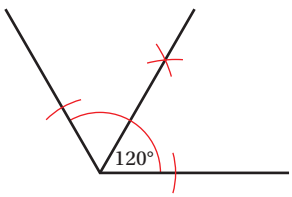
b Sensors would be better placed at C and D to provide maximum coverage.



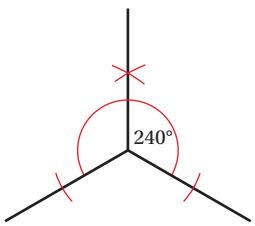
CHAPTER REVIEW

1 a 120° and 240°.

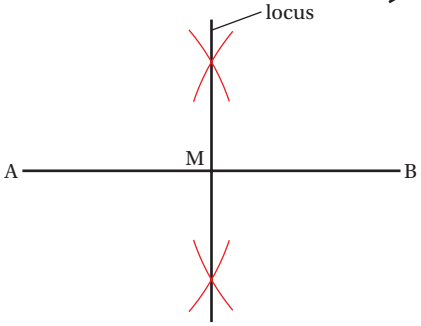
b



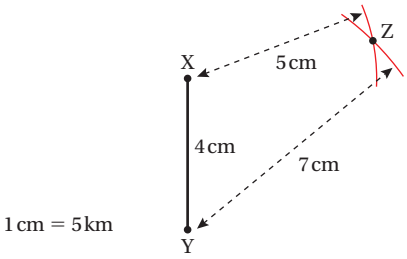
c



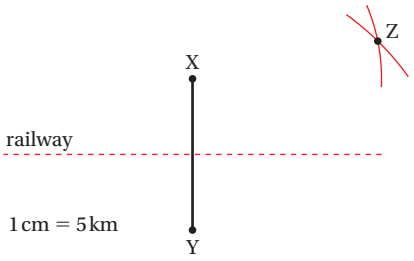
2



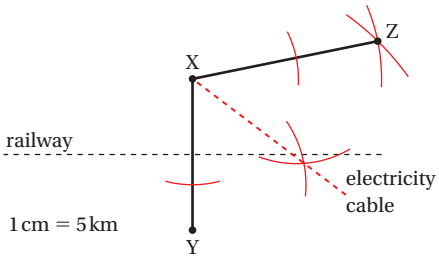
3 a



b



c



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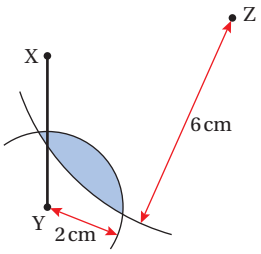

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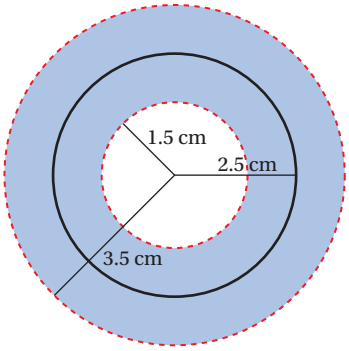
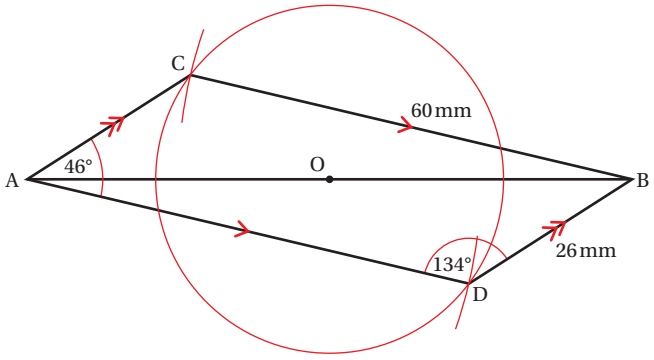


5 Other side 26 mm, internal angles 134° and 46°.



7 Further algebraic expressions

BEFORE YOU START ...

1 a $^{-}6x^2y^2 - xy$

b x^2y^2 and xy are not like terms as they have different powers.

2 $12x + 5y$

3 Yes. Both sides of the identity are equal for all values of b.

4 $x^{\frac{3}{2}} + \frac{36}{y^2}$

5 a $^{-}30a^2$ **b** $21y^3$ **c** $^{-}2a$ **d** $\frac{6}{5}$ **e** $\frac{^{-}5x}{2}$

6 a B **b** C **c** D **d** A

LAUNCHPAD

1 a $x^2 + 8x + 15$ **b** $x^2 + 2x - 15$ **c** $x^2 - 8x + 15$

2 a $(a + 2)(a + 3)$ **b** $(x - 2)(x - 1)$

c $(p - 9)(p + 5)$ **d** $(y + 4)(y - 4)$

3 a $x^2 + 12x - 11 \equiv (x + 6)^2 - 47$

b $x^2 + 8x + 20 \equiv (x + 4)^2 + 4 = (x + 4)^2 - {}^{-}4$

c $x^2 - 5x - 9 \equiv \left(x - \frac{5}{2}\right)^2 - \frac{61}{4}$

4 a $\frac{2x}{5}$ **b** $\frac{4x + 10}{(x + 1)(x + 3)}$

EXERCISE 7A

1 a $x^2 + 7x + 10$ **b** $x^2 - 7x + 10$ **c** $x^2 + 3x - 10$

d $x^2 - 3x - 10$ **e** $x^2 - x - 12$ **f** $x^2 + 2xy + y^2$

2 a $6x^2 + 18x + 12$ **b** $15x^2 + 26x + 8$ **c** $6x^2 - 13x - 5$

d $20y^2 - 11y - 3$ **e** $6a^2 - 13a + 5$ **f** $2b^2 - 11b + 15$

g $6y^2 - 19y + 15$ **h** $4x^2 - 4x - 24$ **i** $20x^2 - 17x + 3$

3 a $6x^2 + x - 2$ **b** $13x^2 + 8x + 5$ **c** $5x^2 + 16x + 3$

4 a $\frac{4}{x^2} + 2 + \frac{x^2}{4}$ **b** $4x^2 - 12xy + 9y^2$

c $4x^2 - 4xy + 4xz + y^2 - 2yz + z^2$

5 $a = {}^{-}1$

EXERCISE 7B

1 a $x^2 - 1$ **b** $a^2 - 4$ **c** $4x^2 - 1$ **d** $x^2 - 4y^2$

2 $a^2 - b^2 = (a + b)(a - b)$ Difference between two squares.

3 a There will be only two terms, separated by a minus sign. Both terms are squares.

b i No **ii** Yes **iii** Yes

EXERCISE 7C

1 a $x^3 + 6x^2 + 11x + 6$ **b** $4x^3 - 16x^2 + 19x - 6$

c $x^3 + x^2 - 4x - 4$ **d** $6x^3 - 19x^2 + x + 6$

2 a $27x^3 - 108x^2 + 144x - 64$ **b** $x^3 + 27$

c $\frac{1}{125x^3} + \frac{1}{27y^3}$ **d** $x^4y^4 - x^4$

3 a $(2x + \frac{1}{2})(x - 2)(x - 2)$ **b** $2x^3 - 7\frac{1}{2}x^2 + 6x + 2$

c These are a perfect square

EXERCISE 7D

1 a $(x + 1)(x + 6)$ **b** $(x + 6)(x + 3)$ **c** $(x + 3)(x + 4)$

d $(x + 2)(x + 15)$ **e** $(x + 5)(x + 4)$ **f** $(x + 5)(x + 18)$

2 a $(x - 1)(x - 2)$ **b** $(x - 3)(x - 9)$ **c** $(x - 5)(x - 6)$

d $(x - 2)(x - 21)$ **e** $(x - 6)(x - 9)$ **f** $(x - 25)(x - 4)$

3 a $2(x + 1)(x + 2)$ **b** $6(x - 3)(x - 1)$ **c** $5(x - 2)(x + 1)$

d $2(x + 2)(x + 5)$ **e** $2(x - 1)(x + 3)$ **f** $3(x + 1)(x - 11)$

WORK IT OUT 7.1

Option B is correct.

Option A is wrong: the brackets should be $(13 + x)(13 - x)$.

Option C is wrong: one bracket must include a '+', one must include a '−'.

EXERCISE 7E

1 a $(x + 6)(x - 6)$ **b** $(p + 9)(p - 9)$

c $(w + 4)(w - 4)$ **d** $(p + 6q)(p - 6q)$

e $(12s + c)(12s - c)$ **f** $(8h + 7g)(8h - 7g)$

2 a $2(2x + y)(2x - y)$ **b** $3(xy + 2z)(xy - 2z)$

c $^{-}4(x - 2)(x - 1)$ **d** $3(x + 2)(x + 6)$

e $7(x - 5 + y)(x - 5 - y)$ **f** $(x + y + 8)(x - y + 2)$

3 a $(100 - 97)(100 + 97) = 3 \times 197 = 591$

b $(50 - 48)(50 + 48) = 2 \times 98 = 196$

c $(639 - 629)(639 + 629) = 10 \times 1268 = 12\,680$

d $(98 - 45)(98 + 45) = 53 \times 143 = 7579$

e $(83 - 77)(83 + 77) = 6 \times 160 = 960$

f $(1234 - 999)(1234 + 999) = 235 \times 2233 = 524\,755$

4 a $(17 - 15)(17 + 15) = 2 \times 32 = 64$ $a = \sqrt{64} = 8$

b $(15 - 4)(15 + 4) = 11 \times 19 = 209$ $a = \sqrt{209}$

c $(20 - 14)(20 + 14) = 6 \times 34 = 204$ $a = \sqrt{204}$

d $(20 - 14.5)(20 + 14.5) = 5.5 \times 34.5 = 189.75$ $a = \sqrt{189.75}$

EXERCISE 7F

1 a $(2x + 5)(x + 1)$ **b** $(3x + 4)(x + 2)$

c $2(x - 5)(x + 9)$ **d** $(2x + 5)(2x + 3)$

e $(2x - 9)(2x - 3)$ **f** $3(x - 7)(x + 5)$

g $3(2x - 1)(2x + 3)$ **h** $(3x - 5)(x + 2)$

i $2(x + 10)(x - 13)$ **j** $(3x + 2)(x - 5)$

2 $5x^2 - 13x + 6$ factorises into $(5x - 3)(x - 2)$, so the length is $(5x - 3)$ cm.

3 $2x^2 + 11x + 12$ factorises into $(2x + 3)(x + 4)$.
$(x + 4)$ is half the base, so $(2x + 3)$ is the height.

4 a Let $a = (x + y)$.
The expression becomes $3a^2 + 13a + 12$, which factorises into $(3a + 4)(a + 3)$.
Replacing a with $(x + y)$: $3(x + y)^2 + 13(x + y) + 12 \equiv [3(x + y) + 4](x + y + 3)$.

b i $(3x - 1)(x - 6)$ **ii** $(20x + 7)(10x + 1)$

EXERCISE 7G

1 a 9 **b** 36 **c** $\frac{25}{4}$

d $\frac{49}{4}$ **e** 36 **f** 25

2 a $x^2 - 2x + 1 \equiv (x - 1)^2$ **b** $x^2 + 2x + 1 \equiv (x + 1)^2$

c $x^2 + 4x + 4 \equiv (x + 2)^2$ **d** $x^2 + 6x + 9 \equiv (x + 3)^2$

e $x^2 - \frac{2}{3}x + \frac{1}{9} \equiv (x - \frac{1}{3})^2$ **f** $x^2 - 2\sqrt{5}x + 5 \equiv (x - \sqrt{5})^2$

g $x^2 - 10x + 25 \equiv (x - 5)^2$ **h** $x^2 - 2\sqrt{11}x + 11 \equiv (x - \sqrt{11})^2$

i $x^2 + 2\sqrt{7}x + 7 \equiv (x + \sqrt{7})^2$

3 a $(x + 1)^2 - 6 \equiv (x + 1 + \sqrt{6})(x + 1 - \sqrt{6})$

b $(x + 1)^2 + 6$

c $(x + 2)^2 - 3 \equiv (x + 2 + \sqrt{3})(x + 2 - \sqrt{3})$

d $(x + 3)^2 - 12 \equiv (x + 3 + 2\sqrt{3})(x + 3 - 2\sqrt{3})$

e $(x - 3)^2 - 3 \equiv (x - 3 + \sqrt{3})(x - 3 - \sqrt{3})$

f $(x - 4)^2 - 21 \equiv (x - 4 + \sqrt{21})(x - 4 - \sqrt{21})$

g $(x + 4)^2 + 9$

h $(x + 6)^2 - 47 \equiv (x + 6 + \sqrt{47})(x + 6 - \sqrt{47})$

i $(x + \frac{11}{2})^2 - \frac{93}{4} \equiv (x + \frac{11}{2} + \frac{\sqrt{93}}{2})(x + \frac{11}{2} - \frac{\sqrt{93}}{2})$

WORK IT OUT 7.2

1 $\frac{3x+2}{2x+3}$

x is not a factor of the numerator or the denominator, so the fraction cannot be simplified.

2 $\frac{x^2-x-6}{x^2+3x+2}=\frac{(x-3)\cancel{(x+2)}}{(x+1)\cancel{(x+2)}}=\frac{x-3}{x+1}$

x^2 cannot be cancelled as it is not a factor of either expression.

3 The expressions must be factorised before common factors are cancelled.

$\frac{x^2-1}{2x+4}\times\frac{4x^2-16}{x+1}=\frac{\cancel{(x+1)}(x-1)}{\cancel{(2x+4)}}\times\frac{\cancel{(2x+4)}(2x-4)}{\cancel{(x+1)}}=(x-1)(2x-4)$

EXERCISE 7H

1 a $\frac{4x}{5}$ **b** 3 **c** $x-3$

d $2x+9$ **e** $\frac{x-3}{x+4}$ **f** $\frac{x+4}{4-x}$

g $\frac{x-1}{x+1}$ **h** $\frac{^-5(5+x)}{3x+4}$

2 a $\frac{2x}{5}$ **b** $\frac{5x}{12}$ **c** $\frac{13}{6x}$

d $\frac{3x+5}{(x+1)(x+2)}$ **e** $\frac{3x+1}{(x-2)(x-1)}$ **f** $\frac{15-2x}{(x-7)^2}$

3 a $\frac{5x+3}{x(x+1)}$ **b** $\frac{x^2-x+6}{(x-2)(x+2)}$

c $\frac{2x+5}{(x+2)(x+3)}$ **d** $\frac{4x}{(2x-3)(2x+3)}$

e $\frac{7p+1}{p(2p+1)}$ **f** $\frac{2p^2+2p+5}{(p-1)(p+2)}$

g $\frac{3x-7}{(x-1)(x-2)(x-3)}$ **h** $\frac{3+x+y}{(x+y)(x-y)}$

EXERCISE 7I

1 a False: Incorrect expansion. Should be $16b-9$. **b** True

c False: Incorrect expansion of brackets.
Should be $9x^2+15x+6$. Or incorrect factorisation, should be $(3x+2)(x+3)$.

d True **e** True **f** False: Numerator should be $8x+7$.

2 a $x^2-25=(x-5)(x+5)$ and $(x-5)\neq(x+5)$. Assuming the sides have to be an expression in terms of whole xs

b Yes: $x^2+10x+25=(x+5)(x+5)$ and $(x+5)\equiv(x+5)$

3 $32x$

4 $(^-2x^2-13x-15)=^-(2x^2+13x+15)=^-(2x+3)(x+5)$, so $(x-1)$ is not a factor

5 $(2x-y)^2-(x+y)(x-y)=(2x-y)^2-(x^2-y^2)=3x^2-4xy+2y^2$

6 a $6x^2+x-2$ **b** $13x^2+8x+5$ **c** $5x^2+16x+3$

7 a $(1.01)^2=(1+0.01)^2=1^2+2\times1\times0.01+0.01^2$
$=1+0.02+0.0001=1.0201$

b $(0.99)^2=(1-0.01)^2=1^2-2\times1\times0.01+0.01^2$
$=1-0.02+0.0001=0.9801$

c $(4.02)^2=(4+0.02)^2=4^2+2\times4\times0.02+0.02^2$
$=16+0.16+0.0004=16.1604$

d $(0.98)^2=(1-0.02)^2=1^2-2\times1\times0.02+0.02^2$
$=1-0.04+0.0004=0.9604$

8 a $x^2+4x+15=(x+2)^2+11\geqslant11$ as $(x+2)^2\geqslant0$

b $x^2+2x+15=(x+1)^2+14\geqslant14$ as $(x+1)^2\geqslant0$

9 Triangle is right-angled if $(x+6)^2+(x-1)^2=(x+8)^2$
Expanding and simplifying gives $x^2-6x-27=0$
Factorising gives $(x-9)(x+3)=0$
$x>1$, so $x=9$ is the only solution.

10 a If perimeter = 20 cm and width is w, length is $\frac{(20-2w)}{2}=10-w$, so area $=w(10-w)$

b $10w-w^2=^-(w-5)^2+25$
As $^-(w-5)^2\leqslant0$, area $\leqslant25$

11 Students' own answer. A possible proof is shown below:
$(x-1)^2\geqslant0$
$x^2-2x+1\geqslant0$
divide by x, as $x>0$ this gives us:
$x-2+\frac{1}{x}\geqslant0$
$x+\frac{1}{x}\geqslant2$
So a positive number plus its reciprocal is greater than or equal to 2.

12 a $\frac{x+2}{x-5}$ **b** $\frac{3x+2}{4}$ **c** $\frac{b^2(a+1)}{a^2(a-b)}$ **d** $\frac{^-(x+3)}{3}$

CHAPTER REVIEW

1 a $3y^2+12y+14$ **b** x^3-7x-6

2 a $(2x+3)(x-7)$ **b** $-2(3x+4)(x+1)$

3 3997

4 Area $=(x-5)(x+2)+2(x-5)=36$
$x^2-3x-10+2x-10=36$
$x^2-x-56=0$
or
Area $=0.5(x-5)(2x+8)$
$36=0.5\,(2x^2-2x-40)$
$36=x^2-x-20$
$x^2-x-56=0$

5 a $\frac{7x-11}{12}$ **b** $\frac{3x+4}{(x+4)(x-4)}$ **c** $\frac{x^2}{6y}$

d $\frac{3(5p-q)}{(4p+q)(p-2q)}$ **e** $\frac{4-17x}{(x-2)(1-3x)}$ **f** $\frac{3x}{(x+5)(x-5)}$

6 a $(x-3)^2-7$, $a=^-3$ and $b=^-7$ **b** $^-7$

8 Equations

BEFORE YOU START ...

1 a D **b** B **c** A **d** C

2 A $6x+1=37$

3 a $7+[^-7]=0$ **b** $[8]-8=0$

c $^-4a+[4a]=0$ **d** $5\times\left[\frac{1}{5}\right]=1$

e $\frac{1}{6}\times[6]=1$ **f** $\left[\frac{1}{12}\right]\times12x=x$

4 a C $(x-2)(x-3)$ **b** A $x(x+3)$

c B $(x+5)(x-5)$ **d** D $(x+\sqrt{5})(x-\sqrt{5})$

5 a $x^2+4x+10=(x+[2])^2+[6]$

b $x^2-8x-5=(x-[4])^2-[21]$

LAUNCHPAD

1 a D $x=12$ **b** B $x=17$ **c** A $x=1$

d E $x=^-3$ **e** C $x=^-2$

f You can check whether a solution is correct by substituting it back into the equation.

2 a $a=5$ **b** $x=-3$ **c** $a=3$

d $x=7$ or $x=^-2$

3 They are equivalent. Both equations have a solution of $x=3$

4 $2x+16=44$ $x=14$

5 $x=3$ or $x=^-1$

6 $x=\pm4$

7 $(x-3)^2 - 11 \equiv x^2 - 6x - 2$

8 $a = 1, b = -6, c = -2$; $x = 6.32$ or $x = -0.32$

9 a $x + y = 6$ has positive integer solutions:
$x = 1, y = 5$; $x = 2, y = 4$; $x = 3, y = 3$; $x = 4, y = 2$; $x = 5, y = 1$

b $x = 5$ and $y = 1$ are the only pair that satisfy the pair of equations simultaneously.

10 a £100 **b** $x = 9$ **c** $y = 115$

11 2 solutions; approximate solution is $x = 4.56$

EXERCISE 8A

1 a $3\frac{1}{2}$ **b** 2 **c** 4 **d** $\frac{1}{2}$ **e** 1 **f** $\frac{1}{3}$

2 a 12 **b** 2 **c** 2 **d** 1 **e** $\frac{-13}{6}$ **f** $\frac{15}{4}$

3 a i $x = 0$.
ii The equation works for all values of x.

b ii is an identity because the equation is true for any value of x.

c If $x = 0$ the equation is true.

EXERCISE 8B

1 a $\frac{9}{8}$ **b** $\frac{-3}{5}$ **c** $\frac{23}{5}$ **d** $\frac{13}{2}$ **e** 7 **f** -5

g 12 **h** -2.8 **i** 2 **j** $\frac{-184}{83}$ **k** 1 **l** $\frac{-1}{7}$

EXERCISE 8C

1 a $3x = 348$; $x = 116$

b $x - 7 = -2$; $x = 5$

c $x + 6 = -4$; $x = -10$

d $4x - 2 = 66$; $x = 17$

e $x + x + 1 = 63$; $x = 31$; numbers are 31 and 32.

f $2x - 3 = -2$; $x = \frac{1}{2}$

g $2x + 1\frac{1}{2} = 4\frac{3}{4}$; $x = \frac{13}{8}$

2 a Lucy is 14. **b** £2.40 **c** 6

d Daughter is 15 **e** 13 **f** 36

3 a $12x$ cm **b** $(6x + 18)$ cm

c $6x + 18 = 12x$; $x = 3$; side of square is 9 cm

4 $4x + 2 = 10x - 1$; $x = \frac{1}{2}$; length = 4 cm, width = 2.5 cm

5 A = 80°, B = 60°, C = 40°

6 37 and 46

7 15 each of apple and orange juice

EXERCISE 8D

1 a $x = -9$ or $x = -3$ **b** $x = 6$ or $x = -5$ **c** $x = \frac{-5}{6}$ or $x = 2$

d $x = \frac{5}{9}$ or $x = -1$ **e** $x = 0$ or $x = -3$ **f** $x = 0$ or $x = \frac{4}{5}$

g $x = 10$ or $x = -10$ **h** $x = \pm\sqrt{5}$ **i** $x = \pm\sqrt{6}$

2 No, it's a sum rather than a difference of two squares.

3 a $x = 8$ or $x = -4$ **b** $x = \frac{-4}{3}$ or $x = \frac{3}{2}$ **c** $x = 8$ or $x = 4$

d $x = 2$ or $x = -4$ **e** $x = 5$ or $x = -6$ **f** $x = \frac{3}{2}$ or $x = 4$

g $x = 3$ or $x = \frac{-1}{2}$ **h** $x = \frac{-7}{6}$ or $x = \frac{-5}{4}$

EXERCISE 8E

1 a $x = 3.70$ or -2.70 **b** $x = 1.37$ or -4.37

c $x = 0.16$ or -6.16 **d** $x = 6.75$ or -7.25

e $x = 1.89$ or 0.11 **f** $x = 5.37$ or -0.37

g $x = 1.30$ or -2.30 **h** $(x = 3)^2 = -4$, we cannot find a real solution to this equation

i $x = 1.62$ or -0.62

2 a $x = 2.28$ or 0.219 **b** $x = 0.631$ or 0.227

c $x = 0.879$ or -0.379 **d** $x = 1.35$ or -2.95

e $x = -2.84$ or -9.16 **f** $x = 6.85$ or 0.146

3 a $x = \frac{-5 \pm \sqrt{5}}{2}$ **b** $x = -1 \pm \sqrt{5}$ **c** $x = -6 \pm \sqrt{33}$

d $x = \frac{-1 \pm \sqrt{22}}{3}$ **e** $x = \frac{-3 \pm \sqrt{29}}{10}$ **f** $x = \frac{3 \pm \sqrt{5}}{4}$

EXERCISE 8F

1 a $x(x + 4) = 140$; $x = -14$ or $x = 10$

b $x(x - 3) = 108$; $x = 12$ or $x = -9$

c $x^2 - 3x = 10$; $x = 5$ or $x = -2$

d $x(x + 2) = 48$; $x = 6$; There are no odd numbers that multiply to give an even number – this is not possible. Even solutions to this question would be 6 and 8.

2 $x = 7$, we can ignore $x = 0$ as this result would not be a triangle.

3 10 cm, we can ignore the result of 45 cm as we cannot cut squares of this size in reality.

4 3 m, modelling the path as square at the corners.
3.10 m, modelling the path as rounded at the corners.

5 $x = 5$, so dimensions of the two rectangles are 18 cm by 2 cm and 9 cm by 4 cm, we can ignore $x = -0.5$ as this result would give us a negative value for one of the edges.

6 160 m and 90 m

EXERCISE 8G

1 a $x = -3, y = -5$ **b** $x = \frac{-1}{2}, y = 5$ **c** $x = 1, y = 2$

d $x = 4, y = 2$ **e** $x = 2, y = 5$ **f** $x = -1, y = 3$

2 a $x = 3, y = 4$ **b** $x = 1, y = 2$ **c** $x = 3, y = 4$

d $x = 7, y = -4$ **e** $x = \frac{-11}{3}, y = 17$ **f** $x = -2, y = 4$

EXERCISE 8H

1 a $x = 4, y = 2$ **b** $x = 3, y = 1$ **c** $x = 2, y = -2$

d $x = \frac{26}{7}, y = \frac{-39}{7}$ **e** $x = \frac{28}{5}, y = \frac{21}{5}$ **f** $x = -1, y = -2$

g $x = 2, y = 3$ **h** $x = 1, y = 3$ **i** $x = 4, y = 1$

2 a $x = 2, y = 3$ **b** $x = 1, y = 1$ **c** $x = 10, y = 5$

d $x = -5, y = -2$ **e** $x = -2, y = 5$ **f** $x = 2, y = -1$

3 a $x = 5, y = 0$

b $x = 3, y = 1$ **c** $x = -2, y = 1$ **d** $x = \frac{45}{17}, y = \frac{38}{17}$

e $x = 3, y = -1$ **f** $x = -1, y = 1$

EXERCISE 8I

1 $a = 70, b = 50$

2 62 and 14

3 £6.20

4 Fizzers cost 20p; Toffees cost 30p

5 Three 5p pieces and fifteen 10p pieces.

6 45 and 219

7 Flash drives £10 and hard drives £25

8 48 blocks (36 blocks of 450 and 12 of 400)

EXERCISE 8J

1 a $x = 1, y = 1$ **b** $x = 2, y = 4$ or $x = -1, y = 1$ **c** no solution

d The two lines will not intersect so there is no solution to the pair of equations.

2 a $(-1, 1)$ **b** $(1, 8)$ or $(-5, 2)$

c $(1, 7)$ or $(-2, 4)$ **d** $(0, 1)$ or $\left(\frac{-1}{2}, 0\right)$

e $(1, 6)$ or $\left(\frac{-1}{3}, 2\right)$ **f** $\left(\frac{-2}{3}, \frac{5}{3}\right)$ or $\left(\frac{-1}{2}, 2\right)$

3 $(1, 4)$ and $(4, 1)$

EXERCISE 8K

1 a 5 miles **b** ~48 minutes **c** 10 miles/hour

2 a ~47 minutes **b** 30 km **c** 90 km/hour

3 a $^{-}2$ **b** 1 **c** 4

4 a 2000 litres **b** 100 minutes
c Students' chosen points and explanations

5 a When 500 units have been sold, and the total revenue is £6 000.
b It tells the business owner how many units must be sold in order to make a profit.

6 $x = 2, y = 4$

7 a y-axis: height of the jump, in metres. x-axis: time in seconds, or distance horizontally (in metres).
b $x = 0$ and $x = 11.5$ seconds.
c These are the roots of the quadratic function.
d ~5.2 m.

8 a The roots of the equation are the values where the graph crosses the x-axis.
b $x = 2$ and $x = 6$
c $y = {}^{-}x^2 + 8x - 12$
d (8.16, $^{-}$13.32) and (1.84, $^{-}$0.68)

9 a Approx $x = 3.2$, $y = 4.1$
b $x = 3.25$, $y = 4.125$
c The accuracy is limited by how accurate the graph is and how well the values can be read from the scale.

10 a 108 cm **b** 95 cm **c** 135 cm

11 a 9 kg **b** 0.057 m
c Approximate final height of 16 kg pendulum = 0.105 m
Approximate final height when there is no weight on the end of the string = 0.025 m
Range = 0.105 − 0.25 = 0.08 m

EXERCISE 8L

1 $x \approx 4.562$

2 $x \approx 4.243$

3 $x \approx 2.56$

4 $x \approx 0.3820$

5 $x \approx 2.29$

EXERCISE 8M

1 $x \approx 0.45$

2 a $x \approx 2.73$ **b** $x \approx 0.7$

EXERCISE 8N

1 a True
b False; $x = \frac{1}{2}$ and $x = 2$
c True, to complete the square of the current equation you would need to subtract 30.25 but for it to be a perfect square you would need to add this value. i.e. $x^2 - 11x + 30.25$ is a perfect square.
d False; $x = \frac{^{-}63}{19}$, $y = \frac{30}{19}$
e False; one solution has a negative value.

2 a $x = 16$ **b** $x = {}^{-}9$

3 a $(x - 5)$ m **b** length = 17.5 m, width = 12.5 m

4 Daughter is 8, father is 36

5 5

6 a Area $= (2y + 1)y + 5y = 2y^2 + 6y$
$2y^2 + 6y = 95$ so $2y^2 + 6y - 95 = 0$
b $y = 5.55$

7 a $x = 4.45$ or $x = {}^{-}0.45$ **b** $n = 5.70$ or $n = {}^{-}0.70$

8 a students' working **b** $x = 1.22$ or $x = {}^{-}3.55$

9 $x = 5$

10 a $a = \frac{4}{3}$ **b** $a < \frac{4}{3}$ **c** $a > \frac{4}{3}$

11 $x + y = 112$ and $x - y = 22$
$x = 67$ and $y = 45$

CHAPTER REVIEW

1 a $x = 7$ or $x = {}^{-}3$ **b** $x = {}^{-}2$ or $x = {}^{-}6$ **c** $x = {}^{-}4$ or $x = {}^{-}5$

d $x = {}^{-}5 \pm \sqrt{30}$ **e** $x = 1 \pm \frac{\sqrt{6}}{2}$ **f** $x = \frac{^{-}1 \pm \sqrt{10}}{3}$

2 a 2 seconds **b** when $t = 3$

3 a 10 and 60
b $y = {}^{-}20(x - 10)(x - 60)$ or $y = {}^{-}20x^2 + 1400x - 12\,000$
c Increasing the selling price increases the profit until the price is 35, but increasing it further decreases the profit.

4 No; one equation is a multiple of the other.

5 $x = 1.28$ and $y = 4.83$
$x = {}^{-}1.88$ and $y = {}^{-}4.63$

6 $x = \frac{^{-}3}{4}$ and $y = 8$ OR $x = 2$ and $y = {}^{-}3$

7 $(x + 1)^2 - x^2 = x^2 + 2x + 1 - x^2$
$= 2x + 1$
$= x + (x + 1)$

8 $^{-}2$ or 8

9 a Students' working **b** $x = 0.826$ or $x = {}^{-}2.83$

10 a 2.414 **b** Students' substitutions

9 Angles

BEFORE YOU START ...

1 a 84 **b** 64 **c** 187 **d** 208

2 a Isosceles right-angled triangle
b Rhombus **c** Rectangle

3 $x = y$ as the triangle formed by the diagonals is isosceles.

4 $a = 35°$, $b = 120°$

LAUNCHPAD

1 a 72°, angles on a straight line sum to 180°
b 60°, angles about a point sum to 360°
c 108°, vertically opposite angles are equal

2 No. Angles on a line are 180°.

3 Choose pairs from $a = c = e = g$, $b = d = f = h$.

4 $c = f$ only if the red line is perpendicular to the two parallel lines

5 44°; angle BCA = 68°; angle CAB = angle BCA= 68°, so $x = 180 - 68 - 68 = 44°$

EXERCISE 9A

1 a 188° **b** 110° **c** 95°
d Reflex; obtuse; obtuse

2 a $x = 49°$, $y = 80°$ **b** $x = 50°$ **c** $p = 60°$

3 Angle ABC = 50 + 65 + 59 = 174°; if AC is a straight line, angle ABC must be 180°.

4 a $a = c = 137°$, $b = 43°$
b $z = 47°$, $w = 84°$, $x = y = 49°$

5 $z = 200°$

6 a $x = 68°$, $y = 68°$, $a = 112°$, $b = 112°$
b $x = 90°$, $y = 90°$, $a = 90°$, $b = 90°$
c $x = y = 146°$, $a = b = 34°$
d $x = 60°$, $y = 60°$, $a = 120°$, $b = 120°$

7 a $x = 20°$ (Vertically opposite angles are equal)
b $x = 68°$ (Angles on a straight line), $y = 26°$ (Angles on a straight line), $z = 94°$ (Angles in a triangle)
c $x = 20°$ (Angles about a point)

EXERCISE 9B

1 $a = 137°$, $b = 137°$, $c = 137°$, $d = 43°$
2 $x = 80°$
3 $a = 125°$, $b = 55°$, $c = 35°$
4 $x = 130°$, y = 130°
5 $a = 138°$, $b = 42°$, $c = 138°$, $d = 138°$, $e = 42°$, $f = 138°$, $g = 42°$
6 $\angle CEG = 32°$
7 $\angle DCF = 54°$
8 a $x = 30°$ **b** $x = 50°$
9 a AB is parallel to DC **b** AB is not parallel to DC
c AB is parallel to DC

EXERCISE 9C

1 a $x = 55°$ **b** $c = 70°$
2 $x = 75°$, $y = 105°$
3 $a = 40°$, $b = 50°$
4 $x = 130°$, $y = 145°$, $z = 85°$
5 $a = 48°$, $b = 84°$, $c = 96°$
6 $x = 18\frac{1}{3}°$
7 $\angle ECD = y$ (corresponding angles), $\angle ACE = x$ (alternate angles)
So $\angle ACD = x + y$
8 a $x = 75°$, $y = 75°$, $z = 35°$
b Assuming AB = AC, $x = 20°$
c $z = 70°$
d $x = 50°$
e $x = 85°$
Students' own reasons

WORK IT OUT 9.1

Option A is correct.
In Option B, the pentagon has not been divided into triangles correctly.
In Option C, 360° has been added unnecessarily.

EXERCISE 9D

1 Students' drawings
2 Students' predictions
3

Number of sides in polygon	3	4	5	6	7	8	10	20
Number of triangles	1	2	3	4	5	6	8	18
Angle sum of interior angles	180°	360°	540°	720°	900°	1080°	1440°	3240°

4 The number of triangles is two less than the number of sides.
5 $n - 2$
6 a 180 times two less than n
b $180(n - 2)$
7 1800°
8 Interior angle = sum of interior angles ÷ 12
Exterior angle = 180 − interior angle

EXERCISE 9E

1 a 1260° **b** 2700° **c** 4140°
2 a 2340° **b** 360° **c** 156° **d** 24°
3 a 45° **b** 135° **c** Octagon
4 $x = 37°$, $y = 97°$, $z = 92°$
5 137° **6** 55°, 110°
7 No. The sum of its exterior angles must equal 360°, so $55 \times n = 360$. This gives $n = 6.55$, n is not an integer so there is no regular polygon with interior angles of 125°
8 a 11 **b** 19
9 1260°

CHAPTER REVIEW

1 a 270° **b** 120° **c** 61°
d 55° **e** 92° **f** 162°
2 Pentagon **3** 360°
4 $z = 105°$ **5** 10 sides, decagon
6 36°
7 I = sum of interior angles, E = sum of exterior angles
We know $I = 180(n - 2)$ formula for sum of interior angles
$I + E = 180n$ the sum of each interior and exterior is 180 and there are n angles
So $E = 180n - I = 180n - 180(n - 2) = 180n - 180n + 360 = 360$
8 $x = 85°$, $y = 15°$
9 Angle x is the same as angle BAC (alternate angles).
Angle ABC equals 180 − 125 = 55° (sum of angles on a straight line is 180°).
Angle ACB equals 80° (corresponding angles).
Angle x = 180 − 55 − 80 = 45° (sum of interior angles in a triangle is 180°).
OR
Angle BDE equals 180 − 125 = 55° (co-interior/complementary angles sum to 180°).
80 + x + 55 = 180 (co-interior/complementary angles sum to 180°).
So $x = 45°$.
10 Sum of interior angles = 360°, adding all angles give $18x = 360$, so $x = 20°$
$\angle ABC = 18$, $\angle BCD = 162$
These are co-interior angles and they sum to 180°, so AB and DC are parallel. "$\angle BAD = 82°$. $\angle BAD$ and $\angle ABC$ are co-interior angles and do not sum to 180°, so BC is not parallel to AD." ABCD is therefore a trapezium.
11 a 22 sides **b** $x = 109.09°$

10 Fractions

BEFORE YOU START ...

1 a 18, 27 and 36 **b** 18, 24, 30 and 36
c 24 and 36 **d** 24 and 32
2 a The mistake was adding the numerators and denominators.
Correct answer: $\frac{17}{12} = 1\frac{5}{12}$
b The mistake was doing $\frac{9}{10} - \frac{4}{5}$ instead. Correct answer: $\frac{^-1}{10}$
c The mistake was to add the numerators rather than multiplying them. Correct answer: $\frac{8}{35}$
d The mistake was to multiply rather than divide. Correct answer: 60
3 a A **b** B **c** B Students' reason for each answer with reference to order of operations.

LAUNCHPAD

1 **a** $\frac{5}{35}$ **b** $\frac{9}{16}$ **c** $\frac{22}{10}$

2 **a** The mistake was adding the numerators and denominators.
Correct answer: $\frac{17}{12} = 1\frac{5}{12}$

b The mistake was doing $\frac{9}{10} - \frac{4}{5}$ instead. Correct answer: $\frac{-1}{10}$

c The mistake was to add the numerators rather than multiplying them. Correct answer: $\frac{8}{35}$

d The mistake was to multiply rather than divide.
Correct answer: 60

3 **a** $\frac{3}{5}$ of 60 **b** $\frac{7}{10}$ of 300 **c** $\frac{1}{2}$ of $\frac{3}{4}$

4 $\frac{13}{16}$

5 $\frac{1}{40}$

EXERCISE 10A

1 **a** $\neq$ **b** $\neq$ **c** $\neq$ **d** $\neq$
e $=$ **f** $=$ **g** $=$ **h** $=$

2 **a** $\frac{8}{32}$ **b** $\frac{48}{192}$ **c** $\frac{27}{108}$ **d** $\frac{13}{52}$

3 Students' own reasoning, but they should realise that they can make an equation and solve for x to find the unknown values.

4 **a** $\frac{1}{5}$ **b** $\frac{2}{3}$ **c** $\frac{1}{4}$ **d** $\frac{1}{2}$ **e** $\frac{1}{3}$ **f** $-\frac{1}{3}$
g $^{-}1\frac{1}{2}$ **h** $\frac{3}{5}$ **i** $\frac{2}{3}$ **j** $\frac{2}{3}$ **k** $\frac{5}{7}$ **l** $\frac{3}{7}$

5 **a** $\frac{1}{4}, \frac{4}{7}, \frac{3}{5}, 1\frac{3}{4}, \frac{9}{4}$ **b** $\frac{3}{4}, \frac{19}{24}, \frac{5}{6}, 2\frac{2}{3}, \frac{11}{3}$
c $\frac{1}{7}, \frac{10}{21}, \frac{8}{14}, \frac{7}{7}, \frac{13}{7}, 2\frac{3}{7}$

6 **a** **i** $\frac{4}{9}$ **ii** $\frac{13}{16}$

b First find the fraction between $\frac{1}{3}$ and $\frac{3}{4}$; this is $\frac{4}{7}$.
Then apply the rule to find the fraction between $\frac{1}{3}$ and $\frac{4}{7}$, and so on.
Three possible fractions are: $\frac{5}{10}, \frac{4}{7}$ and $\frac{7}{11}$.

c Students can place the results on a number line, or cross multiply to show that the fractions are larger/smaller than each other.

d Students' own research.
An internet search for mediant fractions will provide several interesting articles and some proofs.

EXERCISE 10B

1 **a** $\frac{3}{10}$ **b** $\frac{1}{45}$ **c** $\frac{1}{7}$ **d** $\frac{7}{15}$
e $^{-}\frac{1}{24}$ **f** $\frac{2}{5}$ **g** $\frac{4}{33}$ **h** $\frac{5}{28}$
i $\frac{21}{250}$ **j** $\frac{3}{88}$ **k** $\frac{7}{4} = 1\frac{3}{4}$ **l** $\frac{11}{14}$
m $\frac{3}{4}$ **n** $\frac{3}{8}$ **o** $\frac{4}{9}$ **p** $\frac{87}{12} = 7\frac{1}{4}$
q $\frac{49}{10} = 4\frac{9}{10}$ **r** $\frac{87}{20} = 4\frac{7}{20}$ **s** $\frac{232}{35} = 6\frac{22}{35}$ **t** 1
u $\frac{9}{56}$ **v** $^{-}\frac{10}{33}$ **w** $\frac{32}{25} = 1\frac{7}{25}$ **x** $\frac{15}{22}$

2 $4\frac{3}{4}$

3 $8\frac{5}{12}$

4 $6\frac{79}{126}$

5 **a** $4\frac{2}{9}$ **b** $\frac{4}{5}$ **c** $\frac{39}{7} = 5\frac{4}{7}$ **d** $4\frac{3}{4}$
e $\frac{5}{12}$ **f** $\frac{215}{72}$ **g** 0 **h** $\frac{11}{170}$
i $\frac{187}{9} = 20\frac{7}{9}$ **j** $\frac{55}{42} = 1\frac{13}{42}$ **k** $\frac{16}{35}$ **l** $6\frac{13}{54}$

6 $26\frac{5}{12}$ minutes (26 minutes and 25 seconds)

7 $\frac{2}{15}$ **8** $3\frac{5}{12}$ m **9** $\frac{7}{18}$ of a minute

10 $\frac{7}{24}$ **11** $42\frac{1}{2}$ cups **12** 40 meals

13 184 m **14** $28\frac{20}{27}$ m **15** $4\frac{11}{12}$ cm

16 $2\frac{5}{14}$ m **17** **a** $\frac{7}{36}$ **b** 252 pages

EXERCISE 10C

1 **a** 9 **b** 15 **c** 8 **d** 54
e 144 **f** 32 **g** $\frac{3}{8}$ **h** $\frac{1}{10}$
i $\frac{2}{21}$ **j** $\frac{5}{8}$ **k** $\frac{7}{4} = 1\frac{3}{4}$ **l** $\frac{35}{12} = 2\frac{11}{12}$

2 **a** £21 **b** £126 **c** £12 **d** £12
e $1\frac{1}{2}$ cups **f** $2\frac{1}{2}$ cups **g** $\frac{3}{4}$ cup **h** $\frac{7}{4} = 1\frac{3}{4}$
i 1 cup **j** $\frac{8}{3} = 2\frac{2}{3}$ hours **k** $\frac{5}{6}$ hour
l $\frac{15}{4} = 3\frac{3}{4}$ hours **m** $\frac{1}{4}$ hour
n $\frac{7}{3} = 2\frac{1}{3}$ min **o** 12 seconds

3 **a** $\frac{3}{25}$ **b** $\frac{7}{40}$ **c** $\frac{1}{25}$ **d** $\frac{3}{32}$
e $\frac{1}{12}$ **f** $\frac{3}{20}$ **g** $\frac{1}{40}$ **h** $\frac{23}{160}$

4 $\frac{3}{4}$

5 $\frac{11}{75}$

6 General reference 1200, technology 3600, engineering 960, computers 2640.

7 Week 1: 250 m, week 2: 900 m and week 3: 350 m.

8 10 000 first class, 15 000 business class, 22 500 economy and 12 500 no frills.

EXERCISE 10D

1 **a** $\frac{1}{2} + \frac{1}{8}$ **b** $\frac{1}{2} + \frac{1}{10}$ **c** $\frac{1}{4} + \frac{1}{28}$
d $\frac{1}{5} + \frac{1}{45}$ **e** $\frac{1}{2} - \frac{1}{5}$

2 $\frac{1}{4} + \frac{1}{10} + \frac{1}{20}$

3 $\frac{1}{8}$ (factors 1, 2, 8)

4 Yes.
Students can investigate this and may find the theorem that proves this if they are interested.

CHAPTER REVIEW

1 **a** $\frac{1}{6}$ **b** $\frac{39}{46}$ **c** $4\frac{3}{8}$

2 **a** $\frac{3}{7}, \frac{4}{5}, \frac{5}{6}, \frac{8}{9}$ **b** $1\frac{3}{5}, \frac{16}{9}, 2\frac{2}{5}, \frac{23}{7}$

3 **a** $\frac{51}{40} = 1\frac{11}{40}$ **b** $\frac{41}{40} = 1\frac{1}{40}$ **c** $\frac{168}{15} = 11\frac{1}{5}$
d $\frac{194}{35} = 5\frac{19}{35}$ **e** $\frac{22}{15} = 1\frac{7}{15}$ **f** $\frac{53}{44} = 1\frac{9}{44}$
g $\frac{8}{189}$ **h** $\frac{769}{21} = 36\frac{13}{21}$ **i** $\frac{1}{24}$

4 **a** $\frac{35}{78}$ **b** $\frac{119}{3} = 39\frac{2}{3}$ **c** $11\frac{107}{180}$ **d** $\frac{187}{9} = 20\frac{7}{9}$

5 **a** $\frac{17}{100}$ **b** $\frac{3}{10}$

6 $16\frac{2}{3}$ bottles (16 full bottles)

7 30 plots

8 $3\frac{33}{40}$ m deep

11 Decimals

BEFORE YOU START ...

1 **a** 300.098 **b** 0.0398 **c** 19.308
d 0.98308 **e** 13.098

2 **a** < **b** > **c** < **d** > **e** =

3 **a** $\frac{4}{16}$ **b** $\frac{3}{8}$ **c** $\frac{2}{5}$ **d** $\frac{3}{4}$ **e** $\frac{45}{90}$ **f** $\frac{1}{40}$

4 **a** 0.24 **b** ≈ 18 **c** ≈ 4

LAUNCHPAD

1 There are many possible answers. For example:
a 2.155 **b** 2.1555 **c** 0.67535

2 **a** $\frac{9}{100}$ **b** $\frac{2}{1000}$ **c** $\frac{8}{10}$

3 **a** $3\frac{1}{4}$ **b** $\frac{8}{9}$ **c** 0.99

4 **a** C: 21.65 **b** C: 16.05 **c** C: 6.75
d B: 73.28 **e** A: 1.248 **f** C: 19.45

5 **a** $\frac{2}{9}$ **b** $\frac{18}{99} = \frac{2}{11}$ **c** $\frac{120}{99} = \frac{40}{33}$

EXERCISE 11A

1 **a** $\frac{3}{5}$ **b** $\frac{21}{25}$ **c** $1\frac{16}{25}$ **d** $\frac{77}{200}$ **e** $\frac{1}{8}$
f $1\frac{2}{25}$ **g** $\frac{7}{8}$ **h** $\frac{1}{125}$ **i** $3\frac{8}{125}$ **j** $\frac{333}{1000}$

2 **a** 0.6 **b** 0.75 **c** 0.72 **d** 0.95
e 0.68 **f** 0.44 **g** 0.445 **h** 0.152
i 9.25 **j** 2.9 **k** $1.8\dot{3}$ **l** 0.375
m 2.25 **n** $0.\dot{8}$ **o** 2.375

3 **a** Repeating digits of the numerator
b recurring non-terminating decimals
c $0.1\dot{6}$, $0.\dot{3}$, 0.5, $0.\dot{6}$, $0.8\dot{3}$
d $0.\dot{0}\dot{9}$, $0.\dot{1}\dot{8}$
e Should predict $0.\dot{2}\dot{7}$ and $0.\dot{3}\dot{6}$ based on multiples of 9.

4 **a** 8.62, 5.29, 5.2, 4.92, 4.09
b 7.42, 3.219, 0.76, 0.742, 0.421
c 14.89, 14.72, 14.3, 14.07, 14.009
d 0.287, 0.273, 0.26, 0.23, 0.206
e 0.68, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{5}{11}$, 0.45, 0.403
f 0.88, $\frac{7}{9}$, 0.718, 0.625, $\frac{3}{8}$

5 **a** < **b** < **c** < **d** = **e** >
f < **g** > **h** > **i** <

6 There are many possible answers.
a 3.1355 **b** 0.66455 **c** 4.9985

7 **a** Steel Dragon
b California Screaming
c shorter
d Steel Dragon and The Ultimate
e 2.479, 2.268, 2.243, 2.045, 2.0, 1.851

WORK IT OUT 11.1

This is an investigative activity. Students will provide their own answers and explanations.

EXERCISE 11B

1 **a** 1.58 **b** 1.67 **c** 1.7
d 13.35 **e** 22.714 **f** 34.335

2 **a** 66.05 **b** 23.76 **c** 3.61 **d** 22.43
e 332.907 **f** 29.695 **g** 23.959 **h** 78.6
i 109 520 **j** 0.8021 **k** 205.6158 **l** 0.03
m 0.0895 **n** 8.15 **o** 793

3 **a** 1.36 sec **b** 0.54 sec **c** 9.21 sec
d Unlikely. The first runner will start from still so will probably take longer than the other runners who start from a running position.

4 Yes, she has 1.54 litres

5 166.67 mg vitamin C, 7.8 mg boron and 36.85 mg calcium

6 **a** 180.25 kW h **b** 9398.75 kW h

7 £21.52

8 185.9 km

9 42 (with some juice left over)

10 1800

11 69

12 £8.58

13 **a** £78.44
b £392.20
c £550.96
d Toni will not work 365.25 days per year, so the answer to part **a** is not realistic. Her salary will be split equally into 52 weeks, as in part **c**: this will include holiday pay, etc.

EXERCISE 11C

1 Nazeem's hypothesis is correct as long as the original fraction is in its simplest terms.

2 $\frac{3}{6}$ is not in its simplest terms; when reduced to $\frac{1}{2}$ it fits Nazeem's original hypothesis.

EXERCISE 11D

1 **a** 0.375 **b** 0.3125 **c** $0.4\dot{5}$ **d** $0.\dot{4}$
e $2.\dot{5}71428\dot{5}$ **f** $0.4\dot{6}$ **g** $3.\dot{2}85714\dot{2}$ **h** $1.\dot{1}42857\dot{1}$

2 **a** $\frac{8}{9}$ **b** $\frac{25}{9}$ **c** $\frac{812}{99}$ **d** $\frac{35}{11}$
e $\frac{41}{6}$ **f** $\frac{65}{99}$ **g** $\frac{5}{18}$ **h** $\frac{19}{11}$

3 **a** $\frac{4}{9}$ **b** $\frac{74}{99}$ **c** $\frac{79}{90}$ **d** $\frac{103}{900}$
e $\frac{943}{999}$ **f** $\frac{928}{4995}$ **g** $\frac{169}{37}$ **h** $\frac{17}{150}$

4 $6\frac{1}{3}$ cm

5 **a** $\frac{3}{10}$ and $\frac{3}{9}$ **b** $\frac{17}{100}$ and $\frac{17}{99}$ **c** $\frac{173}{1000}$ and $\frac{173}{999}$

6 Students should realise that the recurring version of the terminating fraction has a denominator of $n - 1$.

7 Students' own reasoning and discussion.

CHAPTER REVIEW

1 a 4.08, 4.2, 4.22, 4.8, 4.97
b 2.12, $2\frac{9}{25}$, $2\frac{46}{50}$, 2.955, 2.96
c $\frac{3}{4}$, 0.78, $\frac{4}{5}$, $\frac{5}{6}$, 0.86, 0.91

2 a = **b** > **c** >

3 a $\frac{22}{25}$ **b** $2\frac{3}{4}$ **c** $\frac{1}{125}$

4 a 3.15 **b** 69.67 **c** 32 **d** 0.32

5 a 7.816 **b** 1.092 **c** 876
d 0.01807 **e** 2.884 **f** 13.6

6 Kate has £2.39 more.

7 a $0.\dot{4}$
b $\frac{3}{20}$ terminating; $\frac{17}{60}$ recurring; $\frac{73}{400}$ terminating
c $\frac{3}{11}$

8 a $\frac{2}{9}$ **b** $\frac{6}{11}$ **c** $\frac{77}{90}$ **d** $\frac{1207}{495}$

12 Units and measurement

BEFORE YOU START ...

1 a 10 000 **b** 0.01 **c** 0.1

2 a £140 **b** £90 **c** £220

3 a 180p = £1.80 **b** 15p **c** 60p

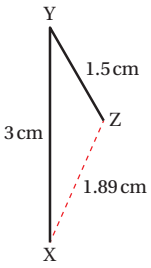
LAUNCHPAD

1 a 11.569 kilograms
b 16 200 seconds
c £1234.56
d 0.0005 m^2

2 a 48 kilometres per hour **b** 13.33 m/s

3 a 24 km/h **b** 90 seconds **c** 167 m

4 800 cm

5 a


b 024° **c** 18.9 km

WORK IT OUT 12.1

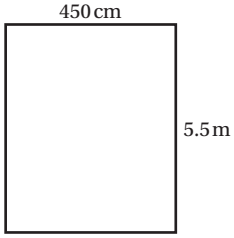
Option C: 2.35 km

EXERCISE 12A

1 10 000 mm = 10 m; 10 000 ml = 10 l; 10 kg = 10 000 g; 0.01 kg = 10 g; 0.1 cm = 1 mm

2 a 2500 **b** 850 **c** 34 000
d 1550 **e** 7 **f** 5400
g 900 **h** 0.102 **i** 0.0145

3 a 8500 ml **b** 2.75 l **c** 25.152 l

4 a


b 20 m
c 17 slabs (20 ÷ 1.2 = 16.67) **d** £77.35 **e** £3.87

5 a 3.6 kg **b** 776.3 cm
c 3.567 tonnes **d** 5 000 000 mm^2
e 96 350 000 cm^3 **f** 0.345 l

6 a <; 50.3 cm **b** >; 100 g **c** <; 15 cl
d >; 4 mm **e** >; 2 cl **f** <; 1 cm
g <; 1.305 kg **h** >; 1976.013 t **i** =

7 a 300 cm **b** 148 000 mm^2 **c** 0.148 m^2
d i 30 047 500 mm^3 **ii** 30.0475 l

EXERCISE 12B

1 1:51:52

2 a 5844 (16 × 365.25 to account for leap years)
b 834.9 **c** 140256 **d** 504 921 600

3 21:05

4 2.5

5 a €1 = £0.79 $1 = £0.59 AS$ = £0.56 1 INR = £0.01
b $212.50
c 4602.60 INR
d £78.22
e You get a lot of Indian rupees for each pound.
f For example, having a weaker exchange rate means that it is expensive to import goods relative to the value of any exported goods.

WORK IT OUT 12.2

Option A: 60 mph

EXERCISE 12C

1 £70.42

2 45p

3 3 bricks

4 21 km/hour

5 6 hours

6 a 10.38 m/s **b** 37.38 km/hour **c** 0.06 m/s

7 0.15 hour or 9 minutes

8 90.4 km/hour

9 18.2 km/hour or 5.06 m/s

10 93.33 km/hour

EXERCISE 12D

1

Material	Answer for part a	Correct answers to part b
Balsa wood	0.2 g/cm^3	200 kg/m^3
Ice	0.9 g/cm^3	900 kg/m^3
Chalk	2.2 g/cm^3	2200 kg/m^3
Tin	7.3 g/cm^3	7300 kg/m^3
Copper	9.0 g/cm^3	9000 kg/m^3
Gold	19.3 g/cm^3	19 300 kg/m^3
Petrol	0.7 g/cm^3	700 kg/m^3
Brick	1.8 g/cm^3	1800 kg/m^3
Aluminium	2.7 g/cm^3	2700 kg/m^3
Iron	7.8 g/cm^3	7800 kg/m^3
Lead	11.3 g/cm^3	11 300 kg/m^3

2 2.38 g/cm^3

3 125 cm^3

4 Block A: 18 N/m^2 Block B: 3 N/m^2

5 60 000 N/m^2

WORK IT OUT 12.3

Student C is correct. The actual distance is 0.85 km.
Student A has divided rather than multiplied by 25 000.
Student B has incorrectly converted mm to km.

EXERCISE 12E

1 **a** 2 km **b** 25 km **c** 24 km

2 **a** 0.0054 km **b** 0.054 km **c** 0.54 km
d 5.4 km **e** 54 km **f** 540 km

3 He is correct. 15 000 cm in real life is represented by 1 cm on a 1 : 15 000 map but it is represented by only 0.1 cm on a 1 : 150 000 map.

4 3 km

5 **a** 530 km **b** 578.18 km/hour

6 **a** 2.5 m **b** 3 cm **c** 1.35 m

7 39 cm

8 **a** 180 km **b** 48.25 mm

9 **a** 1 : 2 000 000 **b** 9.6 cm

EXERCISE 12F

1 **a** Rectangle 16 cm × 8 cm **b** Rectangle 8 cm × 4 cm
c Rectangle 4 cm × 2 cm

2 Students' scaled diagrams.

3 Students' scaled diagrams.

4 **a** 61.25 mm by 47.5 mm **b** 15 mm

EXERCISE 12G

1 **a** 180° **b** 045° **c** 270°

2 **a** 165° **b** 345° **c** 140°
d 025° **e** 250° **f** 205°

3 **a** 288° **b** 108° **c** 148 km

4 **a** 300° **b** 225° **c** 030°

5 **a** ~9.7 km **b** ~090°

CHAPTER REVIEW

1 **a** 259 200 s **b** 182.5 km **c** 5 km **d** 475 litres

2 **a** True
b False. It would take 22.5 minutes
c False. It would be 22 centimetres

3 6 m^2

4 2 000 000 mm^2

5 **a** Scale drawing **b** 67.5 km/hour

6 200 kg

7 **a** Scale drawing **b** 283° **c** 15.3 km
d **i** 10.2 km/hour **ii** 2.83 m/s

8 2 km

9 Approx 41 km apart

13 Percentages

BEFORE YOU START ...

1 **a** 2.1 **b** 0.21 **c** 24 **d** 2.4

2 $\frac{16}{36} = \frac{4}{9}, \frac{15}{35} = \frac{3}{7}, \frac{30}{36} = \frac{5}{6}, \frac{9}{36} = \frac{1}{4}, \frac{39}{52} = \frac{3}{4}, \frac{13}{39} = \frac{1}{3}$

3 **a** F **b** F **c** F **d** T **e** F

LAUNCHPAD

1 **a** $\frac{17}{50}$ **b** $1\frac{3}{20}$

2 **a** 12%, 0.125, $\frac{7}{50}$, 19%, $\frac{5}{12}$
b 2.5%, 12.5%, 1.08, 1.25, 200%, $2\frac{3}{4}$

3 76%

4 64

5 27.2%

6 3.31%

7 £21.90

8 £2590

9 £3.40

EXERCISE 13A

1 **a** 5% **b** 54% **c** 44% **d** 85%
e 50% **f** 66.7% **g** 62.5% **h** 184%
i 30% **j** 4% **k** 47% **l** 112%
m 207% **n** 225% **o** 3.5% **p** 0.7%

2 **a** $\frac{1}{4}$ **b** $\frac{4}{5}$ **c** $\frac{9}{10}$ **d** $\frac{1}{8}$
e $\frac{1}{2}$ **f** $\frac{49}{50}$ **g** $\frac{3}{5}$ **h** $\frac{11}{50}$

3 **a** 0.82 **b** 0.97 **c** 0.45 **d** 0.28
e 0.0005 **f** 0.0008 **g** 0.00006 **h** 0.000 007
i 1.25 **j** 3 **k** 0.0728 **l** 0.09007

4 **a** F **b** T **c** F **d** T **e** F **f** T

5 **a** 6.5% **b** 33.3% **c** 67.5%

6 24%

7 **a** 0.8%, $\frac{1}{20}$, 0.1, 30%, $\frac{3}{5}$ **b** 0.15, $\frac{1}{4}$, 57%, 0.75, 0.88
c 0.25, 60%, $\frac{2}{3}$, 0.75, $\frac{9}{10}$ **d** 0.395, $\frac{3}{7}$, 0.43, $\frac{4}{9}$, 45%
e 49.3%, 55%, $\frac{19}{25}$, 80%, $\frac{5}{6}$

8 $\frac{167}{200}$

9 6%

10 55%

11 **a** 27 out of 30
b 84.7% when calculated as the mean using the percentages. 83.5% when calculated as the total marks scored divided by the total marks available.
Why are the results different and which is the best method?

WORK IT OUT 13.1

B is correct.
A: 9/200 is 4.5% not 9.5%
C: 19/2 is 950%
D: 9.5 is equivalent to 950%
E: the 100 and 400 are in the wrong places

EXERCISE 13B

1 **a** 12.5 **b** 36 **c** 24
d 2925 **e** 9 **f** 270
g 16 **h** 66 **i** 135

2 **a** £9.50 **b** 42 kg **c** 15.75 cm
d 26.1 kg **e** £14 **f** 2.08 mins
g £74 **h** 6.84 m **i** 58 l

3 68 out of 80

4 5.4, so 5 phones

5 **a** 1127 **b** 1323

6 201

7 £18.73

8 **a** 46.5 m^2 **b** 573.5 m^2

9 3164

10 **a** 9 ct = 37.5% gold and 18 ct = 75% gold
b 5.475 g
c 6.1125 g
d Student's own reasoning, but research will show that even if 9 ct is only 37.5 per cent pure gold, gold remains the largest component of the alloy.

WORK IT OUT 13.2

B is correct.
The other answer is wrong. Part of a race cannot be greater than the whole (100%), so 300% can't be right.

EXERCISE 13C

1 **a** 8% **b** 1.5% **c** 15% **d** 4%
e 35% **f** 6.3% **g** 6.25% **h** 18.86%
i 33.3% **j** 27.78% **k** 3.26% **l** 53.33%
m 57.14% **n** 7.14% **o** 26.67% **p** 16.8%
q 25% **r** 64.29% **s** 17.5% **t** 1%

2 Sandra

3 60%

4 6%

5 20%

6 29.63%

7 81.39%

8 **a** 33.33% **b** 0.31%

9 250%

10 $33\frac{1}{3}$

WORK IT OUT 13.3

The answers here will vary from student to student.

EXERCISE 13D

1 **a** £54.72 **b** £945 **c** £32.28
d £40 236 **e** £98.55 **f** £99.68

2 **a** £58.48 **b** £520 **c** £83.16
d £19 882 **e** £76.93 **f** £45.24

3 £129 375

4 £3244

5 £7

6 358

7 £2393.75 at end of first month. £2429.66 at end of second month.

8 £42 430

9 **a** Students' own reasoning, but, for example, the village has only recorded 6 extra crimes whereas the town has 11 however the percentage change makes the village look much worse.
b The village
c Students' own reasoning, but given the numbers, the city is probably still the most risky in terms of crime.

10 **a** £12 **b** 27 739 **c** £114 840

11 It means that compared with the average amount of rainfall (in mm) over the past period, the rainfall increased by almost $\frac{1}{4}$. You'd need to know the average rainfall so you could work out how much more rain actually fell in 2014.

12 6.7

EXERCISE 13E

1 **a** £120 **b** 1500 g **c** 666.67 kg **d** £1739.13

2 **a** £1000 **b** £121.25 **c** £720.83 **d** £45.83

3 £50

4 **a** 1200 students **b** 960

5 £150

6 260 g

7 61.05 kg

8 500 runners

CHAPTER REVIEW

1 **a** $\frac{1}{4}$ **b** $\frac{3}{10}$ **c** $\frac{7}{200}$

2 **a** 5% **b** 12.5% **c** 53.33%
d 50% **e** 125% **f** 0.5%

3 (15×1.015), 15.24, $15\frac{23}{86}$, $15\frac{56}{45}$

4 4%

5 223 435

6 £68.63

7 33.6 hours (33 hours and 36 mins)

8 **a** 12.5% **b** 37.5%

9 £460

10 873.44 to nearest pence

14 Algebraic formulae

BEFORE YOU START ...

1 **a** 7.5 **b** 1.5 **c** $\frac{1}{4}$ **d** −5.5

2 **a** $x = 7$ **b** $x = 8$ **c** $x = \frac{1}{3}$ **d** $x = 10$

3 **a** A = area of a triangle, b = base, h = height
A = area of a circle, r = radius
b $\frac{1}{2}$, π **c** A **d** Use of π

LAUNCHPAD

1 **a** C **b** B **c** E **d** D **e** F **f** A

2 $T = 45w + 20$

3 $-\frac{11}{6}$

4 **a** 4 variables **b** 3 variables **c** 43.2

5 **a** A (area); r (radius); π (constant)
b $r = \sqrt{\frac{A}{\pi}}$ **c** No

EXERCISE 14A

1 **a** $x + y = 2000$ **b** $y = 4x$ **c** $y - 400 = 3x$
d $y = x + 1200$ **e** $\frac{1}{2}x = \frac{2}{5}y$

2 **a** $y = x + 3$ **b** $y = x - 6$ **c** $y = 10x$
d $y = x - 8$ **e** $y = x + x^2$ **f** $y = 3x + 1$
g $y = \frac{2x}{x-2}$ **h** $y = 0.5\pi \times \sqrt[3]{\frac{x}{3}}$

3 **a** 19° C **b** 1000 m

4 **a** $Q = 0.98^n P$ **b** $a = 0.95^n \times 100$

WORK IT OUT 14.1

B is correct.
A has not simplified the calculation.
C has wrong value and incorrect units.

EXERCISE 14B

1 a i 68 **ii** $^{-}18$ **iii** $\frac{^{-}7}{9}$

b i 45 **ii** $\frac{^{-}14}{9}$

2 a $^{-}3$ **b** $\frac{^{-}11}{4}$ **c** 4 **d** $\frac{^{-}3\sqrt{2}}{4}$ **e** 0 **f** $\frac{^{-}9\pi}{14}$

3 $v = 21$

4 a 10 **b** 12.5 **c** 20.9 **d** 10.4

5 a $\frac{4}{3}$ **b** $\frac{30}{17}$

6 a 20.1 m **b** 30.4 m

7 $V = 190\pi$

8 $h = 44.7$

9 28.7 cm

10 99 cm

EXERCISE 14C

1 a $q = \frac{d + ac}{a}$ **b** $p = 2 + 8q$ **c** $x = \frac{4y + 15}{5 + 3y}$

d $h = \frac{2d^2}{3}$ **e** $t = \frac{yp}{2p + y}$ **f** $b = \frac{2 + 5a}{a + 7}$

g $x = \frac{pc}{q - p}$

2 $n = \frac{S}{180} + 2$

a 8 **b** 12 **c** 20

3 $m = \frac{2E}{v^2}$

a 8 kg **b** 3.5 kg **c** 20 kg

4 $C = \frac{5(F - 32)}{9}$

a 20 °C **b** $^{-}5$ °C **c** 100 °C

5 3 cm

6 2.5 cm

7 30.5 m/s

8 $n = 5$

9 $L = 2.45$

WORK IT OUT 14.2

Student B is correct.

EXERCISE 14D

1 a T **b** F, $l = \left(\frac{A}{\pi r}\right) - r$ **c** T

d T **e** F, $V = \frac{1}{2}bhl$

2 16.97 cm^2

3 Total accrued = £6749.18, interest = £749.18

4 This is the formula for solving a quadratic equation.
In order to evaluate the two possible solutions both the positive and negative square root values must be considered.

5 $ax^2 + bx + c = 0 \rightarrow x^2 + \frac{bx}{a} = \frac{^{-}c}{a} \rightarrow \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} = \frac{^{-}c}{a}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

Taking the square root of both sides:

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$

So $x = \frac{^{-}b \pm \sqrt{b^2 - 4ac}}{2a}$

6 a $\sqrt{n}, \frac{n}{2} + 1, n, \frac{36}{n}, n^2, 8n$ **b** $n^2, n, \sqrt{n}, \frac{n}{2} + 1, 8n, \frac{36}{n}$

7 a Yes, gallium melts at 29.76 °C, so it would melt in your hand.
b 2199.2 °F

8 a $Q = ^{-}1.077$ **b** $m = ^{-}6$

9 $l = 13.5$ cm

10 a 30 m **b** 34.125 m **c** 38.5 m
d Stopping distance increases with speed, so in high risk areas it is safer to travel at a slower speed.

11 $a = \frac{(v - u)}{t}$ **12** $s = \frac{\sqrt{n}(b - a)}{3}$

13 30 m/s **14** $x = ^{-}5$ or $^{-}4$

CHAPTER REVIEW

1 a 5 cm **b** 7.4 cm **c** $h = \frac{S}{2\pi r} - r$ **d** $4\sqrt{3}\pi$

e The total surface area is the sum of the area of the curved surface and the area of the flat surface; the area of flat surface is the area of a circle with radius r, which is πr^2, so the total surface area is $2\pi r^2 + \pi r^2 = 3\pi r^2$

2 a $A = \pi r(2C - r)$ **b** $A = \pi(C^2 - h^2)$

3 7.5 km/hr and 5.5 km/hr

4 $A = 96$ cm^2 to the nearest whole cm^2

15 Perimeter

BEFORE YOU START ...

1 a Pentagon **b** Hexagon **c** Octagon

2 a 5000 m **b** 1 200 000 cm
c 8.5 m **d** 4800 mm

3 a True **b** False **c** False

4 $4x$

5 $l = \frac{P}{2} - w$

6 $r = \frac{P}{\pi + 2}$, $r = 2.00$

LAUNCHPAD

1 290 m

2 a 116 mm **b** 317 mm

3 42.25 mm

4 35 mm

5 a 38.33 cm **b** 43.98 cm

6 12 cm

7 8.23 m (to 2 dp)

EXERCISE 15A

1 30 cm **2** 29.96 m

3 a $3a$ **b** $2x + 2y$ **c** $8z$

4 a 120 cm **b** 144 cm

5 144 cm **6** 360 cm

7 a i Both shapes have opposite sides that are equal in length so to find the perimeter you need to add each of these sides twice. $P = 2a + 2b$

ii As a square and a rhombus are specific examples of a rectangle and parallelogram, where $a = b$ this still holds true. $P = 4a$

b Because there is no general relationship between the lengths of the four sides.

8 **a** 516 m **b** 43 posts **c** £1021.25

9 **a** Team A: 2780 m Team B: 3045 m Team C: 3210 m
b 12.84 km/hr

WORK IT OUT 15.1

Option A is correct because there are 16 sides to the perimeter each of 6.5 cm.

EXERCISE 15B

1 Rectangle width 44 mm
Parallelogram length 45 mm
Rhombus side length and width 3.0625 cm
Square length and width 11.82 cm

2 **a** 66 m **b** 45 cm **c** 4.8 cm
d 44.6 cm **e** 280 mm

3 **a** 11 cm **b** 28 cm **c** 35 cm
d 19 cm **e** 17.5 cm **f** 34 cm

4 75 m

5 5.7 m

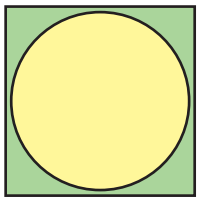
EXERCISE 15C

1 **a** 62.83 mm **b** 43.98 cm
c 5.65 m **d** 6.79 m
e $4\pi x = 12.57x$ cm **f** $\pi x + 4\pi = 3.14x + 12.57$ cm

2 131.95 cm

3 6220.35 mm

4 **a**



b 25.13 cm **c** 26.08 cm
d **i** 59.13 cm **ii** 60.08 cm

5 **a** 6.37 mm **b** 4.84 cm

6 5.8 cm

7 A square plate with side lengths 24.51 cm

8 47 cm

EXERCISE 15D

1 **a** 21.99 cm **b** 17.17 cm **c** 29.60 cm **d** 9.69 m

2 **a** 16.19 cm **b** 22.28 cm **c** 7.24 m **d** 16.54 m
e 44.22 m **f** 54.99 m **g** 1.88 cm **h** 24.56 cm
i 62.46 cm **j** 43.42 m

WORK IT OUT 15.2

All options are incorrect!

Option A is wrong because the whole circumference of each circle was calculated rather than just $\frac{3}{4}$.

Option C is wrong because the wrong formula for working out the circumference was used.

Option B is the nearest to being correct but incorrectly uses a scale of 1 : 25 when the diagram states the scale is 1 cm represents 10 m

EXERCISE 15E

1 **a** 208.75 m **b** 234.85 m

2 **a** Discus **b** 7.85 m
c 6.77 m outer and 6.71 m

3 314.61 m

4 Line A is 45.55 m long. Line B is 30.37 m long.

5 10.47 m and 26.10 m. Answer correct assuming:
1) It is on the 60 m line and
2) 'How far it is from the edge' means along the curve

6 40 074.78 km

7 29.92 cm

8 Answer in range 188 to 201 mm. (194 mm based on measured radius of 15.5 mm and angles of 80° and 50°).

9 129.20 cm

10 1.714 m (Taking top of window as a semicircle).

11 **a** 383.27 m **b** 11.98 m **c** 325.27 m
d 0.213 m/s **e** 2938.95 km
f Based on 50 million people over 14 years.
Answers clearly showing 13 or 15 years also acceptable.
i 3.57 million people per annum **ii** £104.30 million

CHAPTER REVIEW

1 7 cm

2 18 cm

3 18.5 m

4 91.12 m

5 112 cm

6 111.4 cm

7 **a** Main 18.85 m, stage 2 9.42 m, stage 3 4.71 m
b Main 18.85 m, stage 2 15.42 m, stage 3 10.71 m; Total 44.98 m

8 **a** 33.56 mm **b** 94.25 mm **c** 134.25 mm

16 Area

BEFORE YOU START ...

1 **a** Parallelogram **b** Trapezium
c Parallelogram, based purely on the markings. Rectangle, assuming the angles are right angles.

2 **a** 25 **b** 200 **c** 25

3 **a** ±12 **b** ±100 **c** ±0.5

4 **a** $5\text{ m}^2 = 50\,000\text{ cm}^2$ **b** $870\text{ cm}^2 = 87\,000\text{ mm}^2$
c $4\text{ km}^2 = 4\,000\,000\text{ m}^2$

5 8 cm^2

6 $r = \frac{A}{2\pi}$

LAUNCHPAD

1 **a** $A = \frac{4 \times 3}{2}$ **b** $A = 4 \times 3$ **c** $A = 4 \times 3$
d $A = 4 \times 4$ **e** $A = 4 \times 4$

2 **a** Area (A) **b** coefficient **c** radius

3 4.02 m^2 (2 dp)

4 67 cm^2

5 786 m^2

EXERCISE 16A

1 **a** 0.855 m^2 **b** 2100 cm^2

2 12 cm

3 22.5 cm^2

4 0.9 m^2

5 100 cm

6 4.32 m^2

7 a 6688.5 m²
b Red triangle: 0.36 m², green and blue triangles: 0.18 m²
c 0.42 m²
d i 15 592.5 mm²
ii 3500 mm² based on white triangle being approximately $\frac{1}{3}$ of height of large middle triangle.
e i 27 000 mm² **ii** 30 375 mm² **iii** 33 750 mm²
f Students' own working - should include dimensions and method. Answers will be approximately: Red 36.38%, white 41.75%, blue 21.97%

WORK IT OUT 16.1

Option C is correct.
In Option A both dimensions are incorrect; in Option B the height is incorrect.

WORK IT OUT 16.2

Option A is correct.
In Option B the area should be for a rectangle not a triangle; in Option C there are 10 000 cm² in 1 m².

EXERCISE 16B

1 a 60 cm² **b** 703 mm² **c** 308 cm² **d** 3.78 m²
2 a 412.5 mm² **b** 22.5 cm² **c** 64 cm² **d** 10.5 cm²
3 12 cm
4 400 m
5 a 6 cm **b** 4 cm **c** 10 cm
d 5 cm **e** 5 cm
6 a 308 m² **b** 7700 kg of soil, 3080 kg of compost
c 78 m
d i 3.85 **ii** 1
7 a $x(x + 7)$ **b** $8x^2 + 8x - 6$ **c** $6x^2 + 6x$
d 28 **e** $12x^2 + 26x$ **f** $\frac{x}{2}$

EXERCISE 16C

1 a 254.47 cm² **b** 514.72 cm² **c** 153.94 cm² **d** 356.33 cm²
2 a 149.85 mm² **b** 3.67 cm² **c** 3.91 m² **d** 384.34 mm²
3 0.24 m²
4 a 154 cm² **b** 201 cm²
5 a 110.84 cm² **b** 26.39 cm (Circumference of each frame)
6 a 1950.3 m² **b** 4.91 m²
7 Circumference = $2\pi r$ = 75.398 mm. So r = 12 mm.
Area = πr^2 = 452.387 mm²
8 a 479.97 mm²
b diameter 18.33 mm, circumference 57.60 mm
c diameter 12.22 mm, circumference 38.40 mm

EXERCISE 16D

1 a $8 \times 5 + 2 \times 5 = 50$ m²
b $7.2 \times 4.5 + 5.1 \times (7.2 - 1.2 - 2.1) = 52.29$ m²
c $7.2 \times 7.8 - 5.4 \times 3.4 = 37.8$ cm²
d $12 \times 2.4 + 1.2 \times 6 = 36$ cm²
e $2 \times 19.1 \times 3.8 = 145.16$ cm²
f $8.53 \times 7.84 - 0.5 \times 3.71 \times (7.84 - 1.82) = 55.71$ cm²
g $0.5 \times \pi \times 4.3^2 + 0.5 \times \pi \times 2.15^2 = 36.31$ cm²
h $\frac{130}{360} \times 15^2 \times \pi = 255.25$ cm²
2 a 250.47 cm² **b** 13.73 cm² **c** 153.96 cm²
d 149.10 cm² **e** 30.18 cm² **f** 77.43 cm²
g 15.14 cm² **h** 69.53 m²
3 a perimeter = 39.24 m², area = 46.91 m²
b perimeter = 70.69cm, area = 362.57cm²
c perimeter = 26.57cm, area = 24.57cm²
4 188 (assuming can cut to fit)
5 200.86 cm²
6 a 706.95 cm² **b** 678.67 cm²
7 103.87 cm²
8 19.24 m²
9 113.1 cm²
10 6.93 cm
11 a Possible dimensions: rectangles – 3.4 × 6.0 m, parallelograms – 3.4 × 7.0 m
b No because area = base × height
c Possible answers are:
Parallelograms could be better for car parks that do not have edges at right angles. They could also be useful if space is tight as, since vehicles will not have to turn through 90° when reversing, it may be possible to reduce the gap between rows of parking bays.
d Students' sketches and reasoning

CHAPTER REVIEW

1 15.38 m
2 68 cm²
3 13.5 m²
4 660.5 m²
5 372.53 cm²
6 a 0.25 m² **b** 2500 cm²
7 211.25 cm²
8 £40.50
9 a 154.25 cm **b** 1413.72 cm²

17 Approximation and estimation

BEFORE YOU START ...

1 a Correct **b** Incorrect **c** Incorrect
2 a True **b** True **c** False **d** True
3 a 4.0 **b** 3.55 **c** 0.045

LAUNCHPAD

1 a 90 **b** 2000 **c** 134.12
d 20.0 **e** 1000 **f** 235 000 000
2 0.9 m
3 a £5.15 **b** £5.16
4 Around £15
5 Around 11 litres
6 a 14 **b** 4 **c** 2
7 9.5 m ⩽ x < 10.5 m
8 a $17.05 \leqslant (a + b) < 18.15$ **b** $47.925 \leqslant (ab) < 52.925$
c $4.70 \leqslant \frac{a + b}{a} < 5.08$

EXERCISE 17A

1 a B **b** A **c** B **d** A **e** B
2 a i 55 **ii** 11 **iii** 9 **iv** 12
b i 30 **ii** 60 **iii** 110 **iv** 35 810
c i 500 **ii** 5700 **iii** 2400 **iv** 35 800
d i 3000 **ii** 0 **iii** 36 000 **iv** 67 000
e i 100 000 **ii** 1 200 000 **iii** 12 400 000 **iv** 123 500 000
f i 1 000 000 **ii** 1 000 000 **iii** 14 000 000 **iv** 546 000 000

3 **a** £28 **b** 30 **c** £200
d 2m **e** No, it is 63.8 million to the nearest 100 000

4 ± 5

EXERCISE 17B

1 **a** **i** 4.5 **ii** 4.53 **iii** 4.526
b **i** 25.3 **ii** 25.26 **iii** 25.256
c **i** 125.6 **ii** 125.62 **iii** 125.617
d **i** 0.5 **ii** 0.54 **iii** 0.538
e **i** 32.4 **ii** 32.40 **iii** 32.397
f **i** 0.9 **ii** 0.90 **iii** 0.899

2 **a** 19.87 **b** 302.04 **c** 0.29
d 0.21 **e** 21245.84 **f** 0.00
g 0.10 **h** 1.00 **i** 100.00

3 There will be a variety of justifications for answers.
a 24.49 kg **b** 3.14
c 14 km per litre **d** £14.10

4 ± 4.6

5 **a** Rounded to the nearest million

EXERCISE 17C

1 **a** **i** 800 **ii** 4000 **iii** 70 000 **iv** 0.05
b **i** 790 **ii** 3100 **iii** 0.003 3 **iv** 0.000 75
c **i** 789 **ii** 46,700 **iii** 0.004 21 **iv** 753 000
d **i** 38 **ii** $^{-}$4,100 **iii** 3.0 **iv** 2 000 000
e **i** 37.7 **ii** $^{-}$4,130 **iii** 3.04 **iv** 2 000 000

2 Rounding 0.000 134 567 to two decimal places will give 0.00 which doesn't tell us anything. Rounding to two significant figures is a more accurate way to round very small numbers.

3 **a** 3.14 **b** 1.2 kg/m^3
c 300 000 000 m/s **d** 9.81 m/s^2

4 **a** ± 4.6
b Rounding to two decimal places is more accurate than rounding to two significant figures. Rounding in two different ways would mean that the answer is less accurate than if you always rounded to two decimal places.

EXERCISE 17D

1 **a** 37.67 **b** $^{-}$4.12 **c** 3.03 **d** 0.99

2 **a** 4.52 **b** 25.2 **c** 125
d 0.537 **e** 32.3 **f** 200

3 £6.67 Rounding to two decimal places is the most useful way to approximate as this is to the nearest penny. (Rounding up would also always ensure there is enough money to cover the bill.)

4 **a** 1254543.357695 **b** 0.000347 **c** 2.457578

WORK IT OUT 17.1

Estimate 1 is the closest estimate to the actual cost, but students may have justification for choosing a different estimate. (For example always rounding up so you have an overestimate may be good for budgeting.)

EXERCISE 17E

1 **a** $100 \times 4 = 400$ **b** $400 \times 1 = 400$
c $1 \times 20 = 20$ **d** $10 \times 0.5 = 5$
e $3 \times 5 \times 5 = 75$ **f** $5 \times 10 = 50$
g $\frac{200}{20} = 10$ **h** $\frac{60}{0.5} = 120$

2 **a** Answer C, $190 \times 10 = 1900$
b Answer B, $16 \div 8 = 2$

3 **a** $\frac{80 \times 0.5}{40 \times 2} = \frac{40}{80} = 0.5$

b $\frac{20 + 3}{20 - 6} = \frac{23}{14} \approx 1.5$

c $\frac{1000 \div 40}{2 \times 0.2} = \frac{25}{0.4} = \frac{250}{4} = 62.5$

d $\frac{1000 \div 500}{20 \div 40} = \frac{2}{0.5} = 4$

4 **a** $\sqrt{\frac{3 \times 4}{0.4 \times 0.3}} = 10$ **b** $\sqrt{\frac{4 \times 12}{8 \times 0.3}} = \sqrt{20} \approx 4.5$

5 **a** 2160 m **b** 250 seconds

6 **a** $3 \times 3 \times 25 = 225$ Not sensible
b $5 \times 9 = 45$ Not sensible
c $50 \times 9 = 450$ Sensible
d $3 \times 200 = 600$ Not sensible
e $\frac{9}{3 \times 20} = 0.15$ Sensible
f $\frac{0.03}{0.5} = 0.06$ Not sensible

7 **a** 6.9 cm **b** $2(8 + 7) = 30$ cm

8 ± 7.1

9 0.680 87

10 3

EXERCISE 17F

1 **a** $95.5 \leqslant n < 96.5$ **b** $95.95 \leqslant n < 96.05$
c $95.995 \leqslant n < 96.005$ **d** $0.55 \leqslant n < 0.65$
e $0.055 \leqslant n < 0.065$ (rounded to 2dp)
f $0.595 \leqslant n < 0.605$
g $3.1415 \leqslant n < 3.1425$ **h** $9.85 \leqslant n < 9.95$

2 **a** 4.85 cm $\leqslant L <$ 4.95 cm **b** 12.5195 m $\leqslant L <$ 12.5205 m
c 42.95 cm $\leqslant L <$ 43.05 cm **d** 28.5 mm $\leqslant L <$ 29.5 mm

3 **a** Lower bound 35.5 litres
b No. 1.395 m $\leqslant$ length of wood $<$ 1.405 m
c Least weight 43.35 kg;
greatest weight 43.449 999 9 kg (weight $<$ 43.45)

4 $34.5638 \leqslant n < 34.5639$

5 **a** Lower bound: 99.5 m Upper bound: 100.5 m
b Lower bound: 15.25 seconds Upper bound: 15.35 seconds

6 4.45 m $\leqslant L <$ 4.55 m

EXERCISE 17G

1 Upper bound = 50.5 kg − 11.5 kg = 39 kg
Lower bound = 49.5 kg − 12.5 kg = 37 kg

2 **a** LB: 3.605 cm, UB: 3.615 cm and LB: 2.565 cm, UB 2.575 cm
b LB: 9.246825 cm^2, UB: 9.308625 cm^2
c LB: 9.25 cm^2, UB: 9.31 cm^2

3 **a** 11.955 km^2 $\leqslant$ area $<$ 13.075 km^2
b 0.565 km^2

4 £99.5 − £30.5 = £69

5 **a** **i** 24 cm **ii** 7.11 cm (3 sf) **b** 26.25 cm^2

6 $3.16 \leqslant \pi < 3.52$ (3 sf)

7 8.113 m/s $\leqslant s <$ 8.121 m/s (3 dp)

8 5 g

9 0.76885 m/s $\leqslant v <$ 0.90955 m/s

CHAPTER REVIEW

1 **a** F **b** F **c** T **d** T **e** F

2 $\frac{49}{100} \approx 0.5$

3 $\frac{7000}{500} = 14$p

4 5210

5 $\frac{1}{3}$

6 No, a bar could weigh 54.9 g and this is an error of 9.8% which exceeds the tolerance

7 16.4 ohms

8 Various possible answers that should be supported by clear working. Using rounding to 1 sf for each value ~ 25 m/s^2

9 LB: 45.2%, UB: 46.7% (1 dp)

18 Straight-line graphs

BEFORE YOU START ...

1

Term number	1	3	5	10
Term	1	7	13	28

2 a A($^-3$, 4) D(1, $^-4$) E(4, 0)

b i B **ii** F **c** Origin

3 a $x = ^-3$ **b** $x = 42$ **c** $x = ^-0.4$

4 a 1 **b** 1.5

5 a $y = 1 - 2x$ **b** $y = \frac{6 - 2x}{3}$ **c** $y = \frac{x + 2}{2}$

LAUNCHPAD

1 a $x - y = 2$

x	$^-2$	$^-1$	0	1
y	$^-4$	$^-3$	$^-2$	$^-1$

b $x + y = 4$

x	$^-2$	$^-1$	0	1
y	6	5	4	3

c $2x + y + 2 = 0$

x	$^-2$	$^-1$	0	1
y	2	0	$^-2$	$^-4$

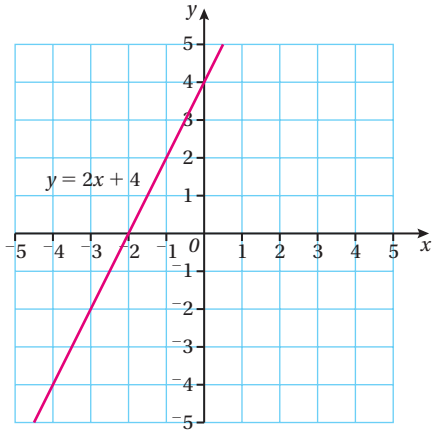
d $x - 2y + 2 = 0$

x	$^-2$	$^-1$	0	1
y	0	$\frac{1}{2}$	1	$\frac{3}{2}$

2 a is (1d) $x - 2y + 2 = 0$

b is (1c) $2x + y + 2 = 0$

3 a



b Gradient = 2 y-intercept (0, 4)

4 $y = \frac{3}{2}x + \frac{5}{2}$ $2y = 3x + 5$

5 $^-\frac{5}{4}$

6 $y = ^-3x + 3$ and $y = 7 - 3x$

7 $y = \frac{1}{2}x + 3$ $2y = x + 6$

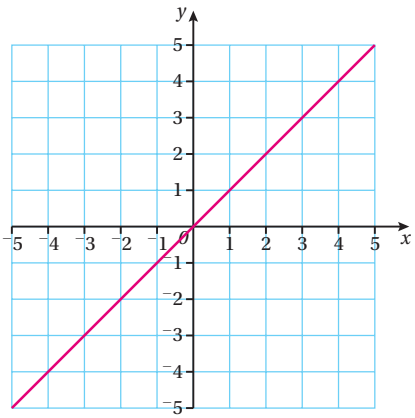
8 The product of the gradients $\left(\frac{1}{2} \times ^-2\right) = ^-1$

9 $y = \frac{3x}{4} + \frac{25}{4}$ $4y = 3x + 25$

EXERCISE 18A

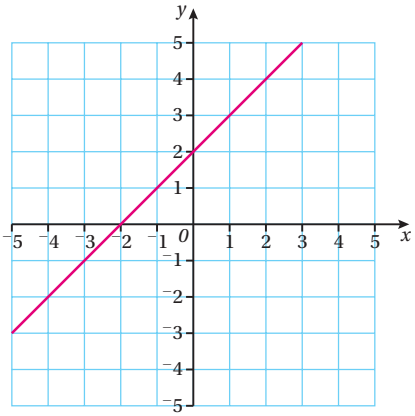
1 a $y = x$

x	$^-2$	$^-1$	0	1
y	$^-2$	$^-1$	0	1



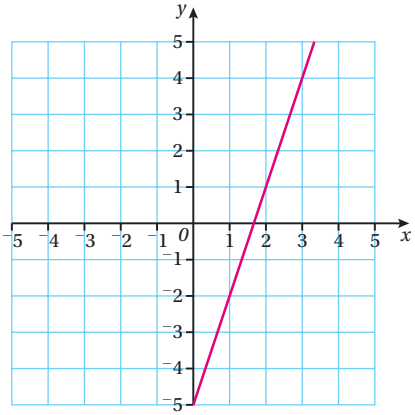
b $y = x + 2$

x	$^-2$	$^-1$	0	1
y	0	1	2	3



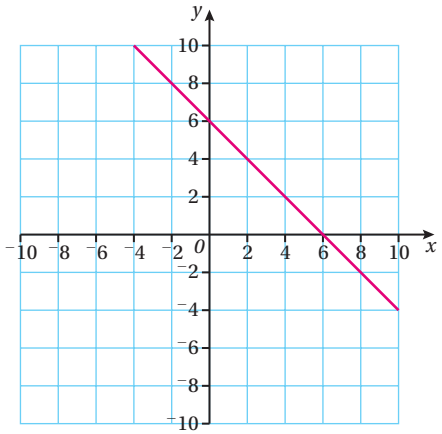
c $y = 3x - 5$

x	0	1	2	3
y	$^-5$	$^-2$	1	4



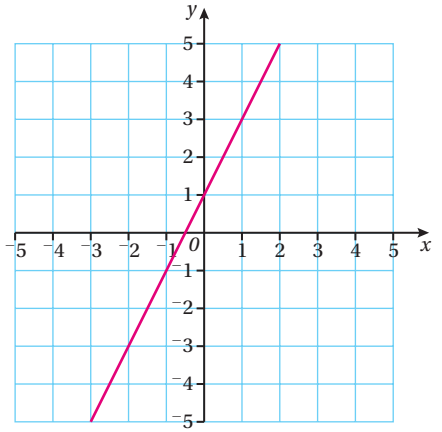
d $y = 6 - x$

x	0	2	6	8
y	6	4	0	$^{-}2$



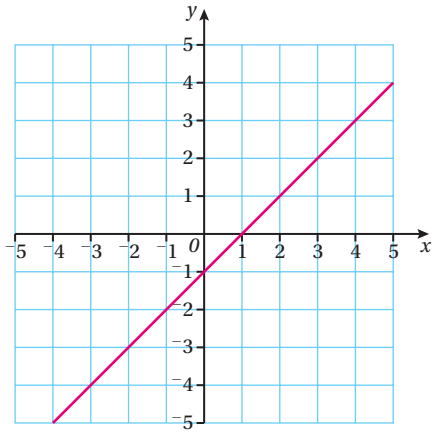
e $y = 2x + 1$

x	$^{-}2$	$^{-}1$	0	1
y	$^{-}3$	$^{-}1$	1	3



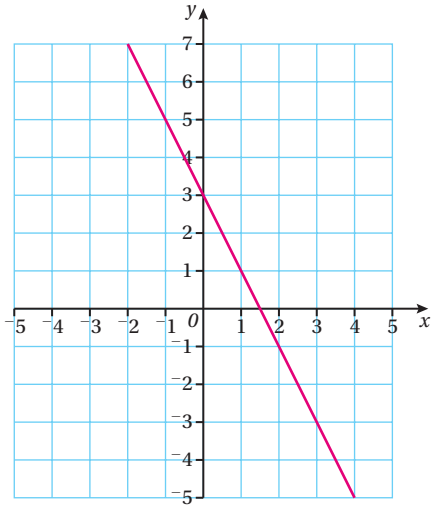
f $y = x - 1$

x	$^{-}2$	$^{-}1$	0	1
y	$^{-}3$	$^{-}2$	$^{-}1$	0



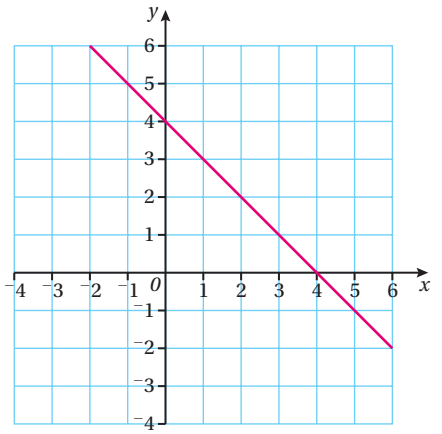
g $y = {}^{-}2x + 3$

x	$^{-}2$	$^{-}1$	0	1
y	7	5	3	1



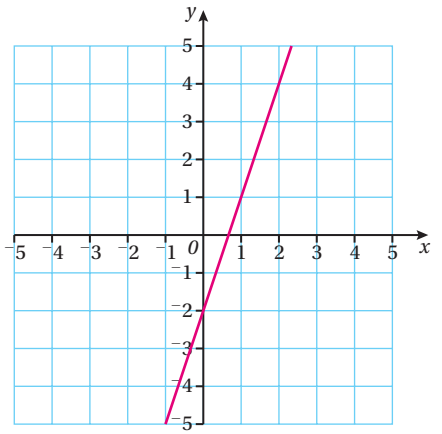
h $y = 4 - x$

x	0	1	3	6
y	4	3	1	$^{-}2$



i $y = 3x - 2$

x	$^{-}1$	0	1	2
y	$^{-}5$	$^{-}2$	1	4



2 Two points are sufficient to define a line.

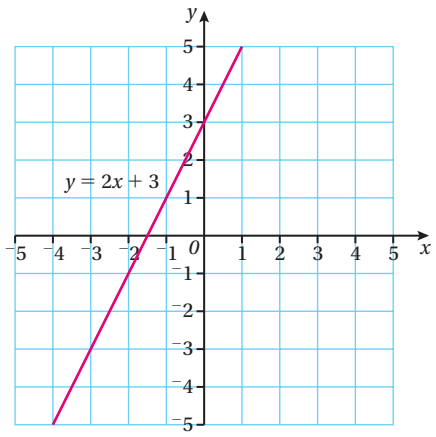
EXERCISE 18B

1 a 2.5 **b** $\frac{-1}{3}$ **c** 1

d 0 (All horizontal lines have a gradient of zero.)

2 $y = 2x + 3$

x	-2	-1	0	1
y	-1	1	3	5



Gradient = 2

3 a 3 **b** 1 **c** -2

d $\frac{-1}{2}$ **e** $\frac{2}{3}$ **f** $\frac{-5}{4}$

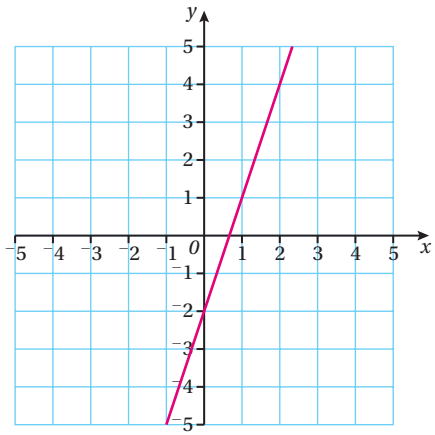
4 a 3 **b** 1 **c** -3 **d** $\frac{7}{4}$

WORK IT OUT 18.1

Option B is correct

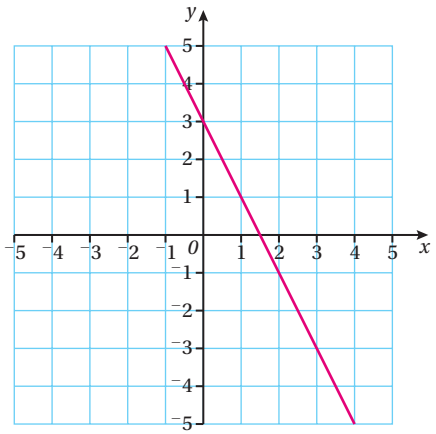
EXERCISE 18C

1 a



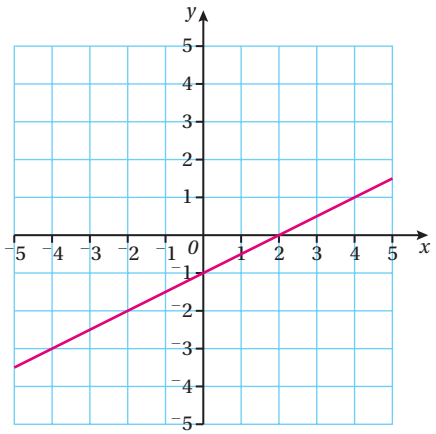
$y = 3x - 2$ this line has a positive gradient of 3 and the y-intercept is $(0, -2)$.

b



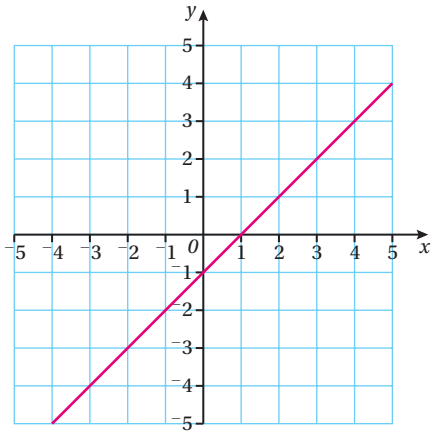
$y = -2x + 3$ this line has a negative gradient of -2 and the y-intercept is $(0, 3)$.

c



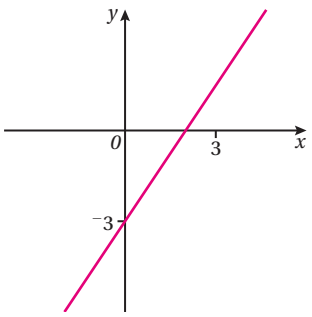
$y = \frac{1}{2}x - 1$ has a positive gradient of $\frac{1}{2}$ and the y-intercept is $(0, -1)$

d

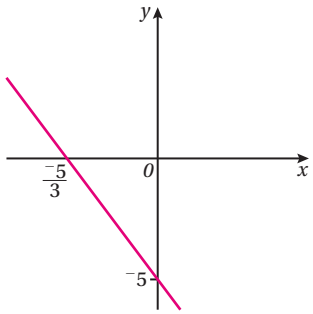


$y = x - 1$ gradient = 1 intercept $(0, -1)$

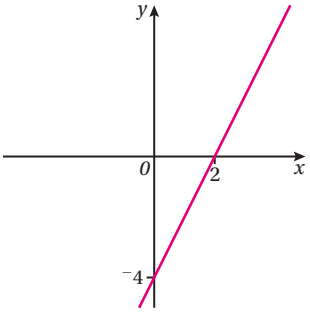
2 a $y = \frac{3}{2}x - 3$



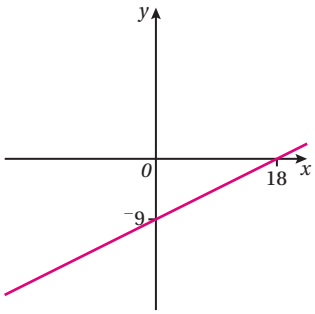
b $y = -3x - 5$



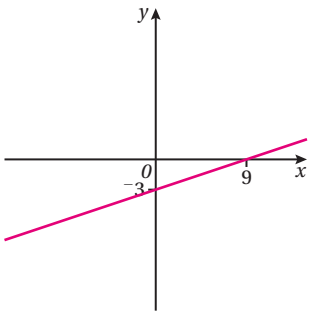
c $y = 2x - 4$



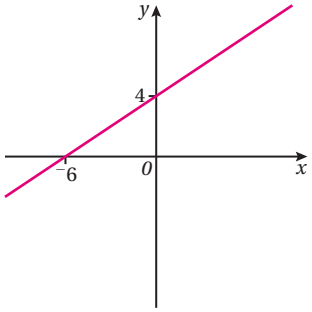
d $y = \frac{1}{2}x - 9$



e $y = \frac{1}{3}x - 3$

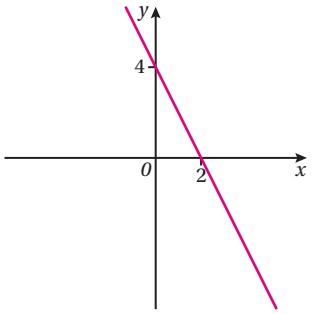


f $y = \frac{2}{3}x + 4$

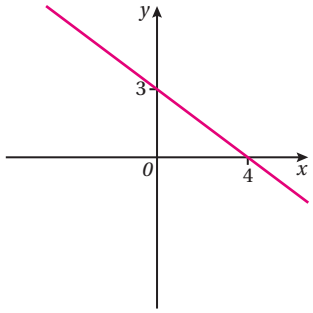


3 a A **b** C **c** D
d B **e** E

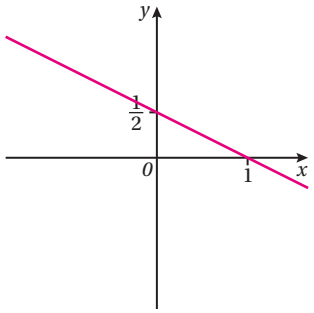
4 a $y = -2x + 4$ gradient $= -2$



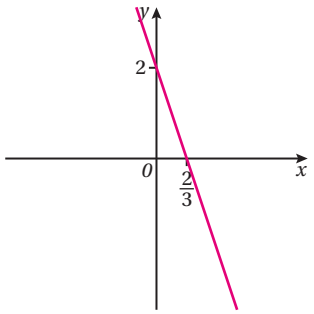
b $y = \frac{-3}{4}x + 3$ gradient $= \frac{-3}{4}$



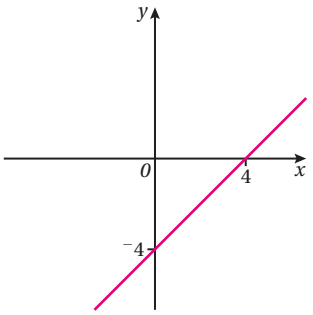
c $y = \frac{-1}{2}x + \frac{1}{2}$ gradient $= \frac{-1}{2}$



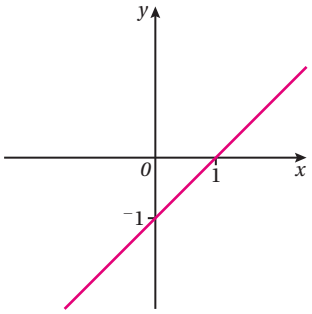
d $y = -3x + 2$ gradient $= -3$



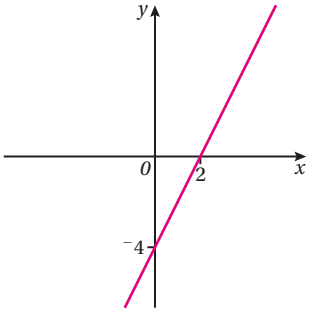
e $y = x - 4$ gradient = 1



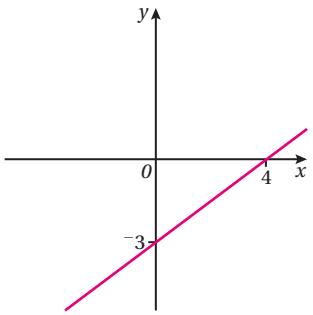
f $y = x - 1$ gradient = 1



g $y = 2x - 4$ gradient = 2



h $y = \frac{3}{4}x - 3$ gradient = $\frac{3}{4}$



5 a 2 **b** $^{-}4$ **c** $^{-}9$ **d** $^{-}8$

6 a $y = \frac{^{-}1}{3}x$ **b** $y = \frac{3}{2}x$

c $y = {}^{-}2x - 9$ **d** $y = {}^{-}2x + 5$

7 a $y = 2x$ when $x = 3$, $y = 6$

b $y = \frac{1}{2}x + 6$ when $x = 8$, $y = 10$

c $y = {}^{-}x + 5$ when $x = 4$, $y = 1$

EXERCISE 18D

1 a $a = 3$ **b** $a = {}^{-}1$

2 $y = {}^{-}2x - 2$

3 a Gradient of AB = 4

b Equation of AB $y = 4x + 2$

c Prove ABCD is a parallelogram

If ABCD is a parallelogram then AB // CD and BC // AD

Gradient of CD $= \frac{8 - 16}{13 - 15} = \frac{^{-}8}{^{-}2} = 4$ CD // AB same gradient

Gradient of BC $= \frac{16 - 14}{15 - 3} = \frac{2}{12} = \frac{1}{6}$

Gradient of AD $= \frac{8 - 6}{13 - 1} = \frac{2}{12} = \frac{1}{6}$ BC // AD same gradient

ABCD is a parallelogram.

4 Lines parallel to the axes have equations such as $x = 2$ and $y = {}^{-}4$.

EXERCISE 18E

1 The line $y = 4x$ is perpendicular to the line $4y + x = {}^{-}2$

2 a Gradient of AB $= \frac{12}{^{-}6} = {}^{-}2$

Gradient of PQ $= \frac{^{-}2}{^{-}4} = \frac{1}{2}$ $\quad {}^{-}2 \times \frac{1}{2} = {}^{-}1$

∴ AB is perpendicular to PQ

b Gradient of MN $= \frac{1.5}{3} = 0.5 = \frac{1}{2}$

gradient of AB × gradient of MN $= {}^{-}2 \times \frac{1}{2} = {}^{-}1$

∴ MN is perpendicular to AB

3 a $y = {}^{-}\left(\frac{2}{5}\right)x - 2$ **b** $y = x - 10$

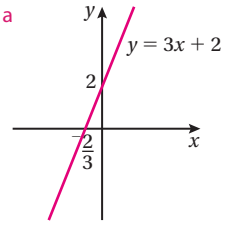
4 Line A $y = 2x + 1$ gradient = 2

Line B $y = \frac{^{-}1}{2}x + 1$ gradient $= \frac{^{-}1}{2}$

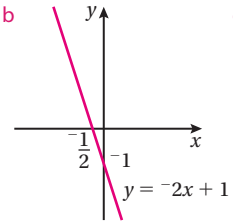
$2 \times \frac{^{-}1}{2} = {}^{-}1$ $\quad$ ∴ Line A is perpendicular to B

5

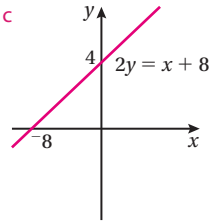
a



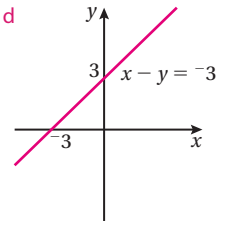
b



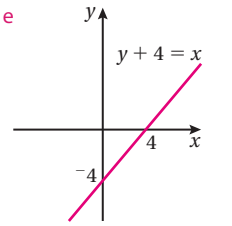
c



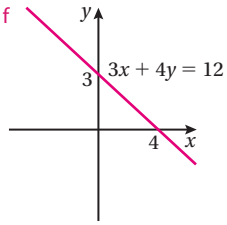
d



e



f



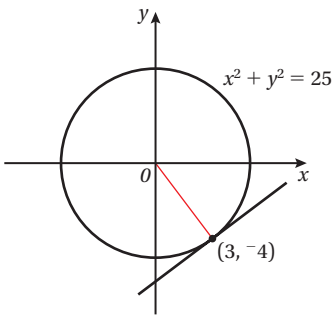
6 Gradient PQ $= \frac{^{-}2}{6} = \frac{^{-}1}{3}$ RP $= \frac{^{-}3}{^{-}1} = 3$

gradient PQ × RP $= \frac{^{-}1}{3} \times 3 = {}^{-}1$

∴ PQ ⊥ RP triangle PQR is right angled.

EXERCISE 18F

1 a



b Gradient of radius line in diagram $= \frac{^-4}{3}$

c $\frac{3}{4}$

d $y = \frac{3}{4}x + \frac{^-25}{4}$ $\quad 4y = 3x - 25$

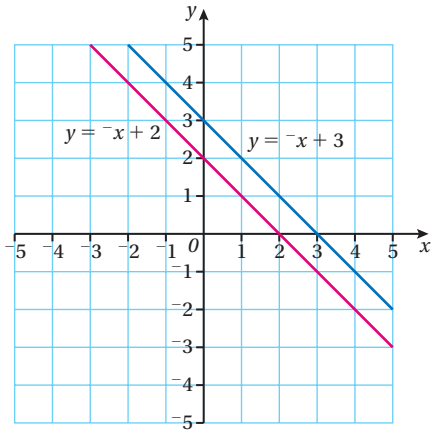
2 a $y = ^-2x + 5$ **b** $y = \frac{1}{2}x + 10$ **c** $y = \frac{^-1}{3}x + 10$

3 $y = 9$ tangent at $(0, 9)$ $\quad x = 9$ tangent at $(9, 0)$

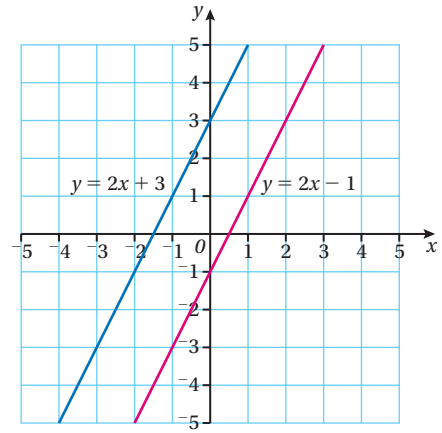
EXERCISE 18G

1 a i $y = x - 2$ **ii** $y = ^-x + 2$

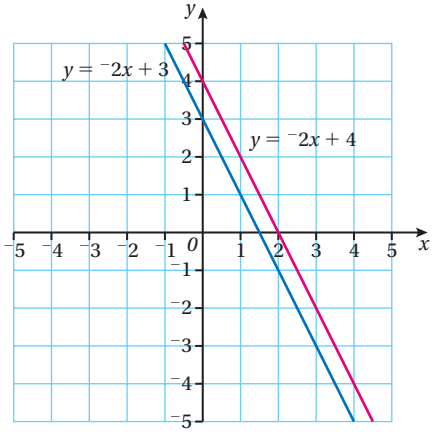
b, c i



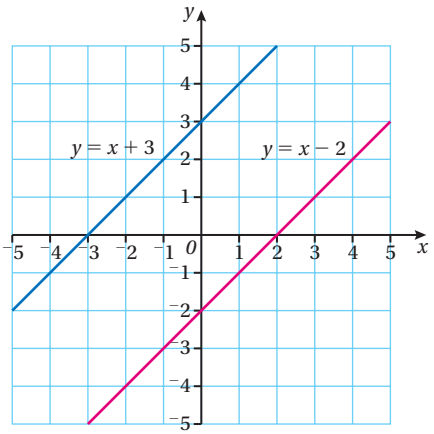
ii



iii



iv



d $1, \frac{^-1}{2}, \frac{1}{2}, ^-1$

2 a $y = 3x + 5$ **b** $y = ^-x + 4$

c $y = \frac{3}{4}x - 2$ **d** $y = \frac{^-1}{7}x$

3 a Any equation with a negative gradient and a negative y intercept, e.g. $y = ^-x - 3$

b Examples: $y = 2x$ and $y = 4x + 3$

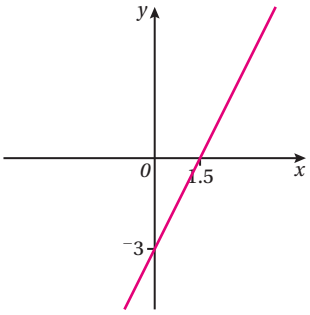
c $y = 3x - 3$.

d Examples: $x = 7$ and $y = 3$

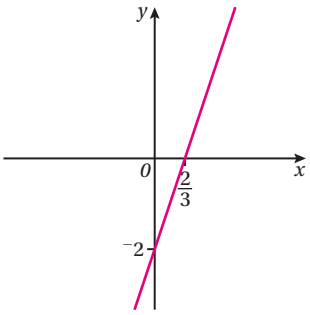
4 a $m = 1$ **b** Values of $m \neq 1$ **c** $m = 2$

5 Gradient $= \frac{1}{2}$, $y = \frac{1}{2}x + 3$

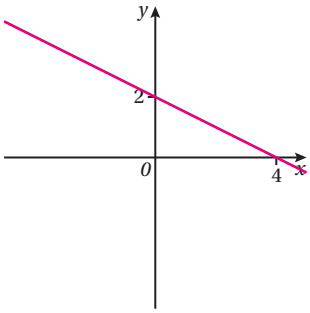
6 a



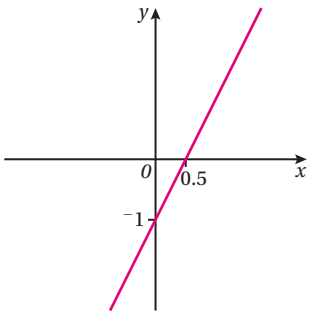
b



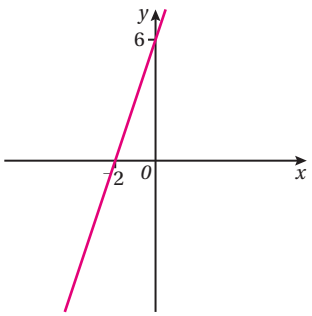
c



d



e



7 **a** $9y + 4x = 74$ **b** $4x + 5y = 32$ **c** $3y = 4x + 26$

8 **a** $y = {}^{-}4x - 6$ **b** $y + 4x = 20$ **c** $y = 5x + 28$

9 $b = {}^{-}4$

10 **a** Gradient is 2, $y = 2x + 1$ **b** $y = {}^{-}x + 1$

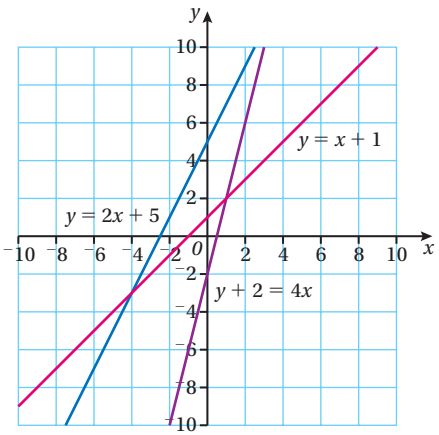
11 $y + x = 3$ $y + x = {}^{-}3$ $y - x = {}^{-}3$ $y - x = 3$

12 The equation of line AB is $y = 5x - 4$
C$({}^{-}2, {}^{-}14)$ ${}^{-}14 = 5 \times {}^{-}2 - 4 \rightarrow {}^{-}14 = {}^{-}14$, hence C lies on the line $y = 5x - 4$

13 $2\sqrt{6}y = {}^{-}x - 23$

CHAPTER REVIEW

1 **a,b,c**

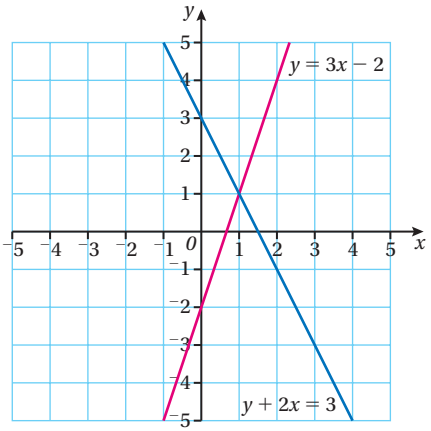


2 **i** **a** $y = 6x - 1$ **b** $y = 1 - x$
c $y = 3x + 1$ **d** $y = {}^{-}x + 2$
ii $y = {}^{-}x - 1$
iii Gradient of d $= {}^{-}1$ gradient of line $2y + 6 = 2x$ is 1
$\therefore$ line d is $\perp$ to the line $2y + 6 = 2x$

3 $y = {}^{-}4x + 12$

4 E; A and F; A and E

5 **a** (1, 1)



b $1 = 3 \times 1 - 2 = 1$ and $1 + 2 \times 1 = 3$

6 **a** Line A $y = {}^{-}\frac{1}{2}x$ Line B $y = x + 4$ Line C $y = 2x - 6$

b Gradient of A $= {}^{-}\frac{1}{2}$ gradient of C $= 2$ ${}^{-}\frac{1}{2} \times 2 = {}^{-}1$
$\therefore$ A $\perp$ C

c Gradient of B $= 1$ gradient of C $= 2$ the gradients are not identical $\therefore$ B is not // C

d x-intercept of B is when $y = 0$ $0 = x + 4$ $x = {}^{-}4$

e y-intercept of C is when $x = 0$ $y = 0 - 6$ $y = {}^{-}6$

f $y = {}^{-}\frac{1}{2}x + 4$

19 Graphs of equations and functions

BEFORE YOU START ...

1 **a** 3 **b** (0,1) **c** $x = {}^{-}5$

d $2y - 6x = {}^{-}4$ becomes $y = 3x - 2$ (make y the subject and divide both sides by 2). The gradient is 3 identical to $y = 3x + 1$ therefore, the two lines are parallel.

2

x	22	21	0	1	2
y	13	4	1	4	13

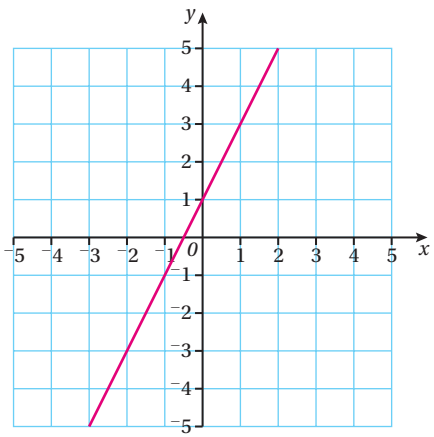
3 **a** $x = {}^{-}4$ $x = 2$ **b** $x = {}^{-}1, x = {}^{-}4$

4 $(x + 2)^2 - 10 = 0$
$x = {}^{-}2 \pm \sqrt{10}$

LAUNCHPAD

1 Three points will give a check point, although a straight line requires only 2 points to be defined.

2 $y = 2x + 1$ is a linear equation with a gradient of 2 and a y-intercept of 1; using two points on the line (0, 1) and (1, 3).

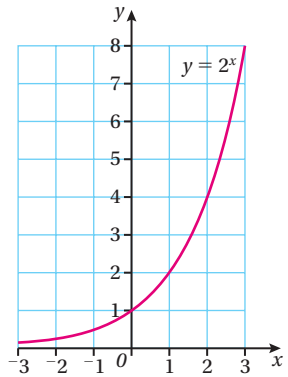


3 a $y = {}^{-}x^2 + 1$
b the coefficient of x^2 is negative which means the parabola will have a vertex that is a maximum
c Maximum
d Vertex (0, 1)
e $x = {}^{-}1$ and $x = 1$

4 a Cubic equation
b At least 5 values, including negative, positive and half values of x

5 a $\frac{1}{0}$ cannot be calculated, undefined
b y gets smaller **c** $\frac{1}{60}$

6



7 a Radius = 7
b $x^2 + y^2 = 49$ the point (1, 7) does not lie on the circle $1^2 + 7^2 \neq 49$.

WORK IT OUT 19.1

Option C. None of the functions has a constant value, so they all pass through the origin.

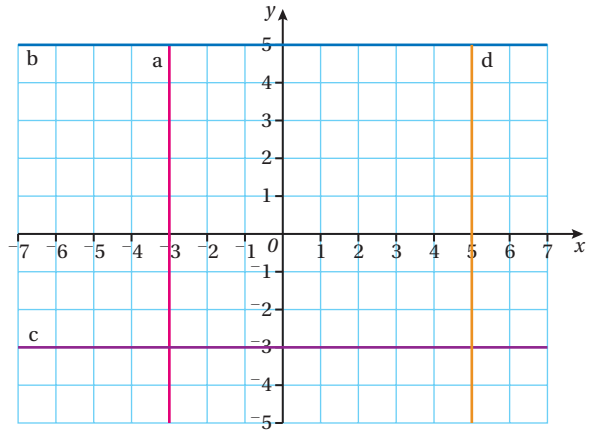
EXERCISE 19A

1 a D **b** B **c** A, B, C and D
d C **e** E

2 a $y = \frac{1}{2}x$ **b** $y = {}^{-}x$ **c** $y = 6x$
d $y = x$ **e** $y = x - 6$

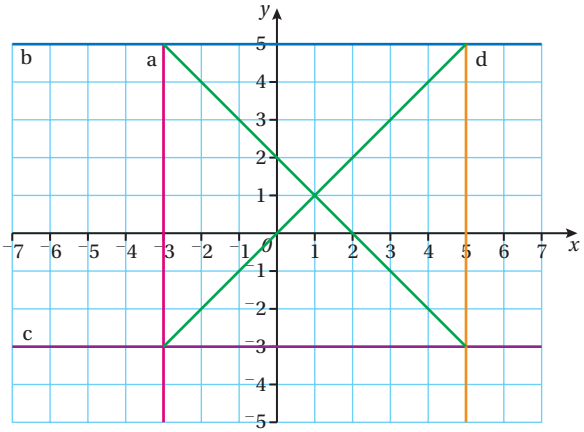
3 a A $x = {}^{-}6$
B $y = 7$
C $y = {}^{-}3$
D $y = {}^{-}4$
E $x = {}^{-}2$
F $x = 4$
b $y = 2$

4



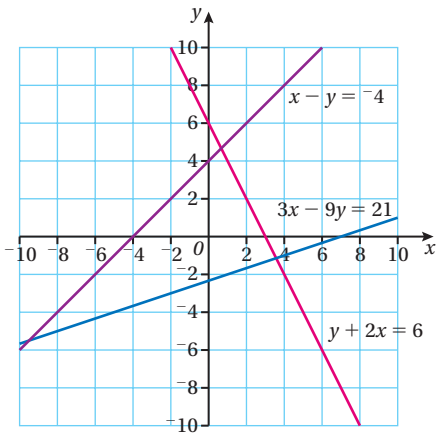
5 A square is created where the lines intersect, because all four sides are 8 units long, there are two pairs of parallel lines and four angles of 90°.

6 a $x = 1$ and $y = 1$
b Mirror line or bisector (line or axis of symmetry is acceptable)
c i

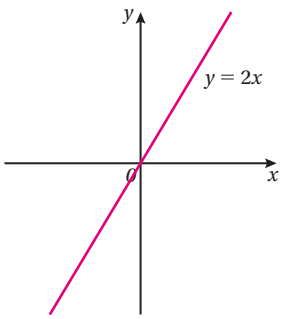


ii $y = x$ and $y - 2 - x$

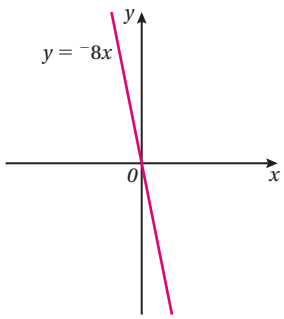
7



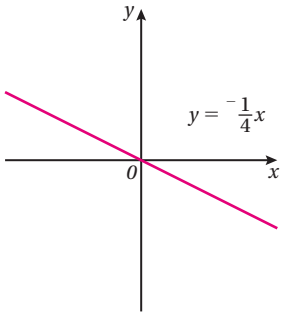
8 a



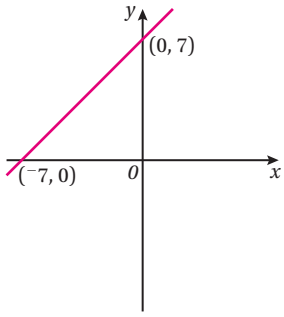
b



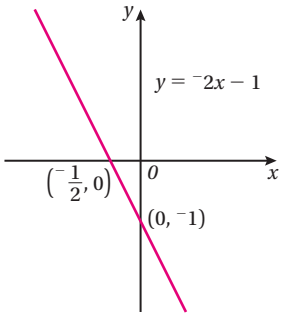
c



d



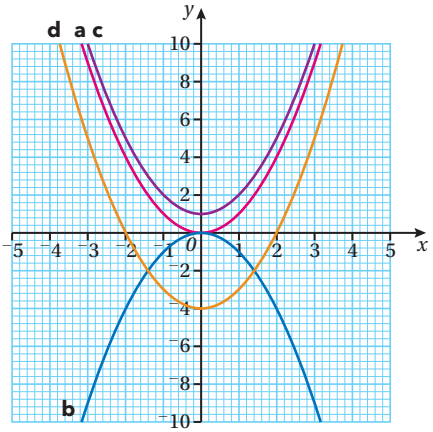
e



EXERCISE 19B

1

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = {}^{-}x^2$	$^{-}9$	$^{-}4$	$^{-}1$	0	$^{-}1$	$^{-}4$	$^{-}9$
$y = x^2 + 1$	10	5	2	1	2	5	10
$y = x^2 - 4$	5	0	$^{-}3$	$^{-}4$	$^{-}3$	0	5



2 $y = x^2 + 2x - 3$ $\quad {}^{-}4 \leqslant x \leqslant 2$

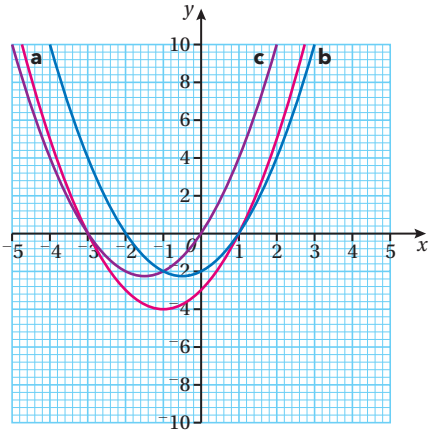
x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2
y	5	0	$^{-}3$	$^{-}4$	$^{-}3$	0	5

$y = x^2 + x - 2$ $\quad {}^{-}3 \leqslant x \leqslant 2$

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2
y	4	0	$^{-}2$	$^{-}2$	0	4

$y = x^2 + 3x$ $\quad {}^{-}4 \leqslant x \leqslant 1$

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1
y	4	0	$^{-}2$	$^{-}2$	0	4



3 a If the coefficient of x^2 is greater than 1 the impact on the shape of the parabola is to increase the steepness (gradient).

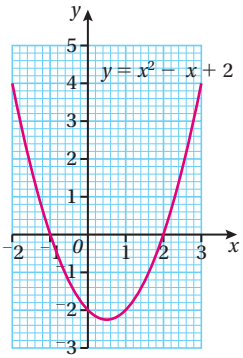
b If the coefficient of x^2 is a value between 1 and 0, the impact on the shape of the parabola is to decrease the steepness (gradient). The parabola widens out.

c If a constant value is added to the graph of an equation such as $y = ax^2$ becomes $y = ax^2 + c$, the graph is moved up the y axis with a positive value of c and down for a negative value.

d When the coefficient of x^2 is negative the basic parabola $y = x^2$ is reflected about the x axis and has a maximum turning point through the origin.

4 $y = x^2 - x - 2$ for $^{-}2 \leqslant x \leqslant 3$.

x	$^{-}2$	$^{-}1$	0	1	2	3
y	4	0	$^{-}2$	$^{-}2$	0	4



a $x = 2$ and $x = {}^{-}1$

b These are the values for x when $y = 0$ – the values where the graph crosses the x axis.

WORK IT OUT 19.2

Option C

EXERCISE 19C

1

Graph	Turning point	Axis of symmetry	y-intercept	x-intercepts	Roots of equation
a	(2, $^{-}9$) minimum	$x = 2$	(0, $^{-}5$)	(5, 0) ($^{-}1$, 0)	$x = 5$ and $x = {}^{-}1$
b	($^{-}2$, $^{-}1$) minimum	$x = {}^{-}2$	(0, 3)	($^{-}3$, 0) ($^{-}1$, 0)	$x = {}^{-}3$ and $x = {}^{-}1$
c	(4, 16) maximum	$x = 4$	(0, 0)	(0, 0) (8, 0)	$x = 0$ and $x = 8$
d	(0, 1) maximum	y axis	(0, 1)	($^{-}1$, 0) (1, 0)	$x = {}^{-}1$ and $x = 1$

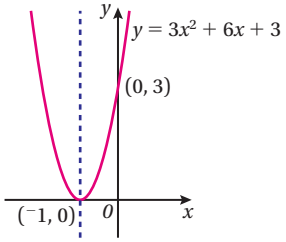
2 $y = 3(x + 1)^2 + 0$

a (0, 3)

b $x = {}^{-}1$ vertex ($^{-}1$, 0)

c ($^{-}1$, 0)

d

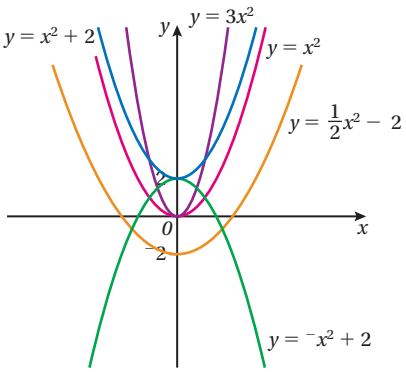


EXERCISE 19D

1 a $y = 6x^2$ **b** $y = x^2 + 4$ **c** $y = \frac{1}{2}x^2$

d $y = {}^{-}x^2 + 2$ **e** $y = x^2 - 1$ **f** $y = {}^{-}7x^2$

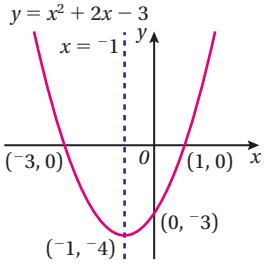
2



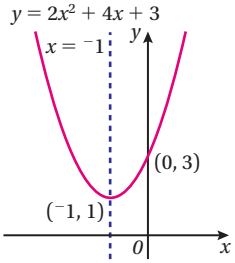
3

	Equation	y-intercept	x-intercept(s)	Symmetry axis	Turning point
a	$y = x^2 + 2x - 3$	$(0, -3)$	$(-3, 0)$ $(1, 0)$	$x = -1$	$(-1, -4)$
b	$y = 2x^2 + 4x + 3$	$(0, 3)$	None	$x = -1$	$(-1, 1)$
c	$y = 4x - x^2$	$(0, 0)$	$(0, 0)$ $(4, 0)$	$x = 2$	$(2, 4)$
d	$y = x^2 + 2x - 8$	$(0, -8)$	$(-4, 0)$ $(2, 0)$	$x = -1$	$(-1, -9)$
e	$y = x^2 - 8x + 12$	$(0, 12)$	$(2, 0)$ $(6, 0)$	$x = 4$	$(4, -4)$
f	$y = -x^2 - 6x - 10$	$(0, -10)$	None	$x = -3$	$(-3, -1)$
g	$y = 2(x - 3)(x + 5)$	$(0, -30)$	$(3, 0)$ $(-5, 0)$	$x = -1$	$(-1, -32)$
h	$y = 4x^2 + 16x + 7$	$(0, 7)$	$\left(-\frac{1}{2}, 0\right)$ $\left(-\frac{7}{2}, 0\right)$	$x = -2$	$(-2, -9)$
i	$x^2 + 3x - 6 = y$	$(0, -6)$	$(-4.37, 0)$ $(1.37, 0)$	$x = -\frac{3}{2}$	$\left(-\frac{3}{2}, -\frac{33}{4}\right)$
j	$2x^2 + x = 8 + y$	$(0, -8)$	$(-2.27, 0)$ $(1.77, 0)$	$x = -\frac{1}{4}$	$\left(-\frac{1}{4}, -\frac{65}{8}\right)$

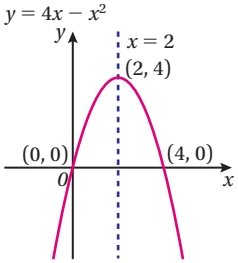
a $y = x^2 + 2x - 3$



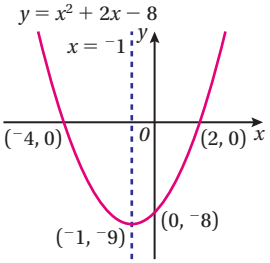
b $y = 2x^2 + 4x + 3$



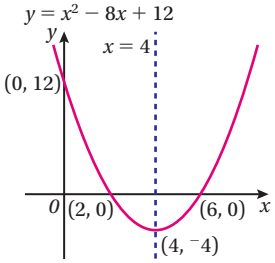
c $y = 4x - x^2$



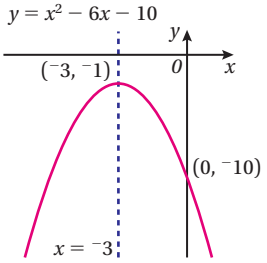
d $y = x^2 + 2x - 8$



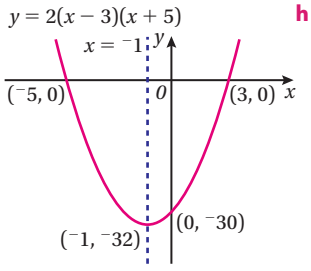
e $y = x^2 - 8x + 12$



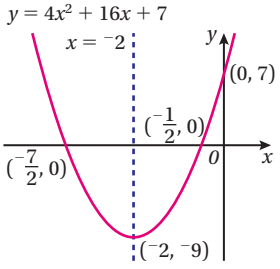
f $y = x^2 - 6x - 10$



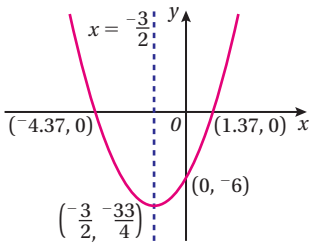
g $y = 2(x - 3)(x + 5)$



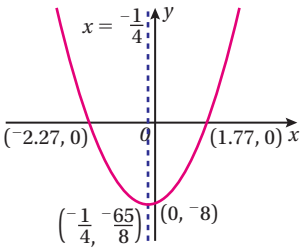
h $y = 4x^2 + 16x + 7$



i $x^2 + 3x - 6 = y$



j $2x^2 + x = 8 + y$

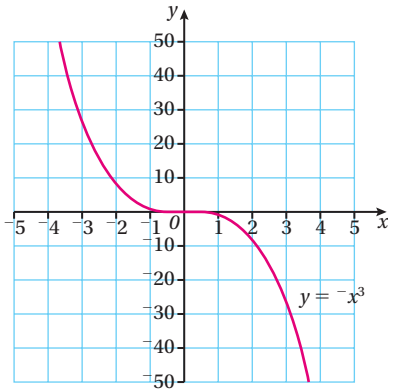


EXERCISE 19E

1

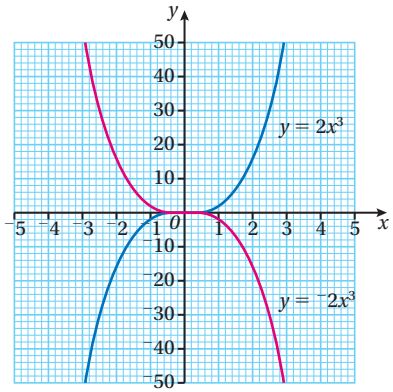
X	−3	−2	−1	0	1	2	3
$y = -x^3$	27	8	1	0	−1	−8	−27

$y = x^3$ ranges of y from negative values for negative values of x to positive values of y for positive values of x and $y = -x^3$ ranges from positive values of y for negative values of x to negative values of y for positive values of x.



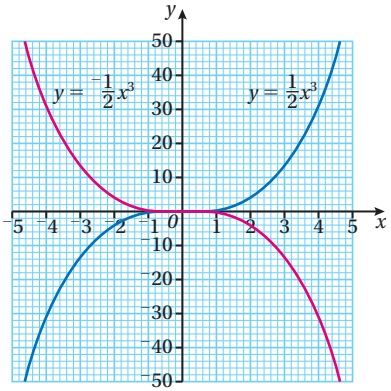
2 a

X	−3	−2	−1	0	1	2	3
$y = -2x^3$	54	16	2	0	−2	−16	−54
$y = 2x^3$	−54	−16	−2	0	2	16	54



b

x	-3	-2	-1	0	1	2	3
$y = -\frac{1}{2}x^3x$	13.5	4	0.5	0	-0.5	-4	-13.5
$y = \frac{1}{2}x^3$	-13.5	-4	-0.5	0	0.5	4	13.5

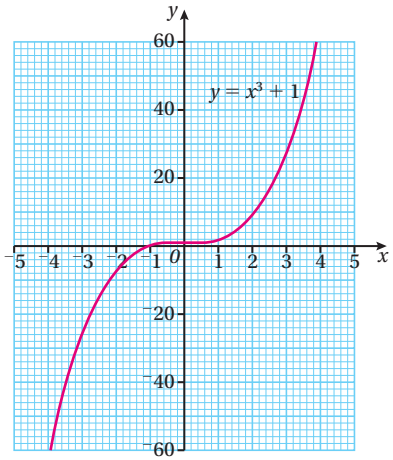


3 Given the graph of $y = 4x^3$ if this graph is reflected about the y axis this will produce the graph $y = -4x^3$

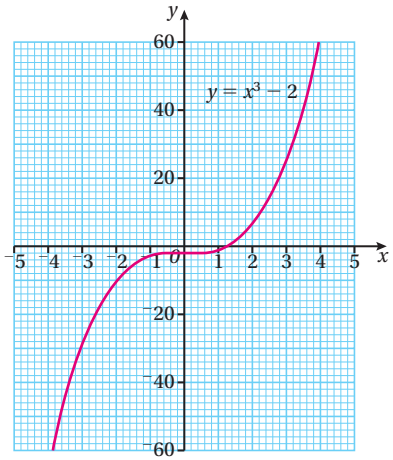
4

x	-3	-2	-1	0	1	2	3
$y = x^3 + 1$	-26	-7	0	1	2	9	28
$y = x^3 - 2$	-29	-10	-3	-2	-1	6	25

a



b

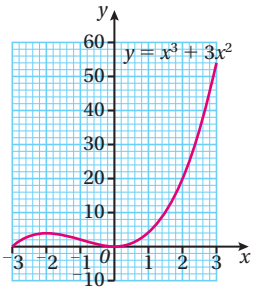


5 **a** $y = x^3 + 1$ y intercept $(0, 1)$
b $y = x^3 - 2$ y intercept $(0, -2)$

6 Line A is $y = x^3 + 5$ Line B is $y = x^3 - 6$

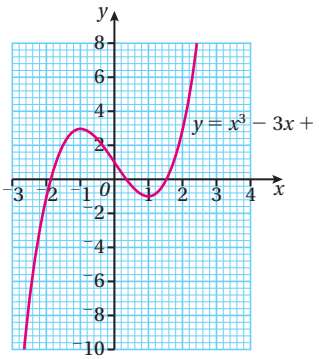
7 **a** $y = x^3 + 3x^2$ $-3 \leqslant x \leqslant 3$

x	-3	-2	-1	0	1	2	3
y	0	4	2	0	4	20	54



b $y = x^3 - 3x + 1$ $-3 \leqslant x \leqslant 4$

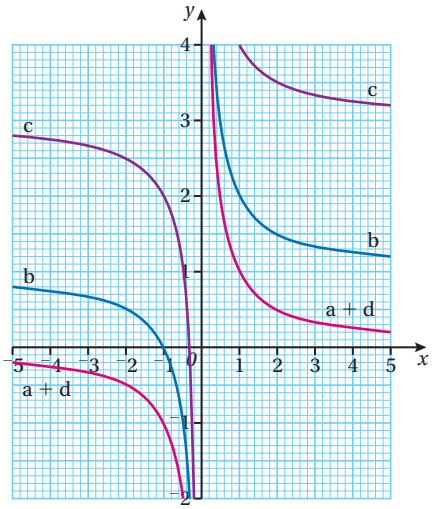
x	-3	-2	-1	0	1	2	3	4
y	-17	-1	3	1	-1	3	19	53



EXERCISE 19F

1

	x	-5	-4	-3	-2	-1	0	1	2	3	4	5
a	$y = \frac{1}{x}$	-0.2	-0.25	-0.333	-0.5	-1	Not defined	1	0.5	0.333	0.25	0.2
b	$y = \frac{1}{x} + 1$	0.8	0.75	0.6667	0.5	0	Not defined	2	1.5	1.333	1.25	1.2
c	$y = \frac{1}{x} + 3$	2.8	2.75	2.6667	2.5	2	Not defined	4	3.5	3.333	3.25	3.2
d	$xy = 1$	-0.2	-0.25	-0.333	-0.5	-1	Not defined	1	0.5	0.333	0.25	0.2



2 The constant in $y = \frac{1}{x} + c$ moves the graph $y = \frac{1}{x}$ the graph in a vertical direction , or translated by $\begin{pmatrix} 0 \\ c \end{pmatrix}$

3 Neo is correct $y = x$ is a line of reflective symmetry for the graph $y = \frac{1}{x}$

4 a

i

x	$^{-}4$	$^{-}2$	$^{-}1$	1	2	4
$y = \frac{2}{x}$	$^{-}0.5$	$^{-}1$	$^{-}2$	2	1	0.5

ii

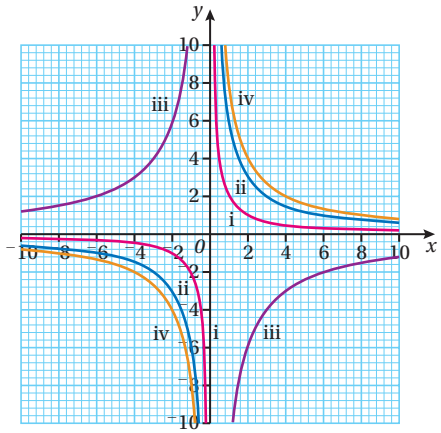
x	$^{-}6$	$^{-}3$	$^{-}1$	1	3	6
$y = \frac{6}{x}$	$^{-}1$	$^{-}2$	$^{-}6$	6	2	1

iii

x	$^{-}10$	$^{-}8$	$^{-}6$	$^{-}4$	$^{-}2$	2	4	6	8
$xy = {}^{-}12$	1.2	1.5	2	3	6	$^{-}6$	$^{-}3$	$^{-}2$	$^{-}1.5$

iv

x	$^{-}8$	$^{-}6$	$^{-}4$	$^{-}2$	1	2	4	6	8
$y = \frac{8}{x}$	$^{-}1$	$^{-}1.333$	$^{-}2$	$^{-}4$	8	4	2	1.333	1



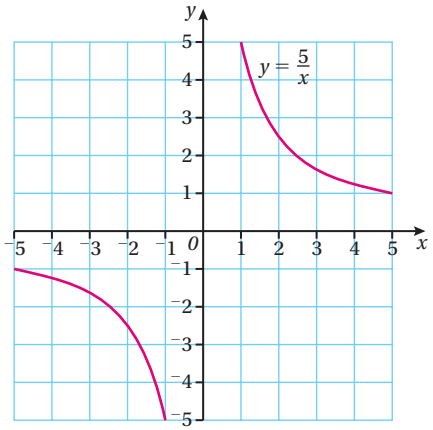
b The constant in the equations affects the graph – when it is positive it moves the graph further away from the origin. When it is negative it moves it away from the origin and the orientation changes to the 2nd and 4th quadrant.

5 a A $y = \frac{2}{x}$ B $y = \frac{4}{x}$ C $y = \frac{8}{x}$ D $y = \frac{10}{x}$

b The graph furthest away from the origin has the greatest value for the constant $y = \frac{10}{x}$.

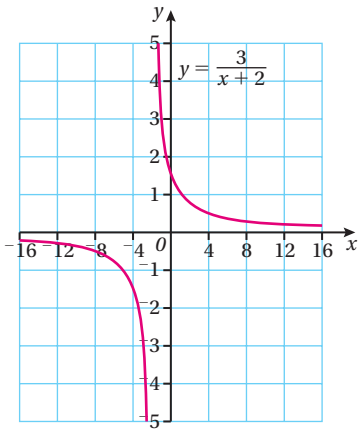
6 a $y = \frac{5}{x}$ $^{-}5 \leqslant x \leqslant 4$

x	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$
y	$^{-}1$	$^{-}1.25$	$^{-}1.667$	$^{-}2.5$	$^{-}5$
x	0	1	2	3	4
y	Not defined	5	2.5	1.667	1.25



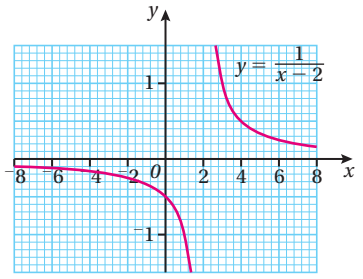
b $y = \frac{3}{x+2}$ $x = {}^{-}16, {}^{-}12, {}^{-}8, {}^{-}4, 0, 4, 8, 12, 16$

x	$^{-}16$	$^{-}12$	$^{-}8$	$^{-}4$	0	4	8	12	16
y	$^{-}0.21$	$^{-}0.3$	$^{-}0.5$	$^{-}1.5$	1.5	0.5	0.3	0.21	0.1667

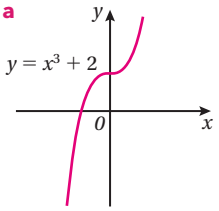


c $y = \frac{1}{x-2}$ $^{-}4 \leqslant x \leqslant 6$

x	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1
y	$^{-}0.1667$	$^{-}0.2$	$^{-}0.25$	$^{-}0.33$	$^{-}0.5$	$^{-}1$
x	2	3	4	5	6	
y	Not defined	1	0.5	0.33	0.25	



7

a


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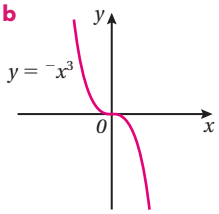

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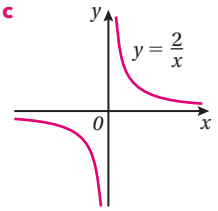
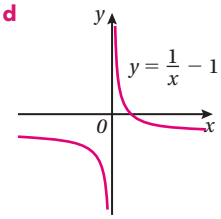
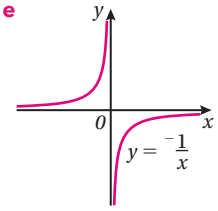


d


e


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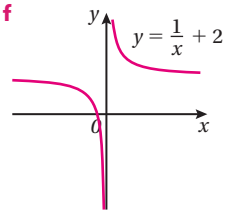
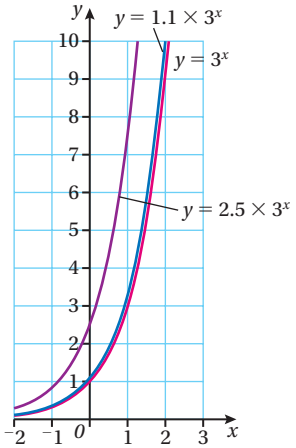


8 $y = \frac{1}{x^2}$

EXERCISE 19G

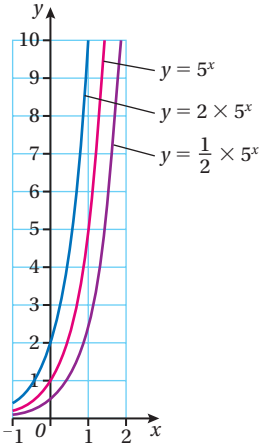
1 a **i** $y = 3^x$ **ii** $y = 1.1 \times 3^x$
iii $y = 2.5 \times 3^x$ values: $-2 \leqslant x \leqslant 3$

x	$^-2$	$^-1$	0	1	2	3
i $y = 3^x$	0.111	0.333	1	3	9	27
ii $y = 1.1 \times 3^x$	0.122	0.367	1.1	3.3	9.9	29.7
iii $y = 2.5 \times 3^x$	0.278	0.833	2.5	7.5	22.5	67.5

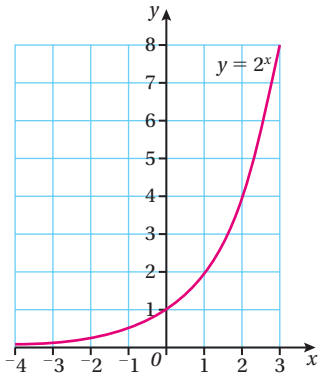


b **i** $y = 5^x$ **ii** $y = 2 \times 5^x$
iii $y = \frac{1}{2} \times 5^x$ values: $^-1 \leqslant x \leqslant 2$

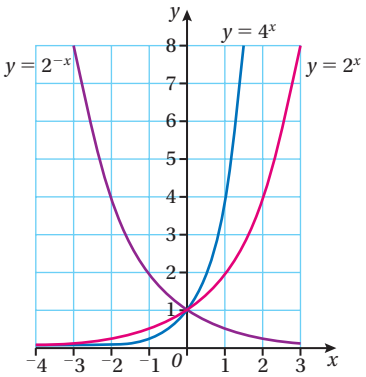
x	$^-1$	0	1	2
i $y = 5^x$	0.2	1	5	25
ii $y = 2 \times 5^x$	0.4	2	10	50
iii $y = \frac{1}{2} \times 5^x$	0.1	0.5	2.5	12.5



2 a

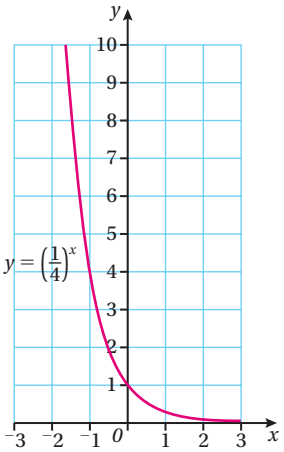


b

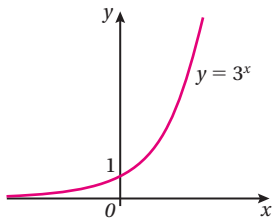


3 $y = \left(\frac{1}{4}\right)^x$ for $-3 \leqslant x \leqslant 3$

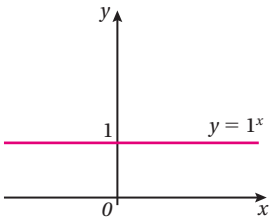
x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	64	16	4	1	0.25	0.0625	0.01563



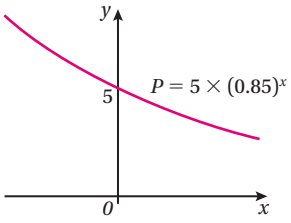
4 a



b



5 a A This is a decreasing graph when $t < 0$ values will be > 5 and when $t > 0$ values will be < 5 but remain positive.
b The graph cuts the y axis when $t = 0$ (0, 5)
c



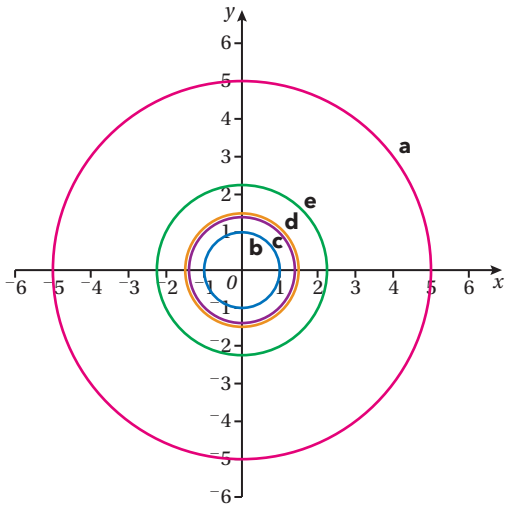
WORK IT OUT 19.3

Option B

EXERCISE 19H

1 a Radius = 5 units
b (3, 4) $3^2 + 4^2 = 9 + 16 = 25$ $(^-3, 4)$ $(^-3)^2 + 4^2 = 9 + 16 = 25$
c (4, 3) $(^-4, ^-3)$ $(^-3, ^-4)$ $(3, ^-4)$
(0, 5) $(0, ^-5)$ $(^-5, 0)$ $(5, 0)$ any four

2

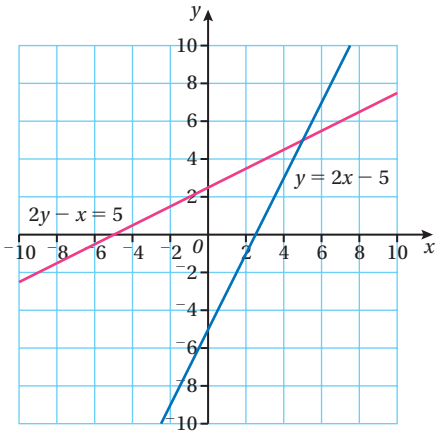


3 a (6, 8) ($^-6$, 8) ($5\sqrt{2}$, $5\sqrt{2}$) (10, 0)
b (5, 12) ($^-5$, $^-12$) ($^-5$, 12) (0, 13)

4 a $x^2 + y^2 = 1$ **b** $x^2 + y^2 = 4$
c $x^2 + y^2 = 9$ **d** $x^2 + y^2 = 16$

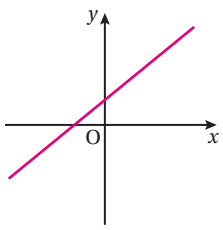
CHAPTER REVIEW

1 Point of intersection of $y = 2x - 5$ and $2y - x = 5$ is (5, 5)

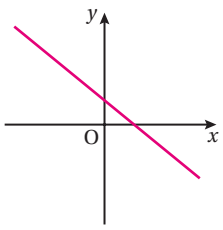


2 a $x = ^-1$ and $x = 3$
b $y = ^-\{(x + 1)(x - 3)\}$
$y = ^-\{x^2 - 2x - 3\}$
$y = ^-x^2 + 2x + 3$
c $x^2 - 2x - 3 = 0 \rightarrow (x - 1)^2 - 4$
So the turning point for $^-x^2 + 2x + 3$ is (1, 4), axis of symmetry $x = 1$.

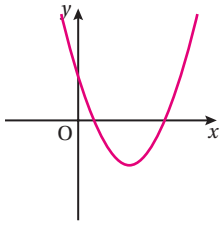
3 a True



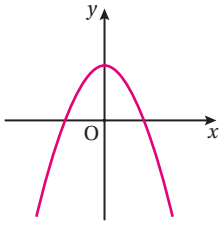
False



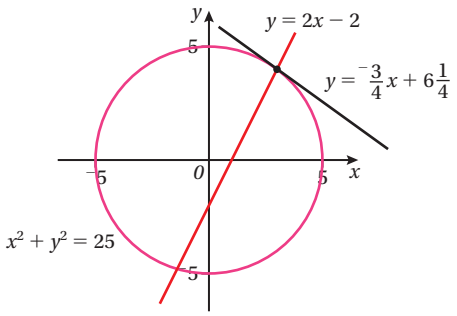
b True



False



4



The point of contact of the tangent $y = ^-\frac{3}{4}x + 6\frac{1}{4}$ with the circle $x^2 + y^2 = 25$ is (3, 4)
$y = 2x - 2$ cuts the circle $x^2 + y^2 = 25$ $x^2 + (2x - 2)^2 = 25$
$5x^2 - 8x - 21 = 0$ solving gives $x = ^-1.4$ and $x = 3$
$y = ^-4.8$ and $y = 4$ the points where the line cuts are ($^-1.4$, $^-4.8$) and (3, 4)
The point (3, 4) lies on the circle $x^2 + y^2 = 25$ $3^2 + 4^2 = 25$
$9 + 16 = 25$
$x = 3$ and $y = 4$ satisfies the equation $y = ^-\frac{3}{4}x + 6\frac{1}{4}$

$y = ^-\frac{3}{4} \times 3 + 6\frac{1}{4}$ $y = -\frac{9}{4} + \frac{25}{4}$ $y = \frac{16}{4}$ $y = 4$

5 a $y = x^2 - x - 12$ **b** $x^2 + y^2 = 16$ **c** $y = ^-x^2$
d $y = \frac{4}{x}$ **e** $y = \frac{1}{2}^x$ or $y = 2^{-x}$ **f** $y = 3 \times 2^x$

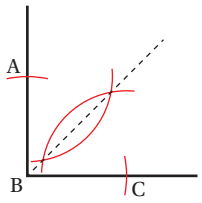
20 Three-dimensional shapes

BEFORE YOU START ...

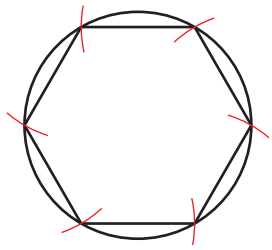
1 a Square-based pyramid **b** Cuboid
c Cube **d** Triangular prism

2 a False; a cube has 6 faces
b True
c True

3



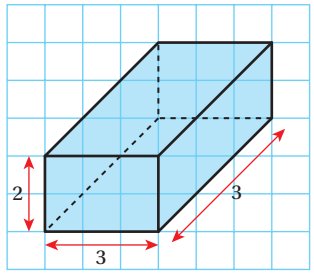
4



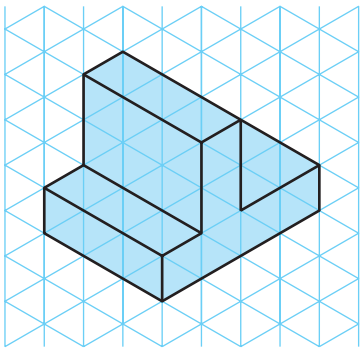
LAUNCHPAD

1 a Cube **b** Pentagonal prism
c Sphere **d** Square-based pyramid

2

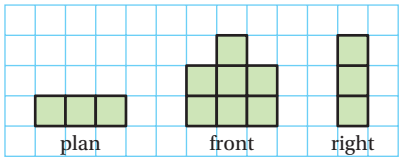


3

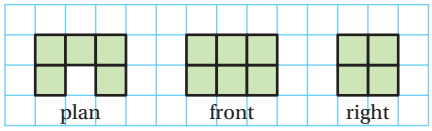


4 C

5 a



b



EXERCISE 20A

1 a Cuboid
b Triangular pyramid
c Triangular prism
d Cube

2 a Cuboid
b Pyramid (triangular- or square-based)
c Cuboid with a square-based pyramid on top
d Cone

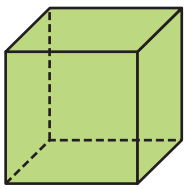
3 Examples:
a ball
b die
c ice cream cone
d tin of baked beans

WORK IT OUT 20.1

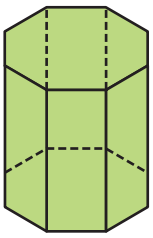
Student A is likely to end up with the correct drawing. The other two students have extended the horizontal part of the shape in wrong directions.

EXERCISE 20B

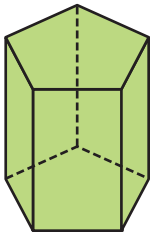
1 a



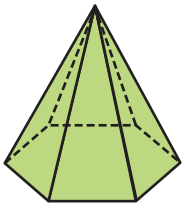
b



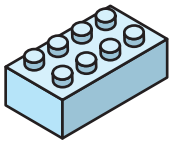
c



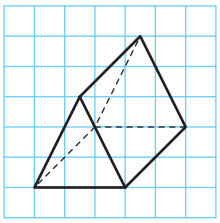
d



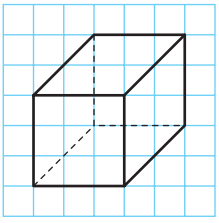
e



2 a i



ii



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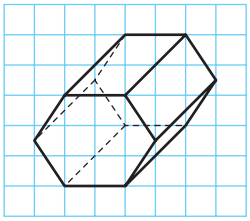
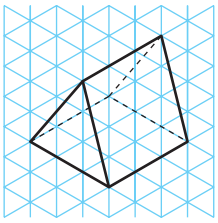
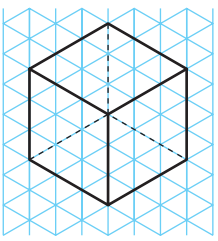


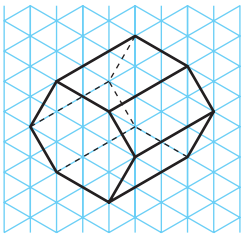
b i



ii



iii

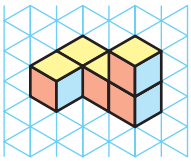


c On a square grid, the objects are drawn as if viewed 'face-on'. On an isometric grid, objects are drawn as if view from one of their edges.

3 a

b

c



4 a 5 **b** 6 **c** 11 **d** 14

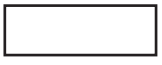
5 a Both
b Both
c Student's own diagrams.

EXERCISE 20C

1 a B **b** B **c** B **d** B **e** A

2 a i B **ii** D **iii** C **iv** A

b i



ii

iii

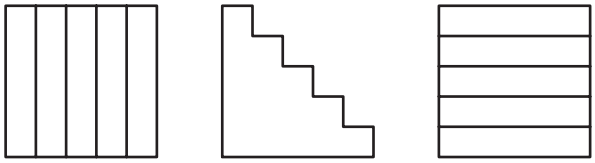
iv

3 a

b

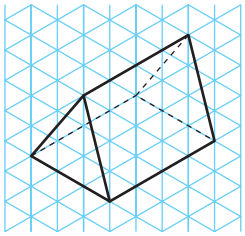
c

4

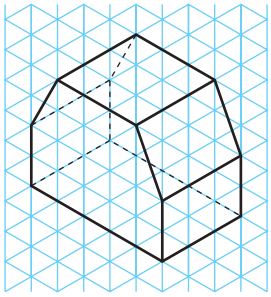


plan front side

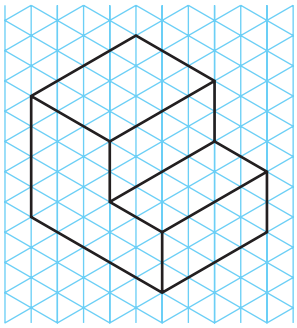
5 a



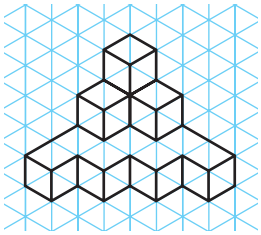
b



c



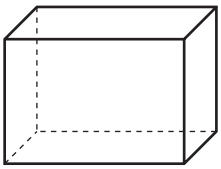
d



CHAPTER REVIEW

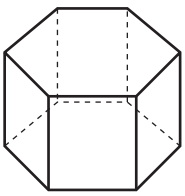
1 a **i** Cuboid **ii** Cylinder **iii** Hexagonal prism
iv Cone **v** Triangular pyramid

b i

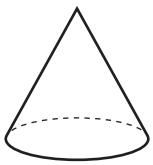


ii

iii



iv



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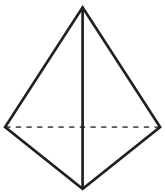
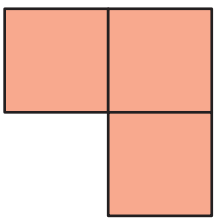


2 a **i** C **ii** D **iii** A **iv** B

b i



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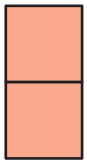

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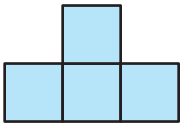

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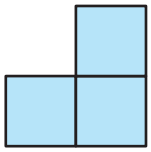

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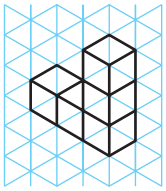
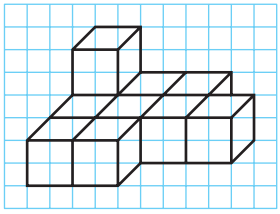
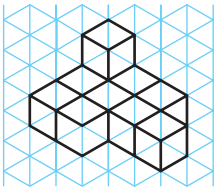


4 a



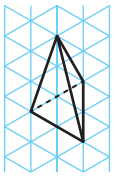
b



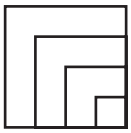
5 a

b

c

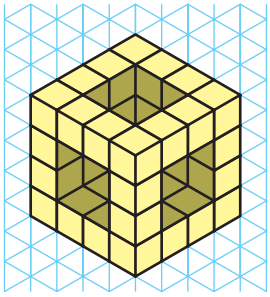


6 a



b 50

7



21 Volume and surface area

BEFORE YOU START ...

1 a Cube **b** Cylinder
c Square-based pyramid **d** Cone
e Polyhedron **f** Triangular prism

2 a Cuboid **b** The shape of the faces.

3 $A = \pi r^2$

4 6 cm^2

LAUNCHPAD

1 125 cm^3

2 120 cm^3

3 Volume = 1.087×10^{12} km^3 surface area = 5.112×10^8 km^2

4 Volume = 8820 m^3

WORK IT OUT 21.1

Calculation A is correct.
B has a − instead of a + in the formula.
C uses the triangle formula.

EXERCISE 21A

1 a Volume = 169.6 cm^3 surface area = 213.37 cm^2
b Volume = 80 cm^3 surface area = 138.16 cm^2
c Volume = 168 m^3 surface area = 244 m^2
d Volume = 141.37 cm^3 surface area = 150.8 cm^2
e Volume = 126 cm^3 surface area = 190 cm^2
f Volume = 42 cm^3 surface area = 96 cm^2

2 720 litres

3 5.24 m

4 2500 m^3

5 33.03 cm^3

6 60 m^2

7 Volume = $w \times b \times h$ the width and breadth of the base remain constant so is proportional to h (height). Volume of oil = 4.05 l

8 6.03 m^2

9 They are the same.

10 a 2764.6 cm^3 **b** 6283.19 cm^3

11 5 cm

12 181.56 m^3

13 127.43 m^3

14 Volume = $x(x + 2)(x + 3) = x^3 + 5x^2 + 6x$
Surface area = $6x^2 + 20x + 12$

15 Volume = $(a + b)^3$

EXERCISE 21B

1 a Volume 109.33 cm^3 surface area 141.37 cm^2
b Volume 45.81 cm^3 surface area 77.75 cm^2
c Volume 65.45 cm^3 surface area 78.54 cm^2
d Volume 56.55 cm^3 surface area 91.4 cm^2
e Volume 134.04 cm^3 surface area 150.8 cm^2

2 2.2×10^{10} km^3

3 a 8659.01 mm^2 **b** 126.68 cm^2 **c** 706.86 cm^2
d 5728.03 mm^2 **e** 1465.74 cm^2 **f** 2026.83 cm^2
g 28.27 cm^2 **h** 17 203.36 mm^2 **i** 153.94 cm^2

4 a Volume 226.72 surface area 235.62
b Volume 9786.68 surface area 2940.53
c Volume 1015.94 surface area 615.75
d Volume 4047.96 surface area 1910.09
e Volume 83 959.91 surface area 23 373.45
f Volume 2069.06 surface area 989.6
g Volume 40 030.35 surface area 7728.32

5 6.12 m^3

EXERCISE 21C

1 a 6082 cm^2 **b** 11 713 mm^3

2 2.487 m^3

3 Volume = 298 174.77 cm^3, surface area = 26 817.48 cm^2

4 a i 13 050 cm^3 **ii** 4180 cm^2
b i 11 281.86 cm^3 **ii** 3602.83 cm^2

5 495 m^3

6 7794.23 cm^3

7 Volume = 130 m^3

8 Assuming a height of 15 cm the radius of a cylinder must be between 4.12 and 4.6 cm.
Assuming a height of 15 cm the radius of a cone must be between 7.14 and 7.98 cm.

EXERCISE 21D

1 a Volume 499.5 cm^3 surface area 423 cm^2
b Volume 480 cm^3 surface area 424.8 cm^2
c Volume 108 cm^3 surface area 171 cm^2
d Volume 28.27 cm^3 surface area 61.6 cm^2

2 2 456 027 m^3

3 54.43 m^3

4 volume = $48x^3$ surface area = $96x^2$

5 0.2887 m^3

6 a volume = 3.182 cm^3 surface area = 15.57 cm^2
b volume = $0.12x^3$ surface area = $1.73x^2$

7 Volume = 152.5 m^3 and surface area = 276.7 m^2

CHAPTER REVIEW

1 11 m^2

2 227.5 cm^3

3 24.64 m^3

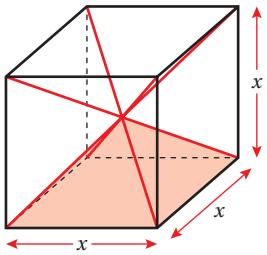
4 49.85 cm^2

5 4186 m^3

6 A

7 a 552.92 cm^3 **b** 276.5 cm^2

8 If the pyramid has a square base with side length x and the height of the pyramid is $\frac{x}{2}$ then you can put 6 of these pyramids together to form a cube as in the diagram below.



The volume of the cube is x^3 and hence the volume of the pyramid is $\frac{1}{6}x^3$. But $\frac{1}{6}x^3 = \frac{1}{3} \times x^2 \times (\frac{1}{2}x) = \frac{1}{3} \times$ (the area of the base) $\times$ (the height).

22 Calculations with ratio

BEFORE YOU START ...

1 a $\frac{2}{5}$ **b** $\frac{3}{4}$

2 28

3 68

LAUNCHPAD

1 4:7

2 8:7

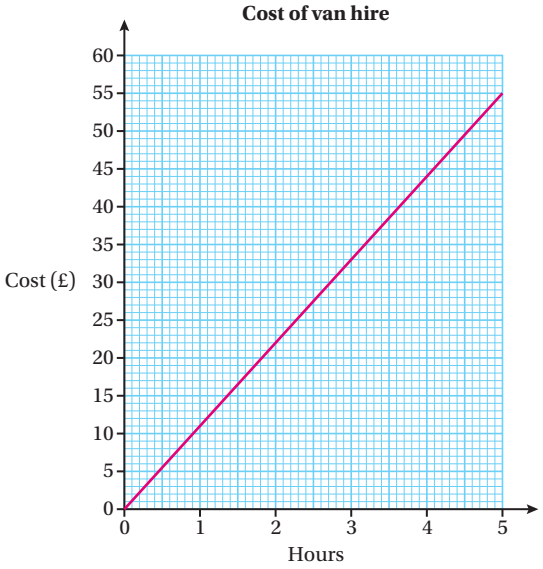
3 15:65 = 3:13

4 10:25

5 345 grams

6 Satsuma delight, Amber, Nectarine night

7 Linear



EXERCISE 22A

1 **a** 45 : 36, simplifies to 5 : 4
b 81 : 9, simplifies to 9 : 1
c 81 : 90, simplifies to 9 : 10
d Yes, 9 pupils per teachers

2 **a** 3 : 5 **b** 1 : 2 **c** 1 : 2

3 **a** 1 : 4 **b** 1 : 2 **c** 2 : 5

4 3

5 1 : 400

6 1 : 950

7 1 : 40

8 **a** **i** 2 : 3 **ii** 2 : 3 **iii** 2 : 3
b They are similar
c 1 : 1

9 11 : : 9

10 3 : 2

11 3 : 2

12 225

13 2 : 5 : 3

EXERCISE 22B

1 **a** 36 : 108 **b** 64 : 80 **c** 132 : 12
d 48 : 72 : 24 **e** 18 : 36 : 90 **f** 16 : 56 : 40 : 32

2 **a** 20 kg **b** $\frac{1}{5}$

3 45 g

4 **a** 200 g flour, 75 g margarine, 75 g lard;
b $\frac{3}{7}$

5 **a** 16 cm by 40 cm
b area is 640 cm^2

6 4.5 litres

7 0.5 litres

8 200 g biscuit, 240 g dried fruit, 80 g butter, 80 g cocoa powder

9 14 oboe players

10 34 pairs

11 18 kg

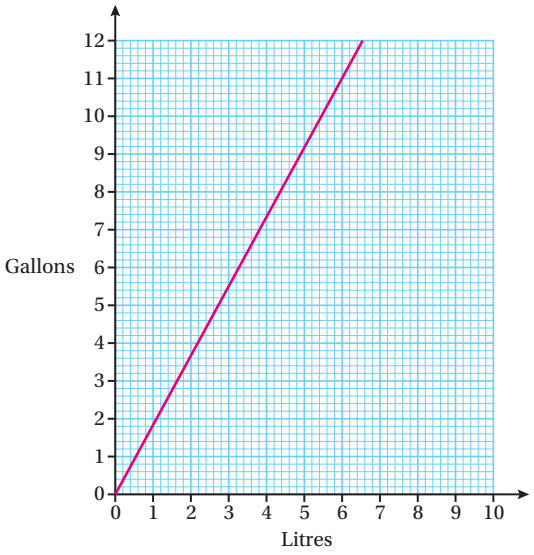
EXERCISE 22C

1 Latte, flat white, cappuccino, espresso, double espresso

2 **a** 5 : 6, 1 : 1.2 **b** 6 : 5, 1 : 0.83

3 **a** 1 : 25 **b** 25 : 1, 1 : 0.04

4 **a**



b 1 : 0.556

5 **a** 75 sweets **b** $\frac{8}{25}$

6 24 km

7 8 sausages, 2 tins of tomatoes; 300 g of potatoes; 6 tsp mixed herbs; 400 ml vegetable stock

8 42 chocolates

9 9 more milk chocolates than dark chocolates

10 **a** 3 : 4 **b** 3 : 2 **c** 1 : 2
d 48 turns **e** 18 turns

11 **a** 5 : 2 **b** 150 cm and 60 cm

12 1 : 2.744

13 1 kg for £1.99

14 **a** 6.7% **b** 145 cm

CHAPTER REVIEW

1 $1 : \pi$

2 It is an isosceles triangle with angles 54°, 54° and 72°.

3 It is a regular pentagon, with each interior angle 108°. The ratio 1 : 1 : 1 : 1 : 1 means all the angles are the same size which means the pentagon must be regular.

4 8

5 It is a right-angled triangles with angles 45°, 90° and 45°; its other sides are $\sqrt{50}$ cm.

6 **a** **i** Joe White 21, Pam Purple 37, Total Purples 77, Total White 53
ii 8 : 7
b 12

7 3 : 4

8 6 white chocolates; 8 milk chocolates; 6 dark chocolates

9 9 : 16

10 **a** 25 : 4 **b** 125 : 8

11 5 : 21

12 2 : 1

13 120, 200, 40

14 1 : 1.6

15 12 silver; 18 blue; 6 red; 9 black; 3 yellow

23 Basic probability and experiments

BEFORE YOU START ...

1 **a** 0.0312 **b** 1 **c** 0.04

2 **a** is correct; **b** and **c** are incorrect. The answer in **b** should be rounded up to 0.317, and **c** should be 1.0 (include the point zero to show the answer is rounded).

3 $\frac{39}{52} = 75\%$ $0.25 = \frac{13}{52}$ $0.077 = \frac{4}{52}$ $50\% = \frac{26}{52}$

LAUNCHPAD

1 **a** 50 **b** $\frac{113}{300}$

c 245 based on the experimental data, 108 based on the theoretical probability

2 **a** $\frac{1}{36}$ **b** 7, $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{6}$

EXERCISE 23A

1 **a**

	Outcome	Predicted probability
i	A total $\leqslant 12$	Certain
ii	An even number	Even chance
iii	An odd number	Even chance
iv	A total of 1	Impossible
v	Exactly 12	Unlikely
vi	A total > 4	Likely or highly likely

b Students' own investigations.

c Students' own conclusions based on their investigations.

2 **a** 0.53 or $\frac{8}{15}$ **b** $\frac{9}{50} = 0.18$ **c** $\frac{9}{20} = 0.45$

3 **a** $\frac{7}{40} = 0.175$ **b** $\frac{33}{40} = 0.825$ **c** $\frac{1}{20} = 0.05$ **d** $\frac{21}{80} = 0.2625$

4 **a** 403

b

Brand	Frequency	Relative frequency
Silk-e-shine	123	$\frac{123}{403} = 0.3052$
Get knotted	105	$\frac{105}{403} = 0.2605$
Goldilocks	83	$\frac{83}{403} = 0.2060$
Bubbly stuff	89	$\frac{89}{403} = 0.2208$
Total	403	

c 0.3052

5 **a**

Result	Frequency	Relative frequency
Spoke to customer	122	0.61
Phone not answered	44	0.22
Left message on answering machine	22	0.11
Phone engaged or out of order	10	0.05
Wrong number	2	0.01

b **i** Likely **ii** Unlikely **iii** Highly unlikely

6 **a** 479 **b** 0.40 **c** 0.64

7 0.668

8 **a** 0.1847 **b** 0.246 **c** 0.3337 **d** 0.754

EXERCISE 23B

1 $\frac{17}{36}$

2 $\frac{1}{2}$

3 Vijay, there are more even number products that odd.

4 **a** No, this is not an equally likely outcome, in English for example, few names start with X. (In China the outcomes might be different!)

b No, the chance of a boy/girl remain the same each time a woman gives birth, in probability the gender at birth is taken to be equally likely but in the real world they are notequally likely outcomes; different countries tend to have different probabilities based on relative frequencies.

c Technically yes, but realistically no, as skill and experience and ranking in the league will all affect the outcomes.

d This could be correct, but unlikely. People are more likey to take holidays in the summer months (which incudes August) than in other months.

e No, will depend on who they are playing, which players are playing and so on.

WORK IT OUT 23.1

Option B is correct

Option A is wrong because the fractions have not been converted to percentages correctly.

EXERCISE 23C

1 0.74 (2 dp)

2 **a** The answer depends on whether the student decides that 0 is divisible by 2 (they should provide reasons for their decision). If they decide 0 is divisible by 2, the answer is $\frac{1}{2}$; if not, the answer is $\frac{49}{100}$.

b 0.91 **c** 0.12 **d** 0.88

3 0.568

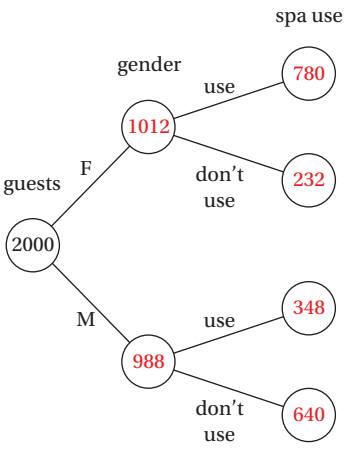
4 **a** 0.16 **b** 0.84 **c** 0.6

d Strawberry 63, Lime 66, Lemon 54, Blackberry 69, Apple 48

5 **a** 0.4 **b** 0.97 **c** 11 **d** 114

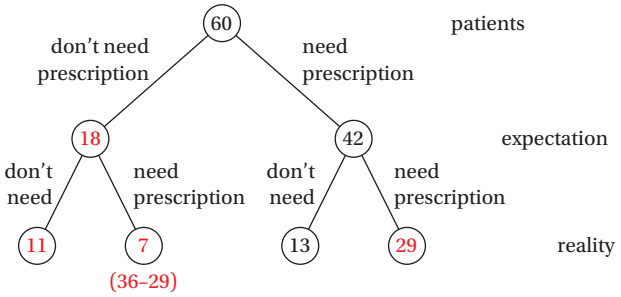
EXERCISE 23D

1 **a**



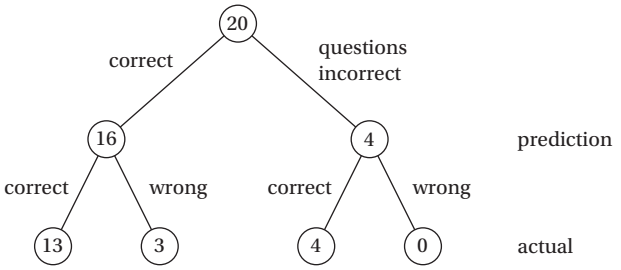
b Female

2 a



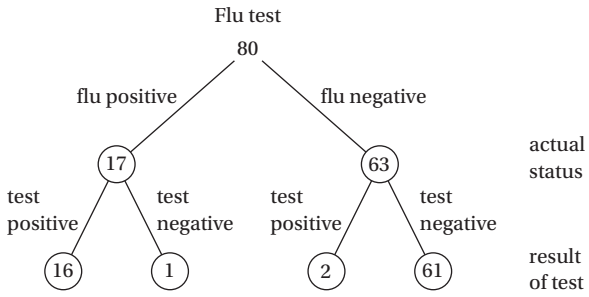
b $\frac{29}{42}$ **c** 38.9%

3 a



b He predicted quite poorly as all the ones he thought he got wrong were actually correct.

4



EXERCISE 23E

1 a No, there are more multiples of three than there are multiples of two. **b** $\frac{3}{4}$

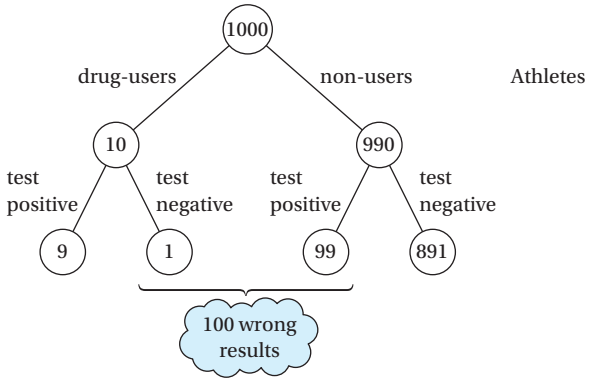
2 Busi is using theoretical probability to argue for an equal number of heads and tails.
Busi is correct to say you *should* get 10 tails, but completely wrong to suggest that you *must* get 10 tails from 20 coin tosses. 20 tosses is too small a trial to decide that the coin is biased.

3 a 0.005
b Students' answers should acknowledge that ½% is a very small margin of error and most tests would be correct. However, parents are correct in claiming that 4 students could incorrectly test positive. The school could agree to retest anyone who gets a positive result for greater accuracy.
c 195

4 a

Status	Test positive (ie fail drug test)	Test negative (ie pass drug test)	Total
Athletes who are using illegal substances	9	1	10
Athletes who are not using illegal substances	99	891	990
Total	108	892	1 000

b



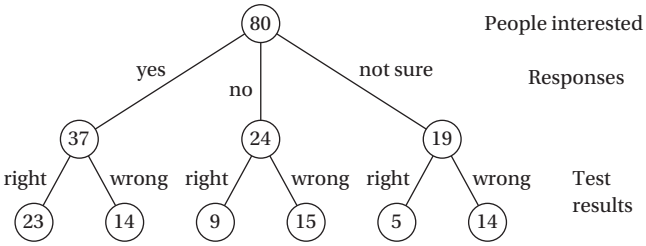
c 91.7%
d No. Diagrams show there are 100 wrong results, so test is only 90% accurate. There is also a 1 in 1000 chance that a drug user will text negative.

5 Students' own opinions, but it seems to be biased in favour of red.

6 a 212 **b** 89
c **i** 0.29 **ii** 0.63 **iii** 0.27
d Students' own ideas.

7 a 42% **b** 0.042 **c** 35 **d** 0.042 or 4.2%
e Outcomes are not equally likely.

8



CHAPTER REVIEW

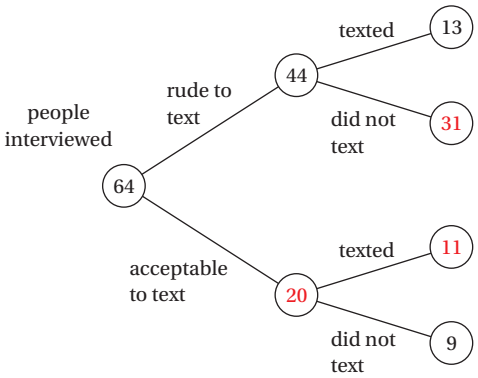
1 a 10 000 times
b Heads: 0.4083; Tails: 0.5917
c 0.41
d She probably is, as this is a large number of coin tosses and tails comes up 59% of the time.

2 a $\frac{1}{2}$ **b** $\frac{9}{10}$ **c** $\frac{9}{10}$ **d** $\frac{1}{2}$

3 5750

4 97.35%

5



6 a Hearts 0.238, Diamonds 0.24, Spades 0.264, Clubs 0.258
b The relative frequencies are all quite close to 25% (theoretical frequency) so it may be fair and unbiased, but you would only be able to tell with a larger number of trials.

24 Combined events and probability diagrams

BEFORE YOU START ...

1 a $\frac{3}{8}$ **b** $\frac{13}{15}$ **c** $\frac{2}{5}$
d $\frac{4}{15}$ **e** 0.18

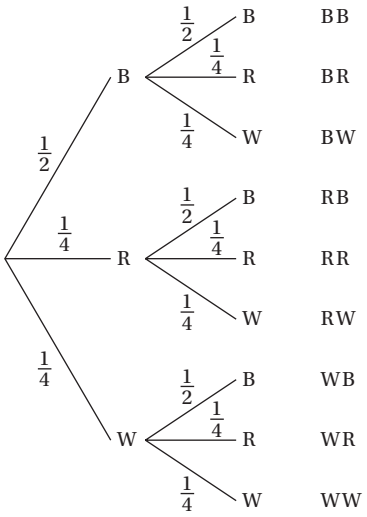
2 a Relative frequency **b** Outcomes
c Event **d** Equally likely **e** Random

3 a FF, FM, MM, MF
b HH, HT, TH, TT
c AB, AC, BA, BC, CA, CB

LAUNCHPAD

1 a 30 **b** 19 **c** 3 **d** $\frac{4}{15}$

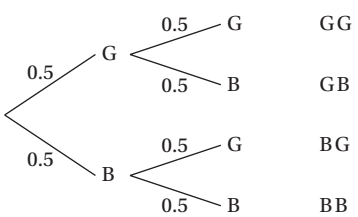
2



3 a $\frac{1}{52}$ **b** $\frac{4}{13}$ **c** $\frac{9}{13}$

4 a $\frac{5}{9}$ **b** $\frac{41}{81}$ **c** $\frac{20}{81}$

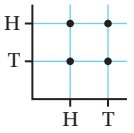
5 a



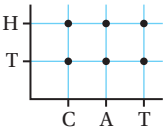
b $\frac{1}{2}$

EXERCISE 24A

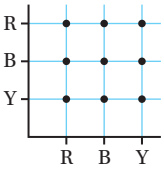
1 a



b



c



2 a i

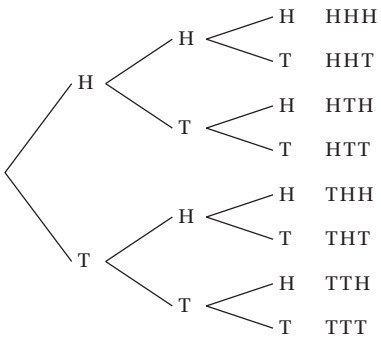
Dice 2 \ Dice 1	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

ii

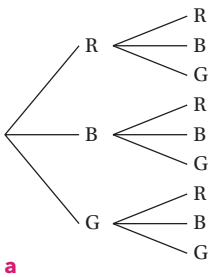
Spinner \ Coin	H	T
A	HA	TA
B	HB	TB
C	HC	TC
D	HD	TD

b Students' own answers.

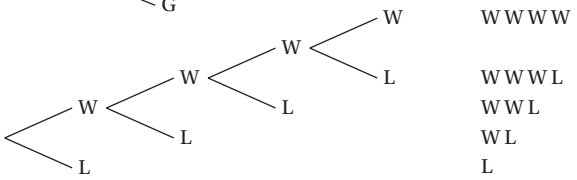
3



4

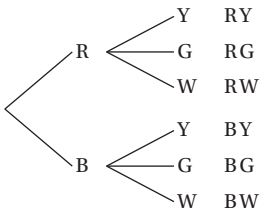


5 a



b 5 **c** $\frac{1}{2}$

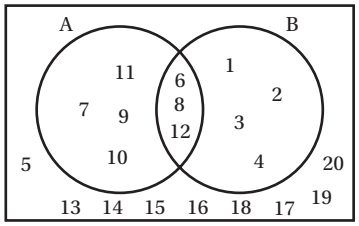
6 a



b No. In reality people have colour preferences, so relative frequency of choices would probably not be equal.

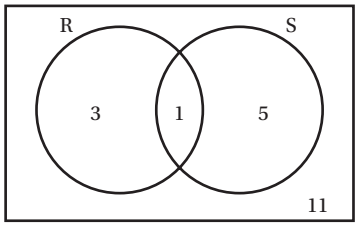
EXERCISE 24B

1 a



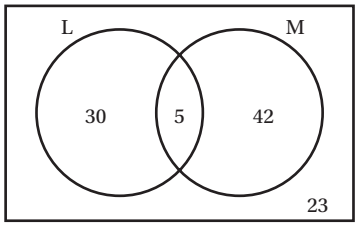
b i {6, 8, 12} **ii** {1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12}
iii 7 **iv** 13 **v** 13

2



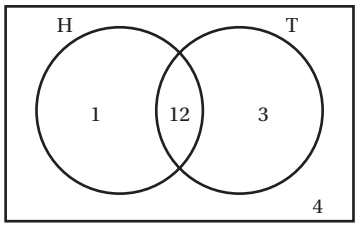
P(not red, not sports shoes) = $\frac{11}{20}$

3



P(tie hair back) = $\frac{1}{20}$

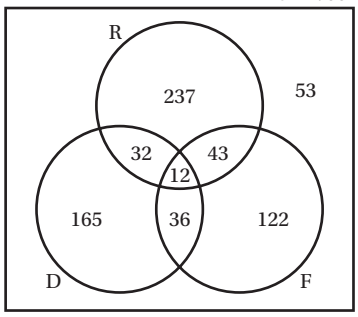
4



12 students wearing headphones and sending texts

5 **a** $\frac{3}{10}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$

6 $\mathscr{E} = 700$



a 12 **b** 36 **c** 165

WORK IT OUT 24.1

Option B is correct
If entrances numbered 1 and 2, and exits labelled A, B and C, the possible combinations are:
1A, 1B, 1C, 2A, 2B, 2C

EXERCISE 24C

1 2 763 633 600 options if the order of the numbers on the ticket matters; 3 276 000 options if the order of numbers does not matter

2 3 838 380

3 45

4 3024

5 1024

6 336 if the order of the fillings matters, 56 if the order of fillings does not matter (the answers assume 3 different fillings)

EXERCISE 24D

1 **a** Possible sample space:

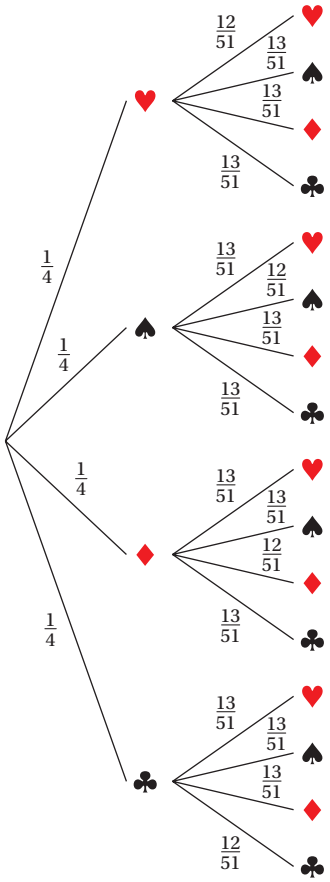
	U	E	E
D	DU	DE	DE
N	NU	NE	NE
D	DU	DE	DE

b $\frac{4}{9}$ **c** $\frac{2}{3}$ **d** $\frac{2}{9}$

2 **a** $\frac{1}{15}$ **b** $\frac{2}{15}$ **c** $\frac{1}{45}$ **d** $\frac{1}{30}$

e $\frac{2}{45}$ **f** $\frac{1}{15}$ **g** $\frac{7}{30}$

3 **a**



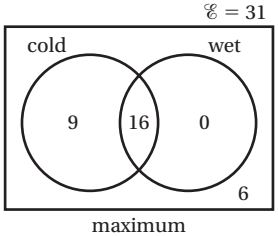
b **i** $\frac{3}{51}$ **ii** $\frac{3}{51}$ **iii** $\frac{13}{51}$

4 **a** **i** $\frac{1}{24}$ **ii** $\frac{1}{24}$ **iii** 0

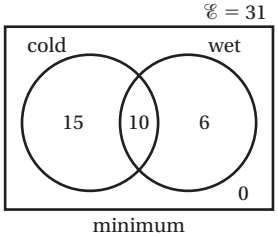
b $\frac{1}{4}$ **c** $\frac{1}{8}$ (ABC, ACD or BCD)

5 **a** $\frac{1}{4}$ **b** $\frac{1}{13}$ **c** $\frac{1}{52}$ **d** $\frac{4}{13}$

6



maximum

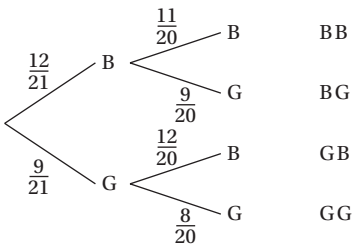


minimum

Maximum = 16 days
Minimum = 10 days

EXERCISE 24E

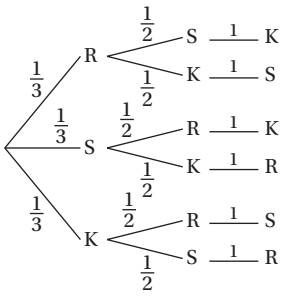
1 a



b i $\frac{11}{35}$ ii $\frac{6}{35}$ iii $\frac{9}{20}$

c i $\frac{6}{95}$ ii $\frac{111}{133}$ iii $\frac{9}{19}$

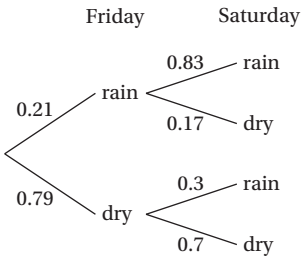
2 a



b Dependent, because one affects the other

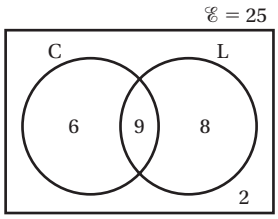
c 1 **d** 6 **e** $\frac{1}{6}$ **f** $\frac{1}{2}$

3 a



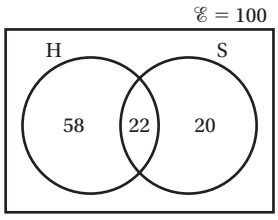
b i 0.1743 ii 0.3

4



a $\frac{3}{5}$ **b** $\frac{9}{17}$

5 a



b i 0.58 ii $\frac{11}{40}$ or 0.275

6 $\frac{4}{15}$ or 0.27 (2 dp)

7 a $\frac{5}{14}$ **b** $\frac{9}{14}$ **c** $\frac{15}{28}$ **d** $\frac{2}{7}$

8 a $\frac{13}{15}$ **b** 0.12 (2 dp)

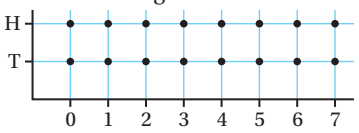
9 a If these two probabilities are the same then the events are independent.

b If the marble is returned, then the probability of a red on the second draw is still $\frac{1}{8}$. If the probability of drawing a blue first is $\frac{x}{y}$, there is no whole number value of y that can lead to a combined denominator of 15 when the two probabilities are multiplied. So, the marble is NOT returned to the bag.

CHAPTER REVIEW

1 Student's own diagrams. Possible answers could be:

a

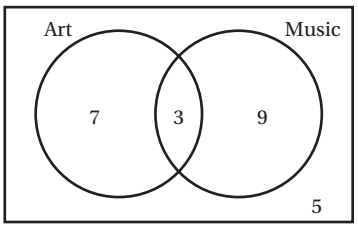


b

	Pink	Yellow
A	AP	AY
B	BP	BY
C	CP	CY

c P A N A N P N A P $3 \times 2 \times 1 = 6$
P N A A P N N P A

d

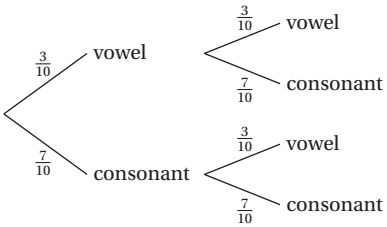


2 Student's own diagrams, but table as per page 433 is most useful.

a $\frac{1}{36}$ **b** $\frac{11}{36}$ **c** $\frac{1}{6}$ **d** $\frac{11}{36}$

3 a $\frac{3}{10}$

b

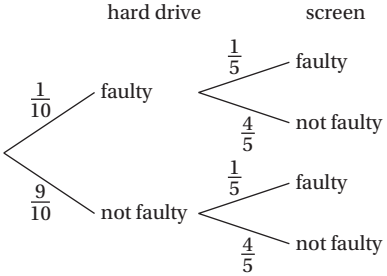


c i $\frac{9}{100}$ ii $\frac{49}{100}$ iii $\frac{21}{50}$ iv $\frac{91}{100}$

d Letter are replaced, so independent.

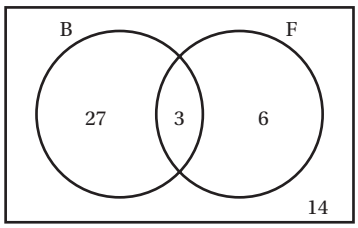
e Remove a letter and don't replace it.

4 a



b $\frac{18}{25}$

5



a $\frac{7}{25}$ **b** $\frac{27}{50}$ **c** $\frac{1}{10}$ **d** $\frac{7}{10}$

6 9 979 200

25 Powers and roots

BEFORE YOU START ...

1 a < **b** = **c** > **d** =

2 a B **b** C **c** C

3 a $\frac{4}{3}$ **b** $\frac{1}{12}$ **c** $\frac{5}{7}$

LAUNCHPAD

1 a 4^5 **b** 8^3 **c** 5^2

d 9^7 **e** $\left(\frac{1}{3}\right)^4$ **f** $\left(\frac{1}{2}\right)^3$

2 a $3^5 = 243$ **b** $4^2 = 16$ **c** $6^0 = 1$

d $6^2 = 36$ **e** $4^{-3} = \frac{1}{4^3} = \frac{1}{64}$ **f** $2^6 = 64$

3 £1610.51

EXERCISE 25A

1 a

Base \ Index	−3	−2	−1	0	1	2	3	4	5
2	$2^{-3} = \frac{1}{8}$	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
3	$3^{-3} = \frac{1}{27}$	$3^{-2} = \frac{1}{9}$	$3^{-1} = \frac{1}{3}$	$3^0 = 1$	$3^1 = 3$	$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$
4	$4^{-3} = \frac{1}{64}$	$4^{-2} = \frac{1}{16}$	$4^{-1} = \frac{1}{4}$	$4^0 = 1$	$4^1 = 4$	$4^2 = 16$	$4^3 = 64$	$4^4 = 256$	$4^5 = 1024$
5	$5^{-3} = \frac{1}{125}$	$5^{-2} = \frac{1}{25}$	$5^{-1} = \frac{1}{5}$	$5^0 = 1$	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3125$

b The negative powers are unit fractions of the matching positive powers.

c Every second power of two is equivalent to a power of four.

d If it does not end in 5 it cannot be a power of 5 except for 5^0, which is 1.

2 a T **b** F **c** T **d** F **e** T **f** F
g F **h** T **i** T **j** T **k** F **l** T

3 a 5 **b** 2 **c** 4 **d** 5 **e** 3 **f** 4
g 7 **h** 50 **i** 5 **j** 30 **k** 20 **l** 25

4 a 8 **b** 36 **c** 1 **d** 512
e 10 000 **f** 1 000 000 **g** 7 **h** 50
i 64 **j** 32 **k** 128 **l** 729
m 2 **n** 16 **o** 49 000 **p** 7 000 000
q 3200 **r** 36 000 000

5 a 4096 **b** 1728 **c** 32 768 **d** 2401
e 3375 **f** 10 000 **g** 784 **h** 15 625

6 a 1472 **b** 159 775 **c** 759 375
d 531 522 **e** 985 **f** 343

7 a > **b** < **c** > **d** < **e** < **f** >

EXERCISE 25B

1 a $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{4}$ **d** $\frac{1}{3^2}$ **e** $\frac{1}{4^3}$ **f** $\frac{1}{3^5}$
g $\frac{1}{3^4}$ **h** $\frac{1}{6^6}$ **i** $\frac{1}{34^5}$ **j** $\frac{1}{x^3}$ **k** $\frac{1}{m^2}$ **l** $\frac{3}{x^4}$

2 a 3^{-1} **b** 5^{-1} **c** 7^{-1} **d** 3^{-2} **e** 4^{-5} **f** 2^{-6}
g 7^{-2} **h** 10^{-5} **i** 2^{-2} **j** 12^{-3} **k** 10^{-4} **l** $3^{-1} \times 2^{-2}$
m x^{-2} **n** x^{-3} **o** $4y^{-2}$

3 a = **b** = **c** = **d** ≠ **e** = **f** =
g = **h** = **i** ≠ **j** ≠ **k** ≠ **l** =

4 a 2^1 **b** 2^4 **c** 2^6 **d** 2^{-3} **e** 2^{-2} **f** 2^0
g 2^{-5} **h** 2^2 **i** -2^3

EXERCISE 25C

1 a 2^7 **b** 10^7 **c** 4^6 **d** 5^7
e 2^{11} **f** 3^{-2} **g** 2^3 **h** 3^2
i 2^{-1} **j** 3^5 **k** 10^1 **l** 10^0

2 a 6^2 **b** 10^3 **c** 6^2 **d** 6^{-2}
e 10^{-2} **f** 3^{10} **g** 3^7 **h** 10^0
i 5^6 **j** 10^{10} **k** 3^1 **l** 2^{-3}

3 a 2^6 **b** 2^9 **c** 2^8 **d** 10^4
e 10^6 **f** 10^8 **g** 2^{-12} **h** 10^{-4}
i 10^{-6} **j** 3^{-8} **k** 2^0 **m** 2^{10}

4 a T **b** F: 3^6 **c** T
d T **e** T **f** T
g F: 3^8 **h** T **i** T

EXERCISE 25D

1 a $\sqrt{3}$ **b** $\sqrt[3]{4}$ **c** $\sqrt[4]{5}$ **d** $\sqrt{6}$
e $\sqrt[9]{4}$ **f** $\left(\sqrt[3]{5}\right)^2$ or cube root of (5^2)
g $\left(\sqrt[8]{4}\right)^3$ or eighth root of (4^3)
h $\left(\sqrt[9]{6}\right)^2$ or ninth root of (6^2)

2 a $6^{\frac{1}{2}}$ **b** $4^{\frac{1}{3}}$ **c** $11^{\frac{1}{3}}$ **d** $9^{\frac{1}{4}}$
e $3^{\frac{4}{3}}$ **f** $7^{\frac{1}{5}}$ **g** $7^{\frac{2}{3}}$ **h** $2(3^{\frac{5}{3}})$

3 a 2 **b** 2 **c** 16
d 36 **e** 64 **f** $\frac{1}{4}$
g $\frac{1}{625}$ **h** $\frac{3}{2}$ **i** $\frac{3}{2}$

EXERCISE 25E

1 Students' estimates may vary slightly.
a 8.49 **b** 5.74 **c** 2.45
d 3.07 **e** $^{-}5.85$ **f** 2.47

2 Answers will depend on students' solutions to question 1

3 Students' estimates may vary slightly.
a $x \approx 4.64$ **b** $x \approx 3.26$ **c** $x \approx 8.13$
d $x = 6$ **e** $x \approx 2.29$ **f** $x \approx 19.13$

4 $a = 9$; $m = 2$, $a = {}^{-}9$; $m = 2$, $a = 3$; $m = 4$, $a = {}^{-}3$, $m = 4$

5 9.49 mm

6 9.28 mm

EXERCISE 25F

1 **a** £4072.24 **b** £3257.79 **c** $F = P(1.03)^{\frac{3}{2}}$ **d** £3345.07

2 **a** Suki is correct. Marie entered the power $\frac{3}{5}$ without brackets, and as exponents get preference in operations, the calculator finds the cube and then divides the total by 5.
b 163.69 mg **c** 133.64 mg **d** 93.3%

3 **a** 0.86 seconds
b **i** Matt's as its falling a shorter distance **ii** 0.14 seconds

4 **a** Students' own answers.
b Elephant: 146.3 seconds, Human: 50.3 seconds. Elephant's blood takes 96 seconds longer to circulate.
c 0.027 kg
d Mouse: 0.00082 m^2; Cat 0.0275 m^2
e Approximately 2925 calories
f The roots and powers do not give exact rational answers. Plus the formulae are based on mean values and all animals are unique.

5 **a** 365.22
b Jupiter 4343.4 days; Uranus 30761.8 days. Uranus takes 26 418 days longer (approx 72.4 years longer).
c Mercury. It is closest, so has the smallest orbit. 88 days.

CHAPTER REVIEW

1 **a** 8^5 **b** 3^3 **c** 8^{-2} **d** $5^{\frac{3}{2}}$

2 **a** $\sqrt{81}$, $2 \times \sqrt{121}$, 4^3, 3^4, 10^2 **b** 96^0, 3^2, 20^2, 5^4, 10^3, 4^5
c $\left(\frac{1}{4}\right)^{\frac{3}{2}}$, $\sqrt[3]{4}$, $4^{\frac{3}{2}}$, 4^2

3 **a** $\frac{1}{3^3}$ **b** $\frac{1}{2^{10}}$ **c** $\frac{1}{5^2}$

4 **a** 4^4 **b** 4^3 **c** 4^4 **d** 4^{-2} **e** 4^6 **f** 4^{-4}

5 **a** 11 **b** 0.5 **c** 5 **d** 2 **e** 3 **f** $\frac{1}{2}$
g 27 **h** 9 **i** 16

6 3.27 cm

7 $\sqrt{2000 \times 24.2} = \sqrt{48400} = 220$ volts

8 **a** About 1 second
b The longer the pendulum, the longer it takes to complete one swing.

9 Yes they will fit. Container has $r = 3.64$ cm, so $d = 7.28$ cm. Biscuit diameter of 7 cm will fit.

10 **a** **i** 1 **ii** $\frac{1}{64}$ **b** **i** 3 miles **ii** 96 feet

26 Standard form

BEFORE YOU START ...

1 **a** 8.7 **b** 8.7 **c** 6.75 **d** 15

2 **a** 510 000 000 **b** 1.10 **c** 0.006

3 **a** False: should be 3^5
b True **c** True
d False: should be x^{-1} **e** True

LAUNCHPAD

1 D

2 C

3 B

4 E

5 5.52×10^{10}

6 0.000 000 25

7 **a** 8×10^5 **b** 4.05×10^{11} (2 dp)
c 7.87×10^6

EXERCISE 26A

1 **a** 3.21×10^5 **b** 1.34×10^3 **c** 4.005×10^4
d 3.01×10^6 **e** 8×10^{-2} **f** 1×10^{-4}
g 3.2×10^7 **h** 9.1×10^5 **i** 3.1255×10^{-5}
j 2.4152×10^{-7} **k** 3.05×10^{-3} **l** 2.01×10^{-1}
m 3.4×10^4 **n** 3.4×10^{-4} **o** 9×10^{-3}
p 2.45×10^0 **q** 4.26×10^{-4} **r** 4.26×10^{-1}

2 **a** 140 **b** 48 000 **c** 2900 **d** 325
e 0.325 **f** 367 000 **g** 45 000 000 **h** 0.0213
i 32 090 **j** 0.003 46 **k** 0.000 189 **l** 0.000 000 7
m 0.0103 **n** 0.001 025 **o** 0.000 020 9

3 **a** 7×10^9 **b** 2.4×10^5 **c** 1×10^{14} **d** 2×10^{-7}
e 1.4×10^8 **f** 1×10^{-10} **g** 6.25×10^6 **h** 7.53×10^{-10}

4 **a** 31 800 000 **b** 0.000 74 **c** 0.000 001 24
d 3 000 000 000 **e** 0.000 12 **f** 778 000 000
g 150 000 000 000
h 0.000 000 000 000 000 000 000 000 000 000 910 938 22

5 3×10^{-3}, 5.7×10^{-2}, 9.9×10^1, 1.75×10^2, 1.75×10^4, 3.654645×10^{20}

EXERCISE 26B

1 Display will vary depending on calculator used.

2 **a** 1.09×10^5 **b** 2.876×10^{-6} **c** 4.012×10^9
d 1.89×10^7 **e** 3.123×10^{13} **f** 2.876×10^{-4}
g 9.02×10^{15} **h** 8.076×10^{-12} **i** 8.124×10^{-11}

EXERCISE 26C

1 **a** 5.62×10^{21} **b** 6.56×10^{-17} **c** 1.28×10^{-14}
d 1.44×10^{13} **e** 1.58×10^{-20} **f** 5.04×10^{18}
g 1.98×10^{12} **h** 1.52×10^{17} **i** 2.29×10^8
j 2.09×10^{-8}

2 **a** 1.3607×10^{18} **b** 1.0274×10^{-15} **c** 1.0458×10^0
d 1.6184×10^{11} **e** 5.2132×10^{19} **f** 3.0224×10^{-16}
g 2.3141×10^{12} **h** 3.8066×10^{17} **i** 3.4760×10^{-3}

EXERCISE 26D

1 **a** 8×10^{30} **b** 4.2×10^{12} **c** 2.25×10^{26}
d 1.4×10^{32} **e** 3×10^{1} **f** 2×10^{1}
g 3×10^{3} **h** 3×10^{42}

2 **a** 8×10^{-20} **b** 6.4×10^{-12} **c** 3.15×10^{-9}
d 3.3×10^{-3} **e** 2×10^{33} **f** 7×10^{-37}
g 5×10^{12} **h** 1.65×10^{1}

3 **a** 1.2×10^{31} **b** 4.5×10^{11} **c** 3.375×10^{36}
d 1.32×10^{-11} **e** 2×10^{26} **f** 2.67×10^{5} (3 sf)
g 1.2×10^{2} **h** 2×10^{-3}

4 **a** 3×10^{9} m **b** 6×10^{9} m
c 3×10^{10} m **d** 6×10^{11} m

5 **a** 3.125×10^{7} times; 31 250 000 times
b 4.5×10^{16} blinks

EXERCISE 26E

1 **a** 5×10^{8} **b** 1.5×10^{-3} **c** 3.15×10^{6}
d 5.6×10^{7} **e** 3.4×10^{-3} **f** -2×10^{-2}

2 **a** The Pacific **b** 5.9×10^{7} km^2
c Total area $= 3.61 \times 10^{8} - (2.71 \times 10^{8}) = 0.9 \times 10^{8} = 9 \times 10^{7}$ km^2

3 9.276×10^{7} miles

4 **a** Virus A **b** 2.7×10^{-7} m **c** 3.3×10^{-7} mm

EXERCISE 26F

1 **a** 1.07×10^{9} **b** 1.10×10^{12}

2 **a** 100 pixels **b** 1.61×10^{-2} cm^2 **c** 4800 pixels
d 4×10^{-8} cm^2 **e** 1×10^{-5} cm^2

3 **a** 3.3×10^{10} nm **b** 2.1×10^{7} nm

4 **a** 500 seconds $= 5 \times 10^{2}$ seconds
b 19 166.67 seconds $= 1.92 \times 10^{4}$ seconds (3 sf)

5 **a** 5.848×10^{11} cells **b** 3.743×10^{13} cells (4 sf)

6 Student's individual problems.

CHAPTER REVIEW

1 **a** 4.5×10^{4} **b** 8×10^{1} **c** 2.345×10^{6}
d 3.2×10^{10} **e** 6.5×10^{-3} **f** 9×10^{-3}

2 **a** 2500 **b** 39 000
c 426 500 **d** 0.000 010 45

3 **a** 9.05×10^{6} **b** 7.848×10^{-1}
c 4.04×10^{10} **d** 3.18×10^{-1}
e 3.429×10^{8} (4 sf) **f** 3.757×10^{10} (4 sf)

4 **a** 5.74×10^{7} **b** 1.764×10^{15}
c 3.4×10^{4} **d** 1.33×10^{1} (3 sf)

5 **a** 8×10^{-8} m **b** 1×10^{-8} m **c** 4×10^{-7} m

6 **a** The Sun **b** 6.051×10^{6}

7 **a** 8.46×10^{6} km^2 **b** Mongolia, India **c** 401 people/km^2

8 **a** 6.35×10^{3} km (3 sf)
b 3.99×10^{4} km (3 sf)
c 1.073×10^{12} km^3 (4 sf)

27 Surds

BEFORE YOU START ...

1 **a** iii **b** ii **c** i

2 **a** $a^2 = b^2 + c^2$ **a** $x^2 + y^2 = z^2$

3 **a** True
b False. Answer is 1, because $y^0 = 1$,not y.

4 **a** Answer should be $3x + 3y$; y terms means $1y$.
b Answer should be $2x^2 + 2x$; x^2 and x are unlike terms so you cannot add them.
c Answer should be $4x - y - 2$; $-2y + y$ gives a negative coefficient for y.

LAUNCHPAD

1 **a** 3.61 cm **b** $\sqrt{13}$ cm
c 13.032 cm and 13 cm; $\sqrt{13}$, the exact length of the square, is irrational, so any decimal value of this number will be an approximation and when squared will only give an approximation of the area of the square

2 **a** Draw a line of 5 cm.
b Construct a right angled triangle with one 1 cm side and one 3 cm side.

3 **a** 3 **b** 4 **c** $2\sqrt{3}$ **d** $3\sqrt{3}$ **e** $\sqrt{3}$

4 **a** $\frac{2\sqrt{5}}{5}$ **b** $^{-}3(1 + \sqrt{2})$

EXERCISE 27A

1 **a** 2.646 **b** 3.464 **c** 7.141
d 8.660 **e** $^{-}1.732$ **f** $^{-}6.856$

2 **a** 2.828 **b** 6.708 **c** $^{-}10.392$
d 14.142 **e** 13.856 **f** $^{-}25.456$

3 **a** 3.146 **b** 1.414 **c** 2.236
d 2.449 **e** 10.172 **f** 3.244

EXERCISE 27B

1 **a** $\sqrt{14}$ cm **b** $\sqrt{20}$ m **c** $\sqrt{17}$ cm

2 $C = 2\pi\sqrt{3}$ cm

3 27

4 $\sqrt{7}$ cm

5 **a** $\sqrt{50}$ m **b** 10 m

6 **a** $\sqrt{2}$ cm **b** 2 cm^2
c **i** 1.41 cm **ii** 1.414 cm **iii** 1.4142 cm
d **i** 1.9881 cm^2 **ii** 1.999396 cm^2 **iii** 1.99996164 cm^2
e Exact area: $2 \times 100 \times 1245 =$ £249 000
Area ii: $1.9881 \times 100 \times 1245 =$ £247 518
Area ii: $1.999396 \times 100 \times 1245 =$ £248 924
Area iii: $1.99996164 \times 100 \times 1245 =$ £248 995
f Nico would get the most profit if he used exact value as this gives the highest price for the metal.

EXERCISE 27C

1 **a** C **b** B **c** B **d** C **e** E

2 **a** $2\sqrt{2}$ **b** $2\sqrt{6}$ **c** $2\sqrt{7}$ **d** $3\sqrt{5}$
e $3\sqrt{6}$ **f** $2\sqrt{17}$ **g** $2\sqrt{15}$ **h** $3\sqrt{14}$
i $3\sqrt{10}$ **j** $10\sqrt{2}$ **k** $3\sqrt{13}$ **l** $9\sqrt{3}$

3 They are all prime numbers, so they do not have factors that are squares.

4 **a** $6\sqrt{2}$ **b** $^{-}8\sqrt{6}$ **c** $10\sqrt{5}$
d $^{-}10\sqrt{15}$ **e** $6\sqrt{14}$ **f** $^{-}16\sqrt{2}$
g $^{-}12\sqrt{5}$ **h** $^{-}6\sqrt{17}$ **i** $42\sqrt{3}$

5 a i $\sqrt{4}\times\sqrt{7}=\sqrt{4\times7}=\sqrt{28}$
ii $^{-}\sqrt{9}\times\sqrt{6}=^{-}\sqrt{9\times6}=^{-}\sqrt{54}$
b Cannot have a square root of a negative number

6 a $\sqrt{18}$ **b** $\sqrt{48}$ **c** $\sqrt{54}$
d $\sqrt{176}$ **e** $^{-}\sqrt{28}$ **f** $^{-}\sqrt{27}$
g $^{-}\sqrt{272}$ **h** $^{-}\sqrt{44}$ **i** $\sqrt{432}$

7 a Students' own descriptions.
b i $2\sqrt{3}, 3\sqrt{3}, 4\sqrt{2}$ ii $5\sqrt{7}, 8\sqrt{3}, 6\sqrt{7}$
iii $2\sqrt{10}, 4\sqrt{3}, 3\sqrt{7}$ iv $6\sqrt{3}, 8\sqrt{2}, 5\sqrt{6}$

WORK IT OUT 27.1

Question 2 is wrong (C is correct). Student has added the numbers inside the root as well as the multiples.
Question 5 is wrong (B is correct). Student has incorrectly combined surds.

EXERCISE 27D

1 Any examples using squares will show this. $\sqrt{4}+\sqrt{9}=2+3=5$ and $\sqrt{13}\approx3.605$, so the expressions are not equal.

2 a $12+3\sqrt{7}$ **b** $4\sqrt{2}+2\sqrt{5}$ **c** $4\sqrt{5}+8\sqrt{3}$
d $2\sqrt{2}+5\sqrt{3}$ **e** $6\sqrt{5}+3\sqrt{2}$ **f** $\sqrt{2}-2\sqrt{3}$

3 a $3\sqrt{2}$ **b** $\sqrt{7}$ **c** $5\sqrt{6}$
d $\sqrt{5}$ **e** $\sqrt{7}$ **f** $8\sqrt{5}$

4 a $6\sqrt{3}$ **b** $6\sqrt{10}-6\sqrt{11}$ **c** $10\sqrt{3}+\sqrt{5}$
d $\sqrt{3}+4\sqrt{5}$ **e** $9\sqrt{6}$ **f** $46\sqrt{2}-\sqrt{6}$

5 $(12+6\sqrt{3})$ cm

EXERCISE 27E

1 a $\sqrt{21}$ **b** $\sqrt{15}$ **c** 6 **d** $6\sqrt{35}$
e $^{-}12\sqrt{33}$ **f** $18\sqrt{5}$ **g** 78 **h** $30\sqrt{2}$
i $20\sqrt{6}$ **j** $18\sqrt{6}$ **k** $12\sqrt{15}$ **l** $24\sqrt{30}$

2 a $\sqrt{7}$ **b** $\sqrt{2}$ **c** $\sqrt{\frac{1}{2}}$ or $\frac{1}{\sqrt{2}}$ **d** $\sqrt{\frac{1}{10}}$ or $\frac{1}{\sqrt{10}}$
e 3 **f** $\sqrt{7}$ **g** $\frac{\sqrt{6}}{2}$ **h** $8\sqrt{3}$
i $6\sqrt{11}$ **j** 6 **k** $^{-}6$ **l** $^{-}3$

3 a $9\sqrt{2}$ **b** $8\sqrt{3}$ **c** $\sqrt{2}$
d 1 **e** $^{-}5\sqrt{2}$ **f** $\frac{^{-}3\sqrt{3}}{4}$

4 a $\sqrt{15}+2\sqrt{5}$ **b** $10\sqrt{3}-6$
c $^{-}\sqrt{6}$ **d** $27+4\sqrt{35}$
e $\sqrt{6}+5\sqrt{2}+3\sqrt{3}+15$ **f** $3\sqrt{5}-7$
g 46 **h** $9+2\sqrt{14}$
i $8-2\sqrt{15}$

5 a $\frac{5\sqrt{3}}{3}$ **b** $\frac{\sqrt{5}}{5}$ **c** $\frac{^{-}2\sqrt{3}}{3}$
d $\frac{\sqrt{6}}{3}$ **e** $2\sqrt{6}$ **f** $\frac{^{-}3\sqrt{7}}{28}$
g $\frac{2\sqrt{3}+3}{6}$ **h** $\frac{2\sqrt{5}+5}{5}$ **i** $\frac{2-\sqrt{2}}{10}$

6 a $\frac{9+3\sqrt{2}}{7}$ **b** $\frac{\sqrt{22}-3\sqrt{2}}{2}$
c $\frac{\sqrt{15}+\sqrt{6}}{3}$ **d** $\frac{9\sqrt{2}-12}{2}$
e $2\sqrt{5}-2$ **f** $\sqrt{15}+2\sqrt{3}-\sqrt{5}-2$
g $\frac{5\sqrt{5}+11}{2}$ **h** $\frac{13+7\sqrt{3}}{22}$
i $\frac{5\sqrt{6}-8\sqrt{3}}{6}$

EXERCISE 27F

1 a $A=18+72\sqrt{2}$ $P=20\sqrt{2}+16$
b $A=11-6\sqrt{2}$ $P=12-4\sqrt{2}$
c $A=\sqrt{6}+\frac{5}{2}$ $P=2\sqrt{2}+2\sqrt{3}+\sqrt{(10+4\sqrt{6})}$
d $A=\pi(19-6\sqrt{10})$ $P=2\pi(\sqrt{10}-3)$
e $A=4$ $P=2\sqrt{7}+2\sqrt{11}$

2 31 cm^2

3 $\sqrt{12}$ cm or $2\sqrt{3}$

4 a $x=210$ mm, $y=420$ mm, $z=594$ mm
b i 10 000 cm^2 ii 1457 mm
c $z=\sqrt{(x^2+2x^2)}=x\sqrt{3}$

5 $552+96\sqrt{15}$ cm^2

6 $\sin A=\frac{1}{\sqrt{2}}$

7 $(x+1)\sqrt{x}$.

8 a 72 cm^2 **b** 12 cm

9 $20\sqrt{6}$ cm

CHAPTER REVIEW

Correct answers are: 1B, 2A, 5A, 6A, 7B, 9A and 10B
Both answers for 3, 4 and 8 are incorrect. They should be:

3 $4\sqrt{5}-\sqrt{3}$

4 $\sqrt{14}+2\sqrt{2}$

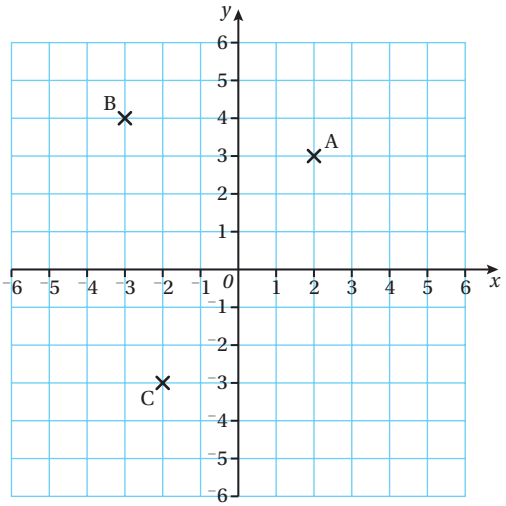
8 $\frac{\sqrt{5}+5}{5}$

11 $3\sqrt{2}+1$

28 Plane vector geometry

BEFORE YOU START ...

1



2 a $^{-}4$ **b** 7 **c** $^{-}23$ **d** $^{-}28$ **e** 27

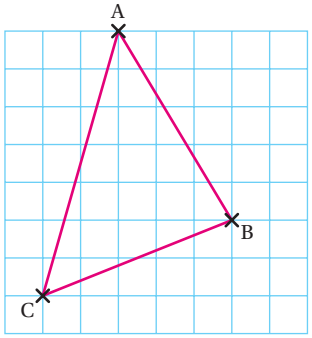
3 a $m=12$ **b** $k=^{-}4$ **c** $d=^{-}7$

4 $x=2, y=1$

LAUNCHPAD

1 $\overrightarrow{HG} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

2



3 a $\mathbf{j} + \mathbf{k} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ **b** $2\mathbf{k} - \mathbf{l} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$

4 f = 3 g = 6

5 a $\overrightarrow{CA} = \begin{pmatrix} -14 \\ -2 \end{pmatrix}$ **b** $\overrightarrow{CA} + \overrightarrow{AB} = \begin{pmatrix} -5 \\ 10 \end{pmatrix}$

6 $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 15 \\ -20 \end{pmatrix}$

7 a $\overrightarrow{BE} = x$ **b** $\overrightarrow{AF} = 2x + y$ **c** $\overrightarrow{AM} = 2x + \frac{y}{2}$

EXERCISE 28A

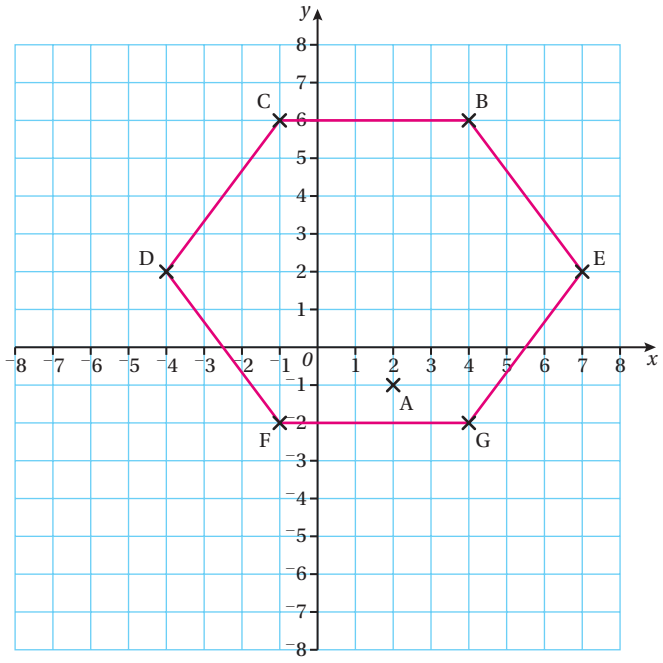
1 **Civ, 2Dii, 3Bi or v, 4Ei or v, 5Aiii**

2 a $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ **b** $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ **c** $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ **e** $\begin{pmatrix} -5 \\ 1 \end{pmatrix}$ **f** $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$

g Vectors $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are the same length and go in opposite directions

h Vectors $\overrightarrow{AB}$ and $\overrightarrow{BH}$ are parallel, $\overrightarrow{BH}$ is 3 times as long as $\overrightarrow{AB}$

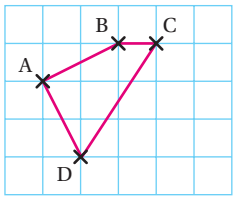
3



4 a $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$ **b** An example is E (0, 0) and F (−4, 6)

5 a $\begin{pmatrix} -6 \\ 10 \end{pmatrix}$ **b** (−5, 4)

6



7 a Bishop can move vectors of the form $\begin{pmatrix} k \\ k \end{pmatrix}$ and $\begin{pmatrix} -k \\ k \end{pmatrix}$, k a positive or negative integer

b King can move $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

c Knight can move $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

8 Consider the vector $\begin{pmatrix} a \\ b \end{pmatrix}$

Using Pythagoras's theorem, the length of this vector is $\sqrt{(a^2 + b^2)}$

Using basic trigonometry, this vector forms an angle θ with the x axis, where $\theta = \tan^{-1}\frac{b}{a}$

WORK IT OUT 28.1

Option C is correct.

EXERCISE 28B

1 a $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ **b** $\begin{pmatrix} 7 \\ -5 \end{pmatrix}$ **c** $\begin{pmatrix} -12 \\ 8 \end{pmatrix}$ **d** $\begin{pmatrix} -12 \\ 21 \end{pmatrix}$ **e** $\begin{pmatrix} -1 \\ -1 \end{pmatrix}$ **f** $\begin{pmatrix} -9 \\ 17 \end{pmatrix}$

Result c (vector **4p**) is parallel to $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

2 Three vectors of the form $\begin{pmatrix} k2 \\ -k3 \end{pmatrix}$ for any three values of k, where k is positive or negative.

3 a $x = 4, y = 0$ **b** $x = 12, y = 5$ **c** $x = 17, y = -5$

d $s = -2, x = -3.5$ **e** $s = -\frac{1}{4}, y = 32$ **f** $s = 3, y = 6$

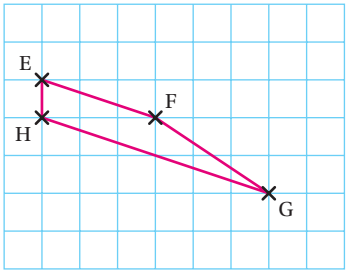
g $s = 3, x = 5$ **h** $s = 4, t = 3$

4 a $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ **b** $\begin{pmatrix} -15 \\ -12 \end{pmatrix}$

5 a EF and HG are parallel

b EFGH will be a trapezium: parallel sides are not equal

c



Vector $\overrightarrow{GF} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$

6 It is a parallelogram, because opposite sides (AB and DC, and DA and CB) are parallel and equal.

EXERCISE 28C

1 a $\begin{pmatrix} -10 \\ -2 \end{pmatrix}$ **b** $\begin{pmatrix} -2 \\ 12 \end{pmatrix}$ **c** $\begin{pmatrix} -1 \\ 6 \end{pmatrix}$

2 a $2\overrightarrow{AB} = \overrightarrow{DE}$

b $2\overrightarrow{AC} = \overrightarrow{DF}$

c The two triangles are similar. DEF is an enlargement of ABC, scale factor 2.

3 a **p**; ABCD is a square, so BC is parallel to AD and of equal length (magnitude)

b $-\mathbf{m}$ **c** $\mathbf{m} + \mathbf{p}$ **d** $\mathbf{m} - \mathbf{p}$

4 a $\overrightarrow{AC} = \mathbf{p} + 2\mathbf{q}$; $\overrightarrow{BC} = 2\overrightarrow{BM} = 2\mathbf{q}$

b $\overrightarrow{DB} = \mathbf{p} - 2\mathbf{q}$; $\overrightarrow{CB} = -\overrightarrow{BC} = -2\mathbf{q}$

c $\overrightarrow{MD} = \mathbf{q} - \mathbf{p}$; $\overrightarrow{MC} = \overrightarrow{BM} = \mathbf{q}$ and $\overrightarrow{CD} = \overrightarrow{BA} = -\overrightarrow{AB} = -\mathbf{p}$

d $\overrightarrow{NM} = \frac{1}{2}\mathbf{p} - \mathbf{q}$; $\overrightarrow{DB} = \mathbf{p} - 2\mathbf{q}$; $2\left(\frac{1}{2}\mathbf{p} - \mathbf{q}\right) = \mathbf{p} - 2\mathbf{q}$, so $\overrightarrow{NM}$ is parallel to DB

5 **a** $^{-}\mathbf{n}$ **b** $^{-}\mathbf{m}$ **c** $\mathbf{n}+\mathbf{m}$ **d** $2\mathbf{n}-\mathbf{m}$

6 **a** $\frac{1}{2}\mathbf{e}$ **b** $\frac{-1}{2}\mathbf{g}$ **c** $\mathbf{g}-\mathbf{e}$ **d** $\frac{1}{2}\mathbf{g}$ **e** $\frac{-1}{2}\mathbf{e}$

Triangle HIJ is similar to EFG. HIJ is an enlargement of EFG, scale factor $\frac{1}{2}$.

7 **a** $5\mathbf{t}$ **b** $4\mathbf{t}+\mathbf{r}$ **c** $\frac{1}{2}(5\mathbf{t}+\mathbf{r})$ **d** $^{-}2\frac{1}{2}\mathbf{t}$

8 **a** $\mathbf{q}-\mathbf{p}$ **b** $\frac{3}{4}(\mathbf{q}-\mathbf{p})$

c $\overrightarrow{OM}=\overrightarrow{OP}+\overrightarrow{PM}=\mathbf{p}+\frac{3}{4}(\mathbf{q}-\mathbf{p})=\frac{3}{4}\mathbf{q}+\frac{1}{4}\mathbf{p}=\frac{1}{4}(3\mathbf{q}+\mathbf{p})$

9 24 m; 26.8 m

CHAPTER REVIEW

1 The coordinate $(^{-}2, 3)$ is a point in two dimensional space

The vector $\begin{pmatrix}^{-}2\\3\end{pmatrix}$ describes movement from one point to another. It has magnitude and direction.

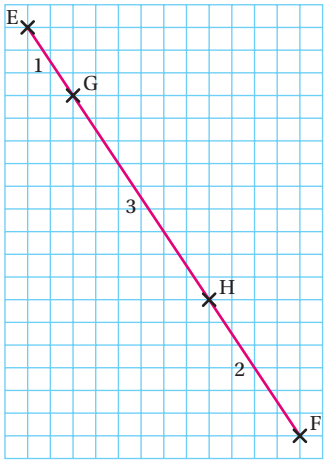
2 **a** and **c**; **b** and **d**; **e**, **f** and **g**

3 **a** $\begin{pmatrix}^{-}1\\^{-}3\end{pmatrix}$ **b** $\begin{pmatrix}2\\^{-}7\end{pmatrix}$ **c** $\begin{pmatrix}^{-}6\\3\end{pmatrix}$

4 **a** $\overrightarrow{EG}=5(2\mathbf{e}+\mathbf{f})=10\mathbf{e}+5\mathbf{f}$

b $\overrightarrow{FG}=3(2\mathbf{e}+\mathbf{f})=6\mathbf{e}+3\mathbf{f}$ **c** $\overrightarrow{OF}=7\mathbf{e}+4\mathbf{f}$

5 **a**



b **i** $6\mathbf{e}-9\mathbf{f}$ **ii** $12\mathbf{f}-8\mathbf{e}$

6 **a** $\overrightarrow{AB}=6\mathbf{b}-6\mathbf{a}=6(\mathbf{b}-\mathbf{a})$

b $\overrightarrow{ON}=3\mathbf{a}+3\mathbf{b}=3(\mathbf{a}+\mathbf{b})$

c **i** $\overrightarrow{AM}=3\mathbf{b}-6\mathbf{a}=3(\mathbf{b}-2\mathbf{a})$

ii $\overrightarrow{OG}=2\mathbf{a}+2\mathbf{b}=2(\mathbf{a}+\mathbf{b})$

d O, G and N are on the same line.

29 Plane isometric transformations

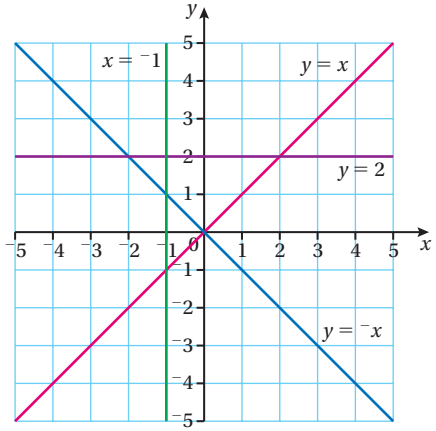
BEFORE YOU START ...

1 **a** 90° anti-clockwise

b 180° clockwise

c 270° clockwise

2



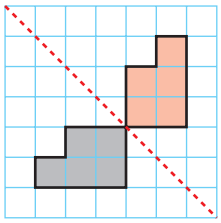
3 **a** The coordinate (3, 2) is a single point in *xy*-space

The vector $\begin{pmatrix}3\\2\end{pmatrix}$ has magnitude and direction, and determines the position of one point in *xy*-space relative to another.

b The vectors are perpendicular to each other.

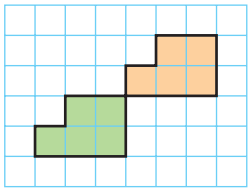
LAUNCHPAD

1



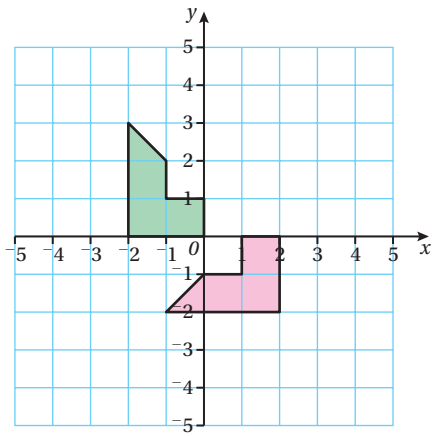
2 $y=^{-}x$

3

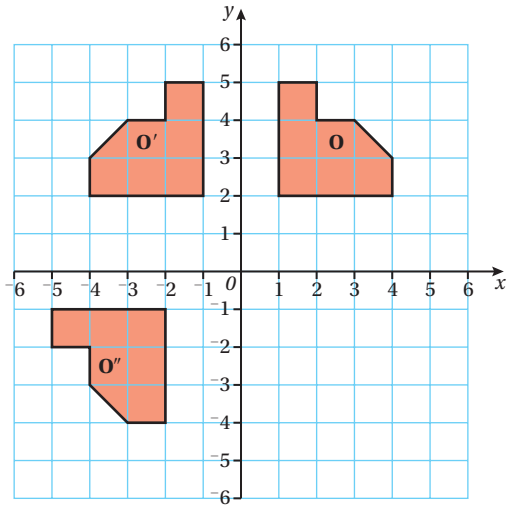


4 $\begin{pmatrix}2\\^{-}4\end{pmatrix}$

5



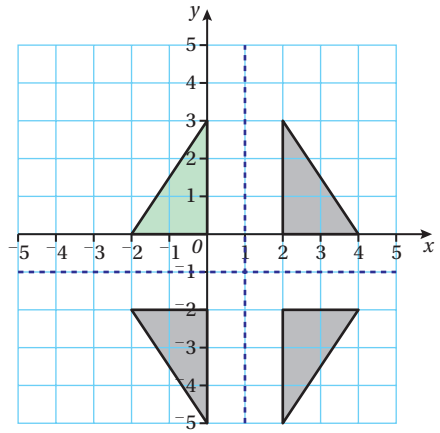
6



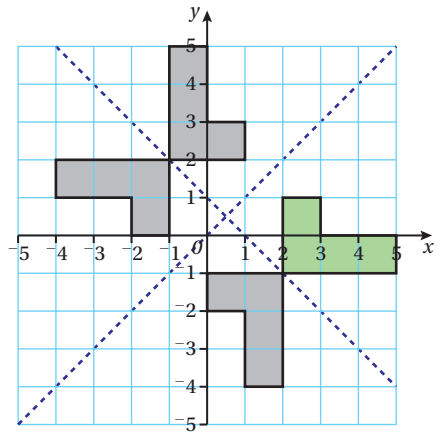
The single transformation that maps **O** to **O″** is a reflection in the line $y=^{-}x$

EXERCISE 29A

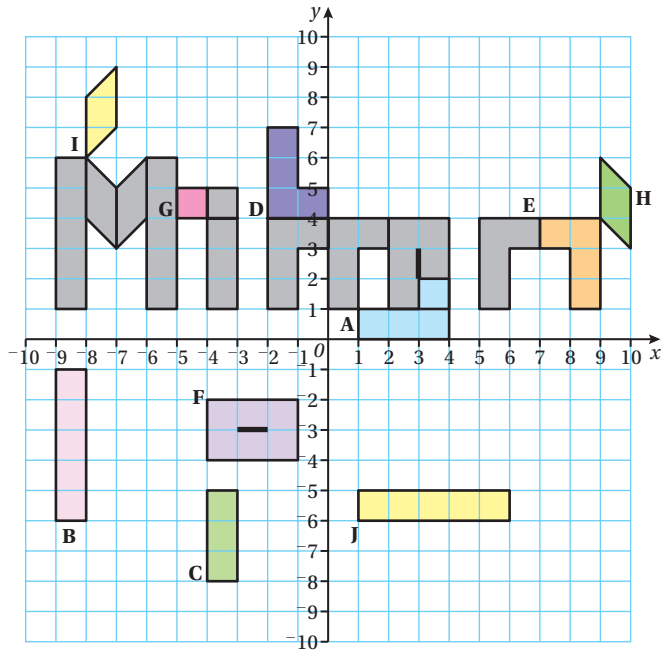
1



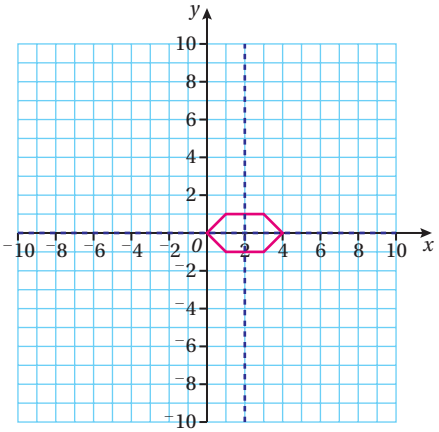
2



3

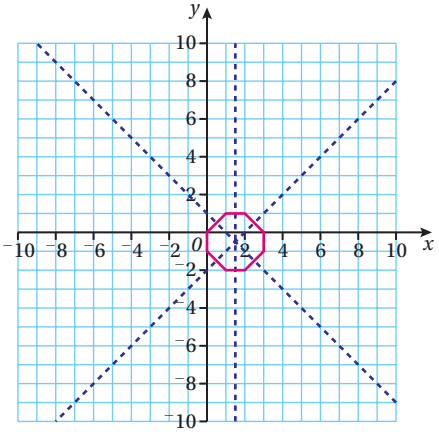


4 a



Reflect in $y = 0$, then reflect in $x = 2$ (or reverse order).

b



First reflect in $x = 1.5$, then reflect in $y = 1 - x$, then reflect in $y = x - 2$.

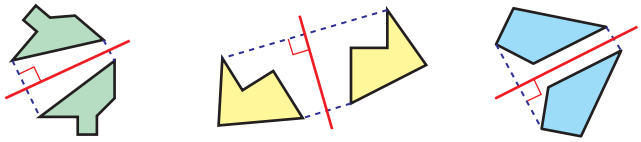
5 Student's own puzzles.

EXERCISE 29B

1 a $y = x - 1$ **b** $y = 2 - x$ **c** $y = 0.5$ **d** $y = 1 - x$

2 a i $y = 2.5$ **ii** $x = 6.5$ **iii** $y = x - 4$ **iv** $y = 9 - x$

3



4 a i One possibility is reflection in $x = 1$, followed by reflection in $y = 1$.

ii One possibility is reflection in $y = {}^{-}x$, followed by reflection in the y-axis and then in $y = {}^{-}0.5$.

b There is more than one answer.

c In some cases the order does matter.

d If the shapes have been 'flipped' as in **i** you need an odd number of reflections, if they haven't been 'flipped' you need an even number of reflections.

WORK IT OUT 29.1

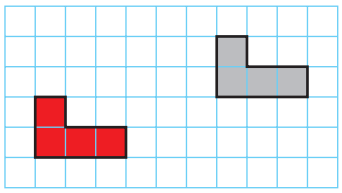
Option B is correct.

Option A is translation $\begin{pmatrix} 5 \\ {}^{-}2 \end{pmatrix}$.

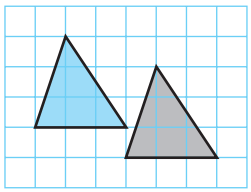
Option C is a translation $\begin{pmatrix} {}^{-}1 \\ 3 \end{pmatrix}$.

EXERCISE 29C

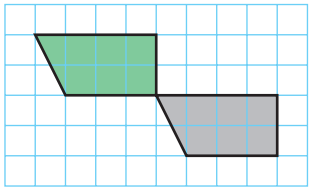
1 a



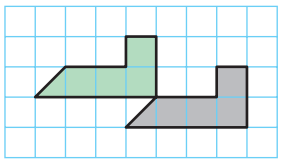
b



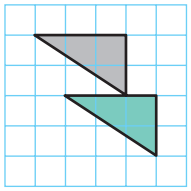
c



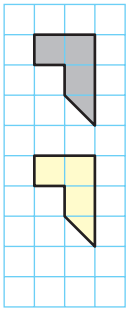
2 a



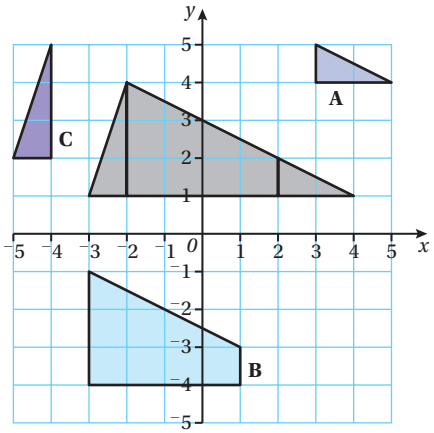
b



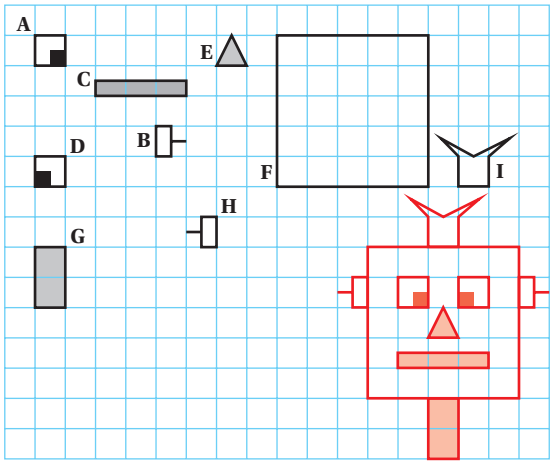
c



3 Shape that is created is a triangle



4



WORK IT OUT 29.2

Option B is a reflection.
Options A and E are translations.

EXERCISE 29D

1 a $\begin{pmatrix}3\\-7\end{pmatrix}$ **b** $\begin{pmatrix}0\\6\end{pmatrix}$ **c** $\begin{pmatrix}-8\\-10\end{pmatrix}$ **d** $\begin{pmatrix}4\\5\end{pmatrix}$ **e** $\begin{pmatrix}-13\\5\end{pmatrix}$

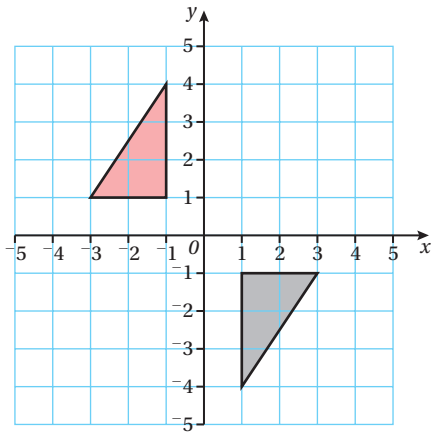
2 Students' own answers.

WORK IT OUT 29.3

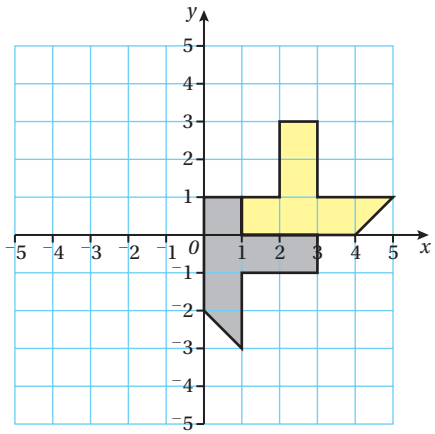
Option B is correct.
Option A is a rotation about the origin.
Option C is a rotation in clockwise direction.

EXERCISE 29E

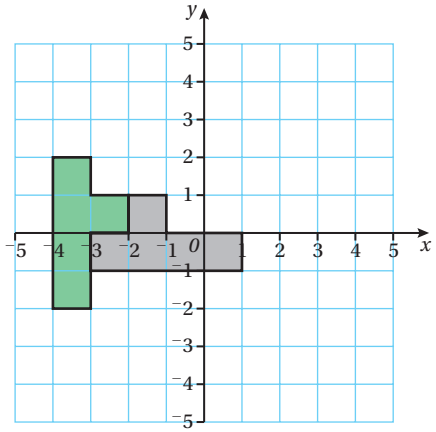
1



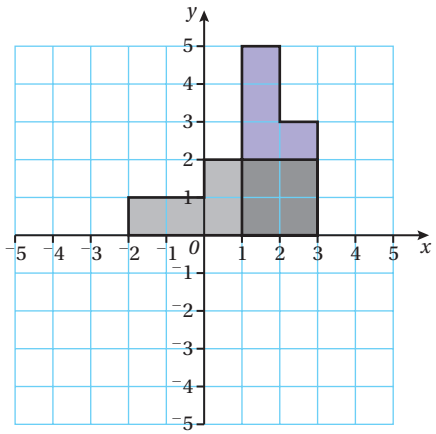
2



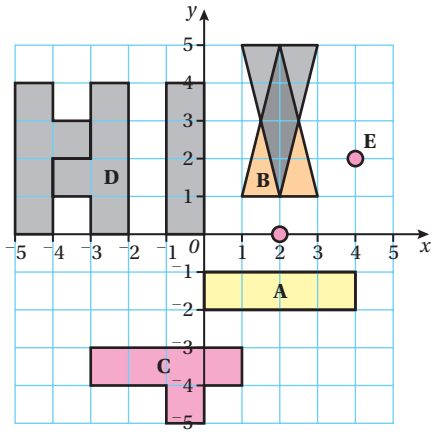
3



4



5



6 Each coordinate is a combination of the same two numerals; for example $(^-1, 5)$, $(5, 1)$, $(1, ^-5)$, $(^-5, ^-1)$. This result is only obtained if the centre of rotation is the origin.

WORK IT OUT 29.4

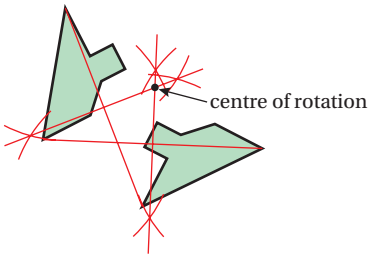
Option A is a rotation.
Options B and F are reflections.
Option D is a translation.
Options C and E are the results of combined transformations. They cannot be described by a single transformation.

EXERCISE 29F

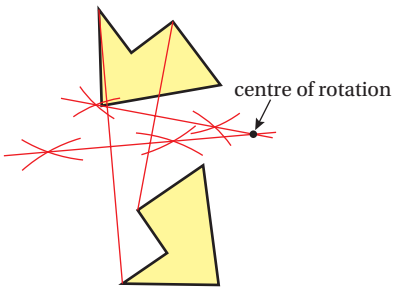
1 a Rotation 180° around the point $(^-1, 0)$.
b Rotation 90° anti-clockwise (or 270° clockwise) around the point $(2, ^-3)$.
c Rotation 90° clockwise (or 270° anti-clockwise) around the point $(^-4, 2)$.
d Rotation 90° anti-clockwise (or 270° clockwise) around the point $(^-2, 3)$.

2 Students' own answers. Students should identify either a line or a 2D shape, and describe the rotation(s) that shape undergoes.

3 a

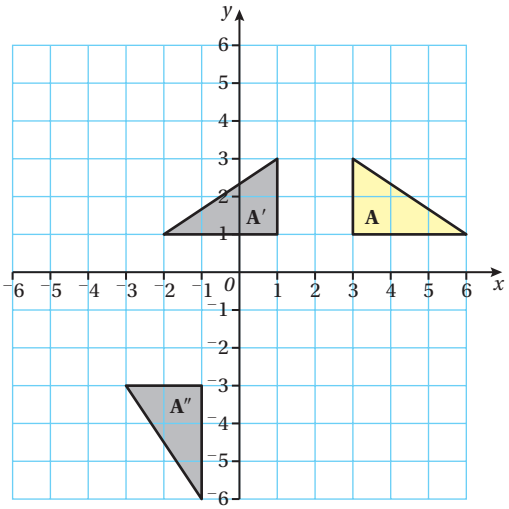


b

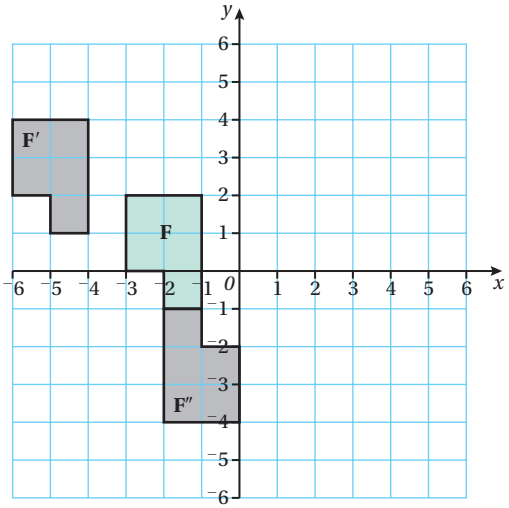


EXERCISE 29G

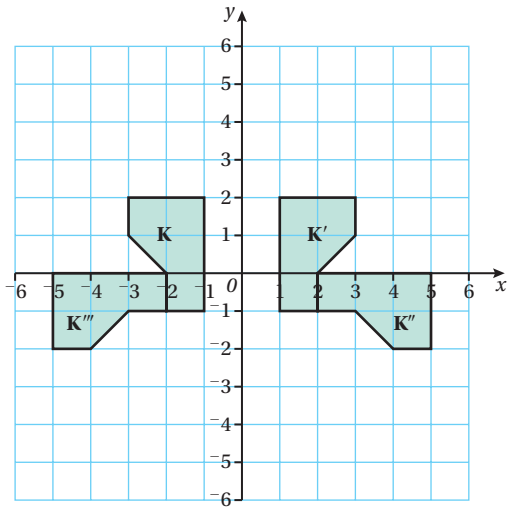
1 The single transformation from **A** to **A″** is a reflection in the line $y = ^-x$



2 The single transformation from F to F″ is a rotation 180° around the point $(^-1.5, ^-1)$.



3 Rotation 90° anti-clockwise about the point $(^-2, ^-1)$



4 a One possibility – rotate 90° clockwise around (1, 1) and then reflect in the line $y = 2$

b Yes, the order matters.

c Students' own answers.

5 Rotation 180° around the origin.

6 Many possible answers. Check students' pairs of transformations.

CHAPTER REVIEW

1 a True **b** True **c** False **d** True

2 a Rotation 180° around (4, 0).

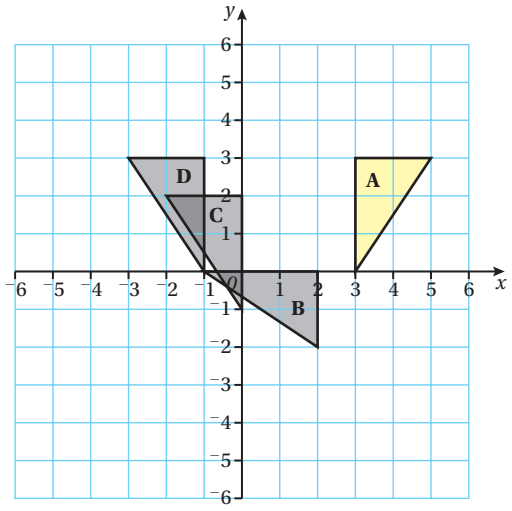
b Rotation 180° around $(3, ^-2)$.

c Translation through vector $\begin{pmatrix}2\\4\end{pmatrix}$.

3 For example:
Translate the original shape through vectors $\begin{pmatrix}2n\\0\end{pmatrix}$ and $\begin{pmatrix}0\\6m\end{pmatrix}$ where n and m are integers.

Rotate the original triangle 180° around (2, 3) and translate this new triangle using the same vectors.

4



The single transformation that maps shape **A** onto shape **D** is a reflection in the line $x = 1$.

5 For example:
Reflect **A** in y axis to give **A′**.
Rotate **A** 90° anti-clockwise around $(^-2, 3)$ and then translate through vector $\begin{pmatrix}0\\^-5\end{pmatrix}$ to give **A‴**.

Reflect **A‴** in $x = 0.5$ to get **A″**.
Rotate **B** 90° anti clockwise around (0, 4) to give **B′**.
Reflect **B′** in the line $x = 0.5$ to give **B″**.
Reflect **C** in the line $x = 0.5$ to give **C′**.
Rotate **C** 180° around $(1.5, ^-1)$ to give **C″**.
Rotate **D** 90° clockwise around the origin to give **D′**.
Rotate **D** 180° around $(^-1, 2)$.

30 Congruent triangles

BEFORE YOU START ...

1 a AB and DE or BC and EF or AC and DF

b Angle EDF

c ACB and DFE or CBA and FED

2 a C **b** B **c** A

3 a True **b** False **c** False **d** True

4 45°

LAUNCHPAD

1 a No; the corresponding sides are not equal.

b Yes; three equal sides.

c Yes, three equal angles and one equal side.

2 a Triangles share the side AC, CD = CB and AB = AD. Hence by SSS the triangle are congruent.

b AC is a line of symmetry hence angle ADC is equal to angle ABC.

3 OPR is an isosceles triangle since it is made up of two radii of a circle.
Hence side OP = side OR.
Triangles OPQ and ORQ are congruent (SSS) share side OQ, side PQ = side RQ, and side OP = side OR.
Hence angle OQP = OQR, since along a straight line each is 90°.

WORK IT OUT 30.1

Option C is correct.
Option A is wrong because for SAS the angle has to be between the two sides.
Option B is wrong because not enough information has been written down to complete the proof.

EXERCISE 30A

1 a RHS **b** SSS **c** ASA **d** SAS

2 a Can't tell. **b** SSA so can't tell.

c ASA. **d** RHS

3 Angle BCA is opposite to angle ECD and hence equal, hence angle CBA and CED are equal.
Triangles are congruent by ASA.

4 SAS – DF shared, angle DFG = angle DFE, and side EF = side FG.
ASA, because DEF and DGF are the same angle (isosceles triangle), and they both have a 9 m side and a right angle.

5 Angle RTS is opposite to angle PTQ and hence equal, angle TRS and TQP are alternate angles and therefore equal. Triangles are congruent by ASA.

6 Angle EAB and DCB are both right angles, since they are co-interior angles with AED and CDE. Side BE = BD. AE = CD. Hence congruent (RHS)

7 Another correct answer is: RHS – ABC and ADC are right-angled riangles AB = AD and BC = CD

SSS – share AC, AD = AB (isosceles triangle) and BC = BD (AC bisects base of the triangle).

8 SAS – PR = SU, shared angle at Q, QU = QR (kite is symmetrical)

9 a Kite so AB = BC and AD = CD, triangles share side BD – SSS congruency.

b Kite so AD = CD, share side ED, angle AED = CED = right angle – RHS congruency.

10 SSS = triangle ADC and triangle ABC, SSS = triangle BAD and triangle BCD,
ASA = triangle EAB and triangle ECD. Other combinations are possible.

11 Side PO = side QO, radii of smaller circle, angle POM = QON, opposite angles are equal, side OM = side ON, radii of larger circle, by SAS congruent triangles.

EXERCISE 30B

1 Triangles share side JL, JK = JM and right angled. Hence congruent (RHS) and KL = ML.

2 a Angle AED = angle BEC vertically opposite angles are equal, third angles in triangles EBC = EDA, so triangles AED and CEB are congruent (SAS), hence side AE = side CE.
Angle AEB = angle DEC vertically opposite angles are equal, side DE = BE, side AE = CE, so triangles ABE and CDE congruent (SAS), hence angle ABE = angle EDC.

b Angle DAC = angle BCA so AD and BC are parallel (alternate angles), from part **a** angle ABE = EDC so AB and DC parallel (alternate angles) hence ABCD has two pairs of parallel sides and is a parallelogram.

3 a QPR = 56 degrees – base angles of an isosceles triangle
PQR = 180 – 2 × 56 = 68 degrees

b Angle QPR = angle PRS = 56 degrees – alternate angles are equal.
Angle SPR = PRS = 56 degrees – base angles of an isosceles triangle.
Angle PSR = 180 – 2 × 56 = 68 degrees – angles in a triangle add up to180 degrees.

4 Angle PRS = 180 – angle QRP and angle PSR = 180 – angle PST (angles on a straight line add up to 180).
PST = PRQ so PRS = PSR and base angles of an isosceles triangle.

5 Angle ABE = angle CBE, BE is a shared side and the angle AEB = angle CEB, as they are both complements of the same angles below (AED and CED) and by ASA AEB and CEB are congruent. This means EA and EC are the same length and ED is a shared side so by SAS triangle EAD and triangle ECD are congruent, and angle EAD = angle ECD.

6 AB = CD, BC = DA and the angle ABC = BCD (both 90°), hence triangles ABC and BCD are congruent (SAS) and AC = BD.

7 a 110°

b AB = DC and parallel (parallelogram), angle BDC = ABD (alternate angles). DX = AD = BC = BY, hence triangles CDX and ABY are congruent (SAS) and CX = AY.

c As with **b** AX = CY, angle CYX = AXY = 110° and triangles share side XY hence congruent (SAS).

d AY = XC and since CYX = AXY = 110°, alternate angles AY and XC are parallel – pair of equal sides are parallel – AYCX is parallelogram.

8 Draw diagonals BP and CQ of rhombus BCPQ.
∠AQB =∠QAB (△ ABQ isosceles), ∠QBC = 2∠QAB (exterior angle), ∠QAB = ∠PBC (diagonal of rhombus bisects angle), BP // AQ (corresponding angles equal), similarly QC // PD. QC and BP meet at right angles at X (diagonals of a rhombus are perpendicular). AR is perpendicular to DR.

9 130° and 110°

10 a Square or rhombus

b Square

c Kite, rhombus, rectangle or square

11 a i ∠ABP = ∠CBP = ∠ADQ = ∠CDQ = 45° (diagonals of square ABCD meet each side at 45°),
BP = DQ (given), AB = BC = CD = DA (equal sides of square ABCD), so ABP △ABP ≡ △CBP ≡ △ADQ ≡ △CDQ (SAS).

ii AP = CP = AQ = CQ (matching sides of congruent triangles), so APCQ is a rhombus.

b ∠FMA = ∠GAM (alternate angles, AG // FM). Also AF = FM (in isosceles △AFM). So 2 adjacent sides of parallelogram AFMG are equal. So AFMG is a rhombus.

12 Angle CDB = 36°, angle BDA = 82°, angle ABD = angle DAB = 49°, angle ABC ≠ 90° so AB and DC are not parallel so ABCD is not a trapezium.

CHAPTER REVIEW

1 a Congruent by SAS. ABC, FED.

b Congruent by SAS. GHI, JKL.

c Not enough information.

d Congruent by SSA. GHI, JKL.

2 Let angle QPS = a
Let angle PQS = b
Triangle PQR three angles are a, b + angle SQR and angle QRP.
Angle QST = angle QPS = a.
Angle TQR = angle PQS = b.
Triangle QST three angles are a, b + angle SQR and angle QTS.
Hence angle QRP = angle QTS.

3 Angle ABE = angle DCE (angles in a triangle add up to 180°), hence angle EBC = angle ECB (supplementary to ABE and DCE).

4 Let angle ABC = a.
Angle ABC = CAB = a base angles of an isosceles triangle.
Angle ACB = 180 – 2a.
Angle ACD supplementary to angle ACB so angle ACD = 2a.
Angle ACD = angle ADC = 2a a base angles of an isosceles triangle.
Angle ADC = EAD = 2a alternate angles are equal.
Hence angle EAD = 2 × angle ABC.

5 UW = UY (Congruent); UWX = UYX
(Supplementary to equal angles); Triangles UWX,UXY Congruent (UX Common); Thus WX = XY so UWXY is a kite

6 MNP congruent to NPQ so angle NPQ is right angle and MN = NP = QP.
Therefore MQ must be equal to NP and all angles right angles. Four equal sides and four equal angles means shape is a square.

31 Similarity

BEFORE YOU START ...

1 **a** BOC **b** 80° **c** 150°

2 Side DF = side AC, angle EDF = angle BAC, side DE = side AB, SAS congruency

3 **a** $x = 8$ **b** $h = 2.5$ **c** $k = 0.25$

4 C

5 $x = 24$

6 **a** $\frac{1}{8}$ **b** 20.25 **c** 3375

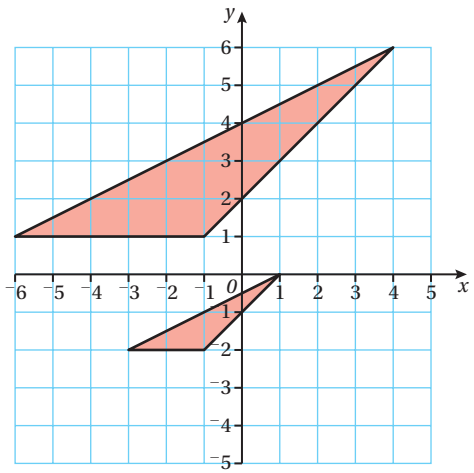
LAUNCHPAD

1 **a** Not necessarily, for example, a square is a special case of a rectangle but is not similar to a rectangle that has different lengths and widths.

b These are similar; corresponding angles are equal in size.

2 AC = 14.1 cm, BC = 4.7 cm, AE = 4 cm

3



4 Enlargement centre (4, 2) scale factor 0.5

5 No. Although angles are the same, the side ratios are not. AB ratio is 2 : 3, but BC ratio is 9 : 14.

6 12 cm

EXERCISE 31A

1 **a** CDE and BDA EDC = ADB (common), DEC = DAB (corresponding), DCE = DBA (corresponding)

b YZV and YWX VYZ = XYW (vertically opposite), YVZ = YXW (alternate), YZV = YWX (alternate)

c PQS and RSQ PQS = RSQ (alternate), PSQ = RQS (alternate), QPS = SRQ (angles in a triangle)

d HFG and NLM FGH = LMN (alternate), extending lines FH to X and LN to Y: GHF = XHN (vertically opposite), XHN = MNL (corresponding) so GHF = MNL, GFH = MLN (angles in a triangle)

2 **a** Yes (angles the same)

b No (angles not the same)

c Yes (ratio of corresponding sides is the same)

3 **a** False (ABC with base angles 30°, WXY with base angles 45°)

b True (all angles are 60°)

c False (two other angles could be 30° and 60° or 45° and 45°, so not the same)

d True (third angle will be 60° for each triangle)

e True (base angles are always both 45°)

f False (ratio of corresponding sides could be in same proportion so corresponding angles could be equal)

4 **a** Triangles NMO, KJO and LJM (JKO = JLM (corresponding), JOK = JML (corresponding) OJK = MJK (common) and JOK = MON (vertically opposite), MNO = JKO (alternate) KJO = NMO (alternate))

b Triangle ABC, BDC and ADB (CBA = 90° (straight line), if CAD = x then DBA = 90 − x and CBD = x and BCD = 90 − x, so angles are the same for all three triangles))

5 $c = 8$ cm, $d = 15$ cm

6 $e = 16$ cm, $f = 13.5$ cm

7 AE = 1.5 cm, CE = 10 cm, AD = 7.5 cm

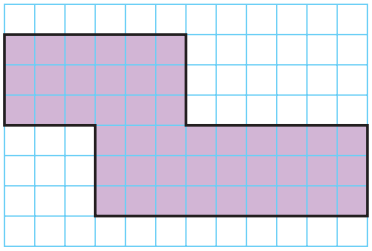
8 YZ = 3 cm, XY = 9 cm

9 2.975 m (3dp)

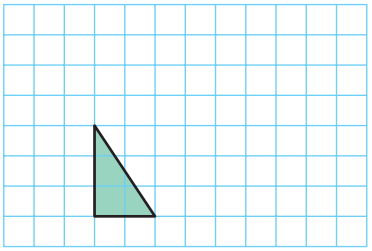
10 192 m

EXERCISE 31B

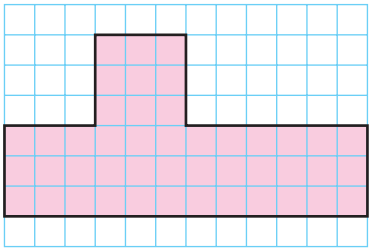
1 **a**



b



c



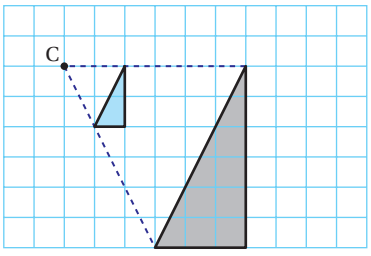
d The enlargement is actually a reduction; the image is smaller than the object.

WORK IT OUT 31.1

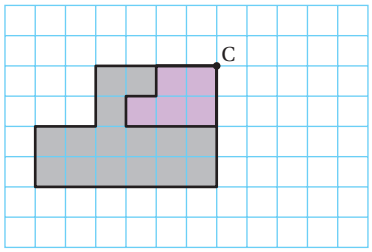
Option A is correct.
Option B is wrong because the base length has not doubled in the image.
Option C is wrong because a scale factor 3 has been used.

EXERCISE 31C

1 **a**



b



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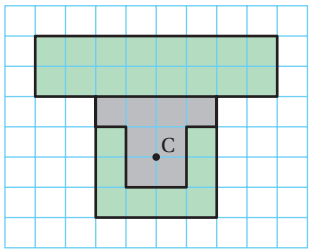

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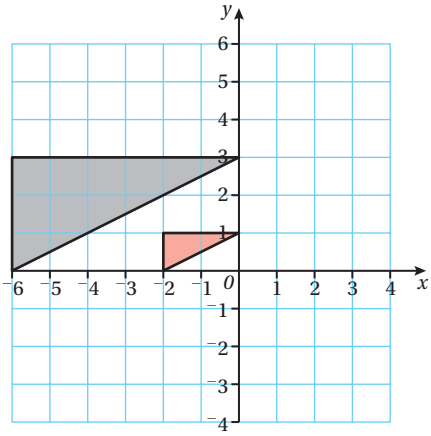

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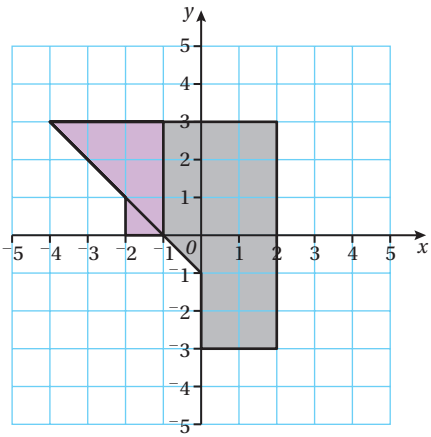

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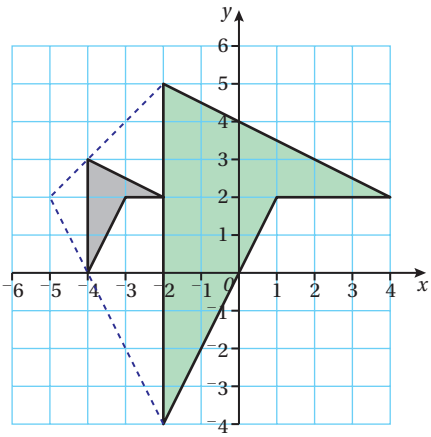

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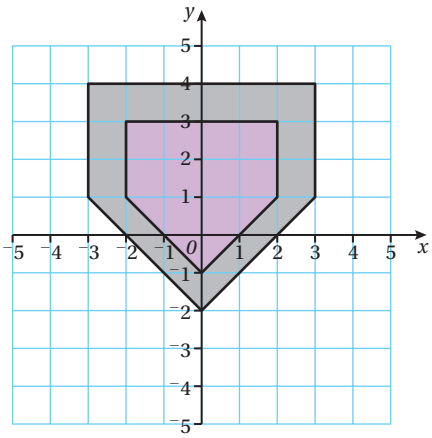
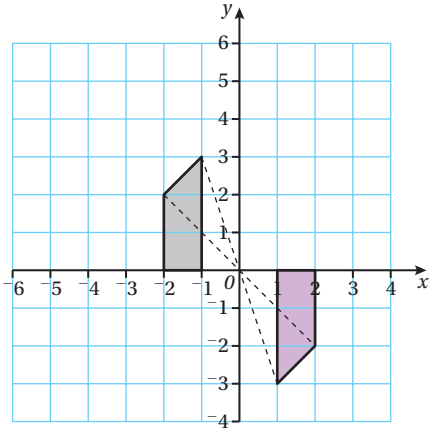


EXERCISE 31D

1



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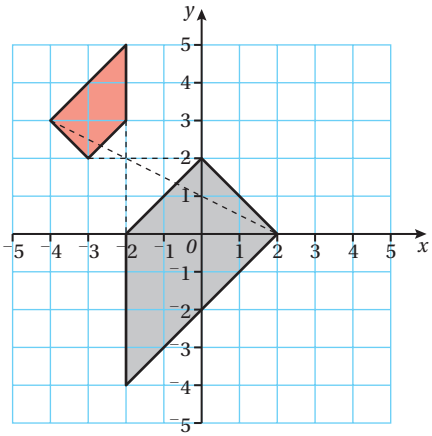

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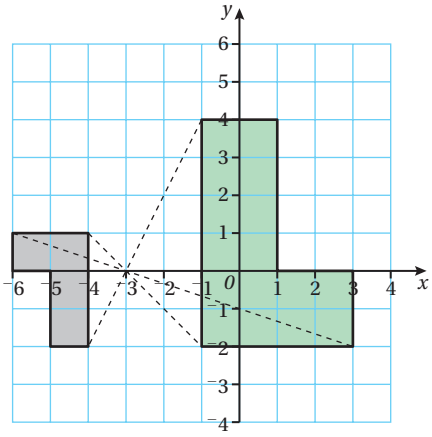

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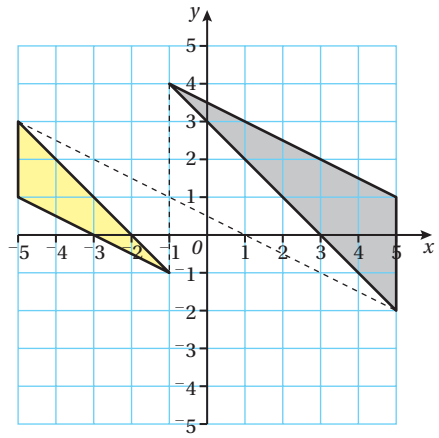


WORK IT OUT 31.2

Ellie got the answer correct.

Always check that each side has been increased by the same scale factor, this doesn't necessarily mean the same number of extra squares. Sides are increased in proportion to each other.

EXERCISE 31E

1 House C, D and F

2 **a** Enlargement scale factor 2 centre $(^{-}4, ^{-}4)$
b Enlargement scale factor 4 centre $(^{-}1, ^{-}4)$
c Enlargement scale factor 3 centre $(^{-}4, 4)$
d Enlargement scale factor $\frac{1}{2}$ centre $(2, 5)$
e Enlargement scale factor $\frac{1}{2}$ centre $(4, 2)$
f Enlargement scale factor $1\frac{1}{2}$ centre $(^{-}5, 4)$

3 **a** Enlargement scale factor $^{-}2$ centre $(2, 1)$
b Enlargement scale factor $^{-}\frac{1}{2}$ centre $(2, 2)$

EXERCISE 31F

1 **a** Yes **b** No **c** No **d** Yes

2 **a** Yes similar **b** No **c** Yes

3 $a = 8$ cm $b = 6$ cm

4 KL = 8 LG = 4 GH = 3.2 HI = 2.4 IJ = 6.4

5 GH = KJ = 7.5cm all other sides = 10.5cm

6 1 : 64

7 **a** 3.5
b Base of 28 units and height 42 units

8 **a** 7.996 cm (3 dp)
b 22.389 cm tall and volume 3446.464 ml

9 **a** Volume 134.041 cm^3 (3 dp), surface area 162.663 cm^2 (3 dp)
b Volume 3619.115 cm^3 (3 dp), surface area 1463.963 cm^2 (3 dp)

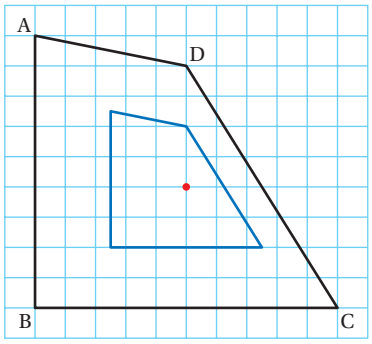
10 **a** Original surface area 2123.717 cm^2 (3 dp), new surface area 33979.466 cm^2 (3 dp)
b Original volume 9202.772 cm^3 (3 dp), new volume 588977.413 cm^3 (3 dp)

CHAPTER REVIEW

1 **a** VWX = VYZ (corresponding), VXW = VZY (corresponding), WVX = YVZ (common), angles are the same, therefore similar
b 3.75 cm

2 6.5 m

3



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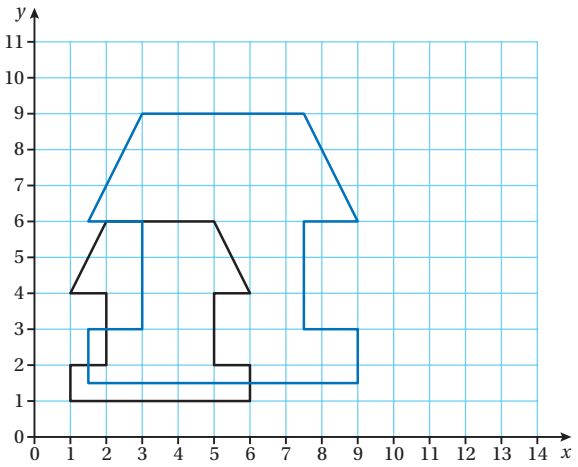


5 Yes – sides all the same length, internal angles all 120 degrees, so they are enlargements.

6 No – different angles are possible in different rhombuses.

7 **a** The scale factor is 2, so the area of the bases would increase by a multiple of 4 (2 squared), not double.
b 60 cm^3

8 48 cm

9 50 cm^3

10 Enlargement scale factor $^{-}\frac{1}{2}$ centre $(^{-}2, 2)$

32 Pythagoras' theorem

BEFORE YOU START...

1 **a** **i** Yes **ii** No **iii** Yes **iv** Yes
b **i** Yes **ii** No **iii** Yes **iv** Yes

2 **a** Acute angle
b Obtuse angle
c Right angle
d Reflex angle
e Straight line, which is two right angles

3 6 $units^2$

4 x and y are equal because it is an isosceles triangle

LAUNCHPAD

1 8.94 m (2 dp)

2 **a** No, it is not a right-angled triangle
b 11.18 cm (2 dp)

3 6.79 m (2 dp)

WORK IT OUT 32.1

Option C is correct
Option A – multiplied by 2 instead of squaring
Option B – substituted the wrong values in the equation

EXERCISE 32A

1 **a** 10 cm **b** 13.42 cm (2 dp) **c** 2.59 cm (2 dp)
d 1.62 cm **e** 7.21 m (2 dp)

2 **a** 2.80 cm (2 dp) **b** 4.47 cm (2 dp) **c** 4.28 cm (2 dp)
d 8.54 km (2 dp) **e** 10.39 cm (2 dp) **f** 8.06 cm (2 dp)

3 93.67 m

4 **a** 8.54 **b** 21.26 **c** 13.42
d 7.30 **e** 10.58 **f** 118.76

WORK IT OUT 32.2

Option C is correct
Option A – added the values instead of squaring and adding
Option B – multiplied by 2 instead of squaring

EXERCISE 32B

1 **a** Yes **b** Yes **c** No
d Yes **e** Yes **f** Yes

2 Any smaller than that are not whole numbers

3 There is no limit as numbers are infinite

4 **a** Yes **b** No **c** No
d Yes **e** Yes

5 **a** Yes **b** No **c** No

EXERCISE 32C

1 4.33 cm (2 dp)

2 7.42 cm (2 dp)

3 14.14 cm (2 dp)

4 Yes

5 **a** $1^2 + 10^2 = 101$, $\sqrt{101} = 10.04897562$ cm = 10.0 cm to 1 dp
b No, because the lengths of the two sides are only equal when when rounded to 1 dp

6 15.26 (2 dp)

7 **a** 6.5 **b** 4.93 (2 dp) **c** 6.35 (2 dp)

8 69.45 units^2 (2 dp)

9 10.2 (2 dp); perimeter is 34.20 units (2 dp)

10 7.07 mm (2 dp)

11 4

EXERCISE 32D

1 17.55 cm (2 dp)

2 Yes, because it is 28.72 cm high (2 dp)

3 42.96 m to the nearest cm

4 No, the longest that will fit is 38.48 cm

5 95.39 mm (2 dp)

EXERCISE 32E

1 16.16 (2 dp)

2 **a** 36.06 inches (2 dp)
b No, because it is 69.28 inches wide

3 Yes, because the longest length is 7.81 m (assuming the ceiling is flat inside)

4 B

5 9.54 cm (2 dp)

6 No, $30^2 + 30^2 = 1800$, length of the brace would be $\sqrt{1800} = 42.43$ cm

7 8.08 cm (2 dp)

8 0.68 m

9 **a** 16.97 m (2 dp)
b 7.89 m (2 dp)

10 **a** Perimeter 54.64 m and 173.2 m^2 to 2 dp and 1 dp
b 19 laps (assuming the horse can trot around the entire perimeter)

11 **a** 141.4 cm (1 dp)
b 6 full rows

12 No, it is 3.1 m high so will not fit into the display area

CHAPTER REVIEW

1 $x = 12.81$, $y = 11.31$

2 95.16 cm

3 6.77 mm (2 dp)

4 2 m

5 1.25 cm

33 Trigonometry

BEFORE YOU START ...

1 **a** $\sqrt{305} \approx 17.5$ (3 sf)
b $2\sqrt{6} \approx 4.90$ (3 sf)

2 7.28

3 ± 9.71

4 $\frac{35}{AC} = \frac{5}{3}$ so AC = 21 cm

LAUNCHPAD

1 5.78 cm (3 sf)

2 8.95°

3 **a** $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ **b** 1 **c** $\sqrt{3}$

4 $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$

5 24.3 (1 dp)

6 **a** PQ = 10.1 cm (3 sf) **b** P = 29°, Q = 51°

7 9.9 m (1 dp)

8 **a** 7 cm **b** 51.1° (1 dp)

9 Pink line is $y = \cos x$ and blue line is $y = \sin x$

EXERCISE 33A

1 **a** PQ hypotenuse, QR adjacent, PR opposite
b XZ hypotenuse, XY opposite, YZ adjacent
c AC hypotenuse, AB opposite, BC adjacent
d RT hypotenuse, RS opposite, ST adjacent

2 **a** 0.530 **b** 0 **c** 0.445
d 0.5 **e** 19.081 **f** 0.656
g 0 **h** 0.848 **i** 1

3 All require the sine ratio to find the opposite length
a 7.5 cm **b** 10.8 cm (3 sf) **c** 2.54 cm (3 sf)
d 5.51cm (3 sf) **e** 6.72 cm (3 sf) **f** 29.0 cm (3 sf)
g 7.99 cm (3.sf) **h** 14.1 cm (3 sf)

4 **a** 18.25 cm **b** 7.83 cm **c** 9.66 mm
d 13.16 cm **e** 4.10 m **f** 14.00 mm

WORK IT OUT 33.1

Option C, using tan, is correct.

EXERCISE 33B

1 **a** **i** 44 **ii** 136 **b** **i** 45 **ii** 135
c **i** 6 **ii** 174 **d** **i** 47 **ii** 133
e **i** 83 **ii** 97 **f** **i** 85 **ii** 95

2 **a** 1° **b** 30° **c** 45° **d** 60°

3 **a** sine angle $= \frac{14}{17.8} = 0.787$ angle = 51.9°

cosine angle $= \frac{11}{17.8} = 0.618$ angle = 51.8°

tangent angle $= \frac{14}{11} = 1.273$ angle = 51.8 °

b There is a slight discrepancy due to rounding

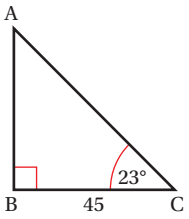
4 **a** 43° **b** 27° **c** 68°
d 15° **e** 76° **f** 38°

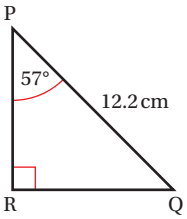
5 **a** 25.9° **b** 44.9° **c** 69.5°
d 79.6° **e** 26.9° **f** 11.5°

6 24.62° (2 dp)

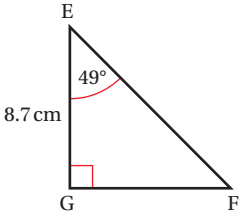
7 42.08° (2 dp)

8 **a** AB = 19.1 units **b** Length QR = 10.23cm (2 dp)

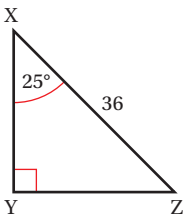




c GF = 10.01cm (2 dp)

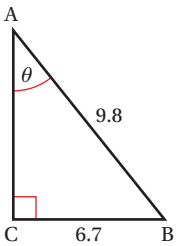


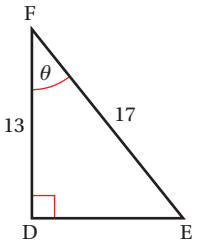
d



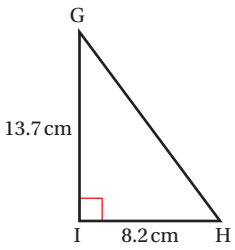
i XY = 32.63 units **ii** YZ = 15.21 units

9 **a** Angle A = 43.13° **b** Angle F = 40.12°



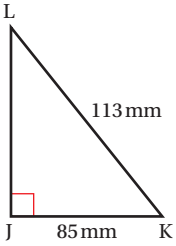


c



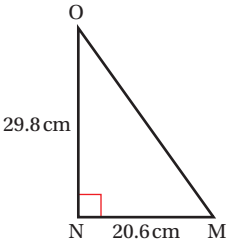
i Angle G = 30.9° **ii** Angle H = 59.1°

d



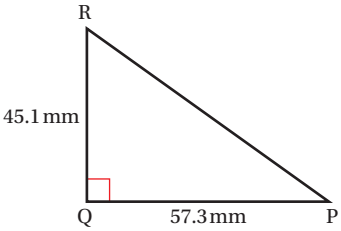
i Angle K = 41.22° **ii** Angle L = 48.78°

e



i Angle O = 34.66°
ii Angle M = 55.34°
iii Length MO = 36.23 cm

f



i Angle P = 38.21° **ii** Angle R = 51.79°
iii Length PR = 72.92 mm

EXERCISE 33C

1

Angle θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	tan 90° is undefined

2 **a** 1 **b** $\sqrt{2}$ **c** 0
Therefore sin x will equal cos $(90 - x)$ as they are complementary angles that add to 90°

3 $\sin 60° = \frac{x}{7}$ $x = 7\frac{\sqrt{3}}{2}$

$\cos 30° = \frac{x}{7}$

$\tan 60° = \frac{x}{3.5}$

4 **a** $x = 4, y = 4\sqrt{2}$ **b** $x = \frac{4}{\sqrt{3}}$

c $x = 16$ **d** $x = \frac{20}{3}\sqrt{3}, z = \frac{10}{3}\sqrt{3}$

e $x = 5, y = 5\sqrt{3}$ **f** $x = \frac{8}{3}\sqrt{3}, y = \frac{16}{3}\sqrt{3}$

EXERCISE 33D

All answers to 2 dp

1 **a** 11.20 **b** 8.58 **c** 25.27

2 **a** $x = 10.65$ cm **b** $x = 5.73$ cm **c** $x = 9.06$ cm
d $x = 5.32$ cm **e** $x = 6.46$ cm **f** $x = 15.5$ mm

3 **a** 54.65° **b** 66.82° **c** 69.75°
d 25.31° **e** 52.70° **f** 50.52°

4 a The ratio of sides to opposite angles is the same (according to the sine rule) and the side opposite Y is shorter than that opposite X.
b Y = 30.95° and Z = 109.05°

5 a 51° **b** 52° **c** 32.35 mm

EXERCISE 33E

1 a 14 cm **b** 8.62 cm **c** 22.3 cm

2 53.8°

3 a 18.7 m (3 sf) **b** T = 52.9°, U = 32.1 (3 sf)

4 X = 60°; Y = 32.2°; Z = 87.8°

5 a 14.42 km (2 dp) **b** 296.3°

EXERCISE 33F

1 a 5 cm^2 **b** 3.27 m^2 (2 dp)
c 22.5 cm^2 **d** 298.58 mm^2 (2 dp)
e 5.79 cm^2 (2 dp) **f** 25.96 m^2 (2 dp)

2 To 2 dp
a 8.16 cm^2 **b** 19.34 cm^2
c 16.01 cm^2 **d** 30.21 cm^2

3 3.91 cm (2 dp)

EXERCISE 33G

1 3.74 m (to 2 dp)

2 35.83 m (to 2 dp)

3 34.34 m (to 2 dp)

4 a 25.98 m (to 2 dp)
b It will decrease to 46.1° (to 1 dp)

5 a 6.99 m (to 2 dp) **b** 4.72 m (to 2 dp)

6 44.48 m (to 2 dp)

7 a Use 10 × tan 20° and add her height up to eye level.
b It would decrease to 10.3°

8 a 020° **b** 281.91 m (2 dp)
c 98 668 m^2 to the nearest square metre

9 $\frac{5a^2}{\tan 36°} \approx 6.88a$ m^2

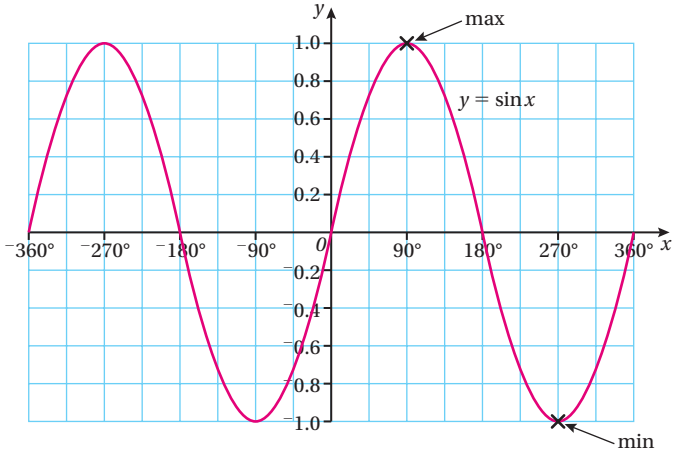
10 15.5° (1 dp)

11 a 7.92 m (2 dp) **b** 56.1°

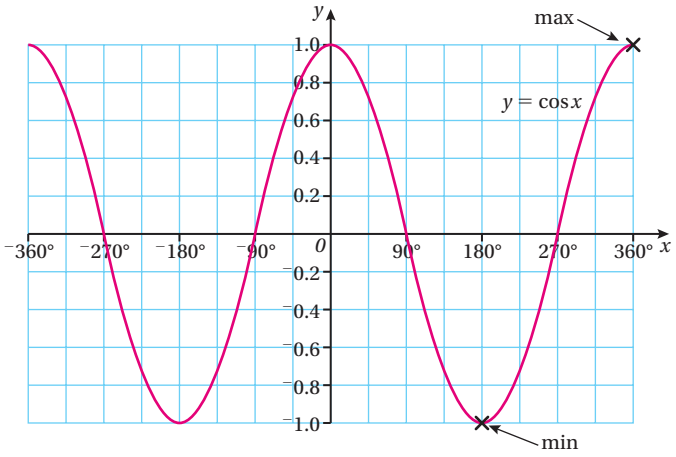
12 a 51.8° (1 dp) **b** 38.2° (1 dp)

EXERCISE 33H

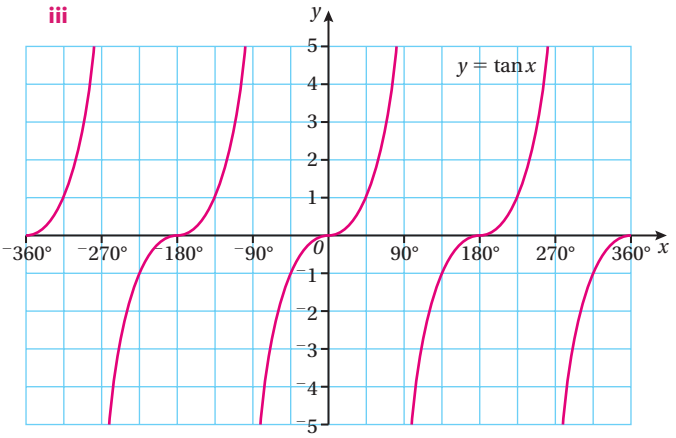
1 a The $y = \sin x$ graph repeats every 360°, the $y = \cos x$ graph repeats every 360° and the $y = \tan x$ graph repeats every 180°.
i



ii



iii



b 45°

2 Graph A resembles $y = \sin x$ because it goes through the origin, at the midpoint of its minimum and maximum values, and is a repeating wave which dips the same below the horizontal axis as rises above it. Graph B is similar to the $y = \cos x$ graph as it is a repeating wave that has its maximum on the y-axis.

CHAPTER REVIEW

1 10.94 m (2 dp)

2 4.33 m (2 dp)

3 78.5° (1 dp)

4 56.96 m (2 dp)

5 Because the scale factor is Triangle PQR = $\frac{3}{2}$ Triangle ABC so PQ = 6 cm and hence $\sin x = \frac{4.8}{6} = 0.8$ as required. In addition because the triangles are similar $\frac{AC}{AB} = \frac{PR}{PQ}$ and hence $\sin x = \frac{AC}{AB} = \frac{3.2}{4} = 0.8$.

6 138 miles

7 7.4 cm

8 74.1 or 105.9 degrees

9 9 cm

10 a 5.16 m **b** 3.1 m^2 (1 dp)

11 a i The sum of the squares of the two shorter sides is 348.34 which is not equal to 376.36, the square of the larger side.
ii 36.2° (1 dp)
b 10.9 cm^2 (1 dp)

12 a 15.8 cm (1 dp) **b** 16.1 cm (1 dp) **c** 10.7°

13 a 7 cm **b** 51.1° (1 dp)

14 a Blue curve is $y = \sin x$; red curve is $y = \cos x$
b 45° and 225°

34 Circle theorems

BEFORE YOU START ...

1 **a** False **b** True **c** False **d** True

2 **a** Arc length $= \frac{6}{5}\pi \approx 3.8$ cm, area $= \frac{9}{5}\pi \approx 5.7$ cm^2

b Arc length $= \frac{175}{4}\pi \approx 137.4$ mm, area $= \frac{4375}{8}\pi \approx 1718.1$ mm^2

c Arc length $= 10\pi \approx 31.4$ cm, area $= 60\pi \approx 188.5$ cm^2

3 **a** Correct **b** Incorrect **c** Correct

LAUNCHPAD

1 A circumference, B chord, C diameter, D radius

2 12,756 km

3 **a** 65°

b Angle APB = 90° because the angle at the circumference subtended from the diameter is a right angle. Angle AQB = 90° because opposite angles in a cyclic quadrilateral sum to 180°. Angle PBA = Angle BAQ and Angle PAB = Angle ABQ because alternate angles are equal. Hence Angle PAQ = Angle AQB = 90° and hence as all four angles are right angles the shape APBQ must be a rectangle.

4 Angle ADB = 90° because the angle on the circumference subtended from the diameter is 90°. From first principles: angles in a triangle sum to 180° so $x + y + y + x = 180°$ and hence $2x + 2y = 180°$ so therefore $x + y = 90°$ and hence Angle ADB $= x + y = 90°$.

EXERCISE 34A

1 **a** E radius **b** F major segment **c** A sector
d B tangent **e** D circle **f** C minor arc

2 **a** Diameter **b** Radius **c** Large **d** Minor arc
e Major arc **f** Chord **g** Centre, EF **h** AB

3 **a** They are equal because base angles in an isosceles triangle are equal.

b As OE = OF, because both are radii of the smaller triangle, triangle OEF is an isosceles triangle and the base angles OFE = OEF. Similarly, OB = OC as they are both radii of the larger circle and hence angles OBC = OCB. Triangles OBC and OEF both share angle EOF and hence they are similar triangles because they share the same angles and angle OEF = OFE = OCB = OBC.

EXERCISE 34B

1 See proof on page 618. Let Angle OAC $= x$ and angle OAB $= y$. Angle OAC = OCA as base angles in an isosceles triangle are equal. Similarly, Angle OAB = OBA. Therefore angle CAB $= x + y$. As angles in a triangle sum to 180° angle AOC $= 180 - 2x$ and angle AOB $= 180 - 2y$. Angles around a point sum to 360° and hence COB $= 360 - (180 - 2x) - (180 - 2y) = 2x + 2y = 2(x + y) = 2$CAB as required.

2 See proof on page 619. Let OXY $= x$ and OZY $= y$. Angle OXY = OYX as base angles in an isosceles triangle are equal. Similarly, angle OZY = OYZ. As angles in a triangle sum to 180° $x + x + y + y = 180°$ and hence $2x + 2y = 180°$ and hence $x + y = 90°$. Therefore angle XYO + OYZ $= x + y = 90°$ so angle y is a right angle as required.

3 By the theorem proved in question 1 angle DOC = 2DAC = 50°. By the same theorem angle 2DBC = DOC and hence angle DBC $= \frac{1}{2}$DOC $= \frac{1}{2} \times 50 = 25°$ as required.

4 236°

5 35°

6 Angle BCD $= 180 - 42 = 138°$

7 CAB = 60° and CBA = 30°

EXERCISE 34C

1 18 units

2 10 units

3 CAD = CED
AEC = ADC = DAT = NAC = ACD
ACE = ADE
DCE = DAE = ATE
CAT = AED = NAD

4 $a = 66$, $b = 24$

EXERCISE 34D

1 **a** Angle BAN = ACB $= x$ and angle TAC = CBA $= x$ by the alternate segment theorem and hence angle TAC = ACB and angle NAB = ABC. Therefore the alternative segment theorem holds and hence CB is parallel to TN.

b Angle ACB = CBA and hence triangle ACB is an isosceles triangle because base angles I an isosceles triangle are equal. Therefore as triangle ACB is an isosceles triangle AC = AB.

2 **a** x **b** $180 - 2x$ **c** $2x$
d $90 - x$ **e** $90 - x$

3 **a** $a = 78$ because opposite angles in a cyclic quadrilateral sum to 180°, $b = 102$ because co-interior angles sum to 180°, $c = 78$ because opposite angles in a cyclic quadrilateral sum to 180°.

b $x = 36$ because $2x + 3x = 180$ because opposite angles in a cyclic quadrilateral sum to 180°, $3x + y = 180$ because angles on a straight line sum to 180° so $y = 72$, $z = 96$ because opposite angles in a cyclic quadrilateral sum to 180°.

c $a = 62$ because opposite angles in a cyclic quadrilateral sum to 180°, $b = 90$ because angles subtended from the diameter at the circumference are right angles, $c = 28$ because angles in a triangle sum to 180°.

d $p = 120$ because LM = LP and hence angle LMP = LPM because base angles in an isosceles triangle are equal and angles on a straight line sum to 180°.

WORK IT OUT 34.1

Option A is incorrect because it states angle OCB = ABC which is not true angle OCB = OBC.

Option B is correct.

Option C is incorrect because angle BAC is not equal to OCB.

EXERCISE 34E

1 **a** 14 **b** 36 **c** 64 **d** 80 **e** 60

2 Example reasons given, there may be more than one way of finding the missing angle.

a Angle EHO = 90°, radii bisect chords at 90° only.

b Angle GFB = 62°, because of the alternate segment theorem.

c Angle GBF = 28°, angles in a triangle sum to 180° and the angle on the circumference subtended from the diameter is 90°.

d Angle FEG = 28°, the angle on the circumference subtended from the diameter is 90° (angle BEF) and BEG is 62° because of the alternate segment theorem.

e Angle DBF = 62°, since GF and BD are parallel angle DBG = 180 − angle BGF = 90° (complementary angles). Thus, angle DBF = 90 − angle GBF = 90 − 28 = 62°.

f Angle GEH = 90°, since opposite angles in a cyclic quadrilateral sum to 180°. Cyclic quadrilateral here is BGED, so angle GEH = 180 − angle GBD = 180 − 90 = 90°.

3 $2x$

4 **a** $90 - x$ **b** $180 - 2x$ **c** $2x - 90$

5 **a** Diameter = 17.3 cm, because OC is the hypotenuse of triangle OCF and radii bisect chords at 90 degrees.

b Diameter = 4.25 m, because OB is the hypotenuse of triangle O(mid point of AB) B and radii bisect chords at 90 degrees.

c Diameter = 31.1 mm, OB is the hypotenuse of the right angled triangle OFB and OF = AF = FB = $\frac{1}{2}$AB.

6 **a** $2x$ **b** $90 - x$ **c** x

7 **a** $a = 70$ **b** $b = 125$

c $c = 60$, $d = 60$, $e = 80$ and $f = 40$

CHAPTER REVIEW

1 Angle ACD = 54° because angles on the circumference in the same segment subtended from the same chord are equal.

2 **a** $x = 140$ because the angle subtended from the chord at the centre is twice the angle subtended from the same chord at the circumference in the same arc.

b $y = 110$ because opposite angles in a cyclic quadrilateral sum to 180°.

3 **a** $a = 54$ because because as it is an alternate segment to CEB

b $b = 102$ as the angle EBT = 48 as alternate segment to ECB, then 180 − a − EBT = 78 as they are angles on a straight line, then use opposite angles in a cyclic quadrilateral add up to 180 for 180 − 78 = 102

35 Discrete growth and decay

BEFORE YOU START ...

1 **a** 0.05 **b** 1.9 **c** 0.004 **d** 0.125

2 **a** $53.68 **b** £36.93 **c** £7.28 **d** $156.40

3 **a** 100 **b** number doubles

4 **a** Initial number of bacteria

b Each hour the number of bacteria is multiplied by 2

LAUNCHPAD

1 **a** £573.76 **b** 500×1.035^n **c** 6 years

2 **a** 3.1 grams (1 dp) **b** 28 years **c** 4×0.975^n

WORK IT OUT 35.1

Tom and Zac are correct, Kayleigh has used simple percentage increase, but it should be compound.

EXERCISE 35A

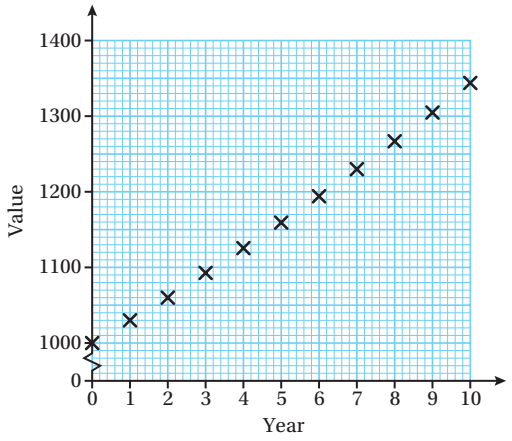
1 **a** £306 **b** £318.36 (2 dp)

c £351.50 (2 dp) **d** 300×1.02^n

2

Investment	Interest rate	1 year	2 years	$5\frac{1}{2}$ years	n years
£250	2%	£255	£260.10	£278.77	250×1.02^n
£1500	4.5%	£1567.50	£1638.04	£1910.87	1500×1.045^n
£50	3%	£51.50	£53.05	£58.83	50×1.03^n

3



4 256 **b** 100×1.04^n

5 **a** 5 444 617 **b** 844 617 **c** 4 447 500

6 £668.68 **b** 450×1.02^n

7 4 months
Month 1 £200 + 8% = £216
Month 2 £216 + 8% = £233.28
Month 3 £233.28 + 8% = £251.94
Month 4 £251.94 + 8% = £272.10

8 After 5 years the population will be nearly 6 and a half million (6 400 000) and after 10 years there will be well over twenty million million (roughly 2.048×10^{13}).
With no outside influences the biologist should be concerned as the predicted spread is large. They should carry out further studies. It is unlikely that the initial 2 mussels are the only ones in the lake.

9 **a** Over 5 years simple interest will get a total of 30%, compound interest 30.70% – compound better

b Over 4 years simple interest will get a total of 24%, compound interest 23.88% – simple better

10

Investment	Rate	1 year	2 years	3 years	n years
$600	1.5%	$609	$618.14	$627.41	600×1.015^n
£500	6%	£530	£561.80	£595.51	500×1.06^n
$6000	10%	$6600	$7260	$7986	6000×1.1^n
£750	20%	£900	£1080	£1296	750×1.2^n

11 **a** $12 889.46 (compound interest $62 889.46, simple interest $50 000)

b 6.29% (2 dp)

12 £296 023.73

13 Model 1 12.476% Model 2 12.4864% Model 3 11.7% Model 2 best

14 15 hours

WORK IT OUT 35.2

Jenny and Ethan are correct, Bob has used simple percentage increase instead of compound.

EXERCISE 35B

1 **a** £10 120 **b** £8565.57
c £5645.41 **d** $11\,000 \times 0.92^n$

2

Initial cost	Depreciation rate	1 year	2 years	6 years	n years
£400	2%	£392	£384.16	£354.34	400×0.98^n
£2 500	15.0%	£2125	£1806.25	£942.87	2500×0.85^n
£50 000	3.5%	£48 250	£46 561.25	£40 376.98	$50\,000 \times 0.965^n$

3 **a** 82 g
b Mathematically speaking there will always be DDT present, however the actually amount will become negligible and too small to be measured.

4 Graph B

5 **a** 20% drop **b** 1342 **c** 20 hours

6 685.7 Pa (1 dp)

7 **a** 7 137 564 **b** 10 years

8

Initial cost	Depreciation rate	1 year	2 years	6 years	n years
$7500	7.5%	$6937.5	$6417.19	$4697.99	7500×0.925^n
£650	5%	£617.50	£586.63	£477.81	650×0.95^n
$34 000	11%	$30 260	$26 931.40	$16 897.36	$34\,000 \times 0.89^n$
£12 000 000	7.5%	£11 100 000	£10 267 500	£7 516 776.59	$12\,000\,000 \times 0.925^n$

9 5% loss each year, 8 years to cost less than two thirds of today's price

CHAPTER REVIEW

1 **a** £85 **b** £765 **c** £153
d £38.25 **e** £153

2 £2903.70

3 £4188.80

4 9

5 5.7% per year

6 **a** 0.95 represents 5% less **b** 12 350 **c** 2025

36 Direct and inverse proportion

BEFORE YOU START ...

1 **a** 30 **b** 15 **c** 20 **d** 12

2 **a** $\frac{1}{12}$ **b** $\frac{2}{5}$ **c** $\frac{9}{10}$

3 **a** 21 **b** 24 **c** 0.4

LAUNCHPAD

1 **a** 450 g **b** 100 km **c** €17.14

2 **a** $c = 6.5a$ **b** £208 **c** 50.6 m^2

3 **a** $c = 15.25s^2$ **b** £857.81 **c** 6.50 m

4 **a** 12 days **b** 5 days

EXERCISE 36A

1 Students' own answers, such as 9 km in an hour, 1 km in 400 seconds, ...

2 £5.50 per hour

3 **a** 60p **b** £3.60 **c** £12 **d** 350 g

4 **a** £6.30
b Ben: 35p per minute; Danny: 32p per minute.
Danny's phone is better value.

5 36 people

6 **a** 640 km **b** 160 km **c** 80 km **d** 5.33 km
e 0.89 km **f** 125 hours (5.2 days)

7 500 metres

8 AU$316.75

9 £26.19

10 **a** **i** £9 **ii** £12 **iii** £60 **iv** £300
b No, the price per litre usually drops as the size increases.

11 **a** £1 = €1.21 **b** 1 = €0.83

12

	Accommodation	Food	Ski rental	Flights
Bun di Scuol	£340	£65	£300	£69
Flims-Laax-Falera	£300	£100	£111	£144

Bun di Scuol = £774

Flims-Laax-Falera = £655, cheaper by £119

13 UK 100 g = £7.69, Spain 100 g = £7.80: cheaper in the UK

WORK IT OUT 36.1

Option A – Option B does not have a fixed gradient and neither Option C nor Option D passes through the origin

WORK IT OUT 36.2

A No. Graph would not go through origin.
B Yes. Graph is straight line and would go through origin.
C Yes. This can be rewritten as $s = 7t$
D No. This would give a curved graph.

EXERCISE 36B

1 $c = 4t$

2 Number of people and area of field

3 **a** 3 km **b** 2.25 hours **c** Runner B
d A 6 km/h, B 8 km/h
e The athletes run at a constant speed

4 **a** £2 **b** cost = 2 × length **c** $c = 2l$

EXERCISE 36C

1 **a** $w = 4m^3$ **b** 500 **c** 2.5

2 **a** $r = 2\sqrt{s}$ **b** 8.94 (2 dp) **c** 36

3 **a** $j = 70I^2$ **b** 2.39 amps (2 dp)

4 **a** $t = 0.32\sqrt{l}$ **b** 39.0625 cm

5 **a** $m = 19.3s^3$ **b** 8.63 cm (2 dp)

WORK IT OUT 36.3

The area is the product of x and y, which is shown by Option C.
The values of x and y can't be zero and the graph extends off before reaching either axis.
Option A has a constant value for y and Option B has a constant sum of x and y, not a constant product.
Option D has x and y values equal to 0, which is impossible.

EXERCISE 36D

1 **a** \$12 **b** 15 **c** \$1.50

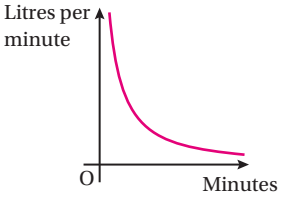
2 **a** $t = 180/s$ **b** 45 mph
c 2 hours 24 minutes **d** 80 mph

3 **a** 300 000 **b** 3 000 000 **c** $p = \frac{3\,000\,000}{t}$

4

m minutes	10	20	30	40	50	60	70	80	90	100
r litres per minute	60	30	20	15	12	10	8.57	7.5	6.67	6

$r = 600/m$



5 **a** $a = 50/b^2$ **b** 12.5 **c** 10

6 $r = 6.63/\sqrt{m}$ (2 dp)

7 0.069 minutes (4.11 seconds)

CHAPTER REVIEW

1 **a** £3.60 **b** 611 grams

2 10 people

3 $c = 2.2m$

4 $y = 5x^2$

5 **a** $F = 3\sqrt[3]{g}$ **b** 15 **c** 343

6 20 minutes

7 $t = \frac{100}{w}$

8 **a** $d = \frac{24}{e^3}$ **b** 0.89 (2 dp) **c** 3.63 (2 dp)

37 Collecting and displaying data

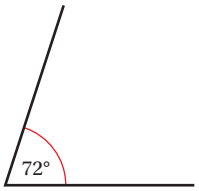
BEFORE YOU START ...

1 Every 5 cm from $1.35 \leqslant h < 1.40$, $1.40 \leqslant h < 1.45$ etc, or every 10 cm $1.30 \leqslant h < 1.40$ etc

2 **a** 25 **b** 20, 40, 60, 80

3 **a** 120°, 30° and 17°

b



LAUNCHPAD

1 **a** Ask a sample of students from each year group.
b Use a sample that is representative of different groups by selecting a stratified sample.

2 **a** Greece and Croatia
b Greece and Croatia, Russia and Portugal
c Germany, Holland, England and Bosnia

3 **a** 31–40
b 18 people

4 **a** It is a histogram with unequal intervals.
b $11 \geqslant \text{age} > 16$
c 15

5 **a** February
b November to December
c April to May

WORK IT OUT 37.1

A Option D is the closest to random.

B A: Only asking women and children means no males are sampled.
B: Only surveying between 8 and 8.30am means only those awake are questioned.
C: Standing outside a bookshop means the survey is biased towards those who buy books.

EXERCISE 37A

1 **a** Not random (might be a very wealthy street).
b Not random (excludes people who work or are out during the day).
c Not random (young people are more likely to wear trainers).
d Random (a person's name does not determine any other characteristic of that person).
e Random (the chance of picking a given name is the same as picking any other name).

2 **a** Any sensible suggestion where parents may be found e.g. soft play area; a park; nursery; baby food section of a supermarket.
b Approximately 700.

3 **a** 15/28 would mean about 161 machines.
b No, because not all the members are likely to be there at once.
c Sample the members present at different times of the week.

4 **a** 1.722 million **b** 574 000 **c** 2.296 million

5 **a** The street may favour a particular income group if the houses are the same.
b All answers should be rounded to the nearest whole number.

Radio Uno	1138
Ears On	546
Hip and Happening	501
Classic Numbers	273
R Town Radio	1047

6

	Year 7	Year 8	Year 9	Year 10	Year 11
Boys	7	8	13	11	10
Girls	5	9	12	12	12

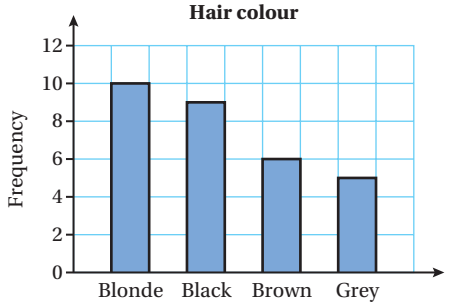
7 Approximately 160 badgers

WORK IT OUT 37.2

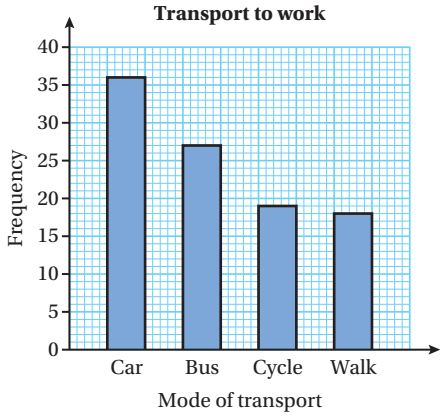
Graph C is the best as it shows the frequency of the numbers of mistakes he makes. Graph A shows the changes in numbers but lines between the points don't mean much, and Graph B shows too much data about each individual test and the axes are not labelled.

EXERCISE 37B

1



2



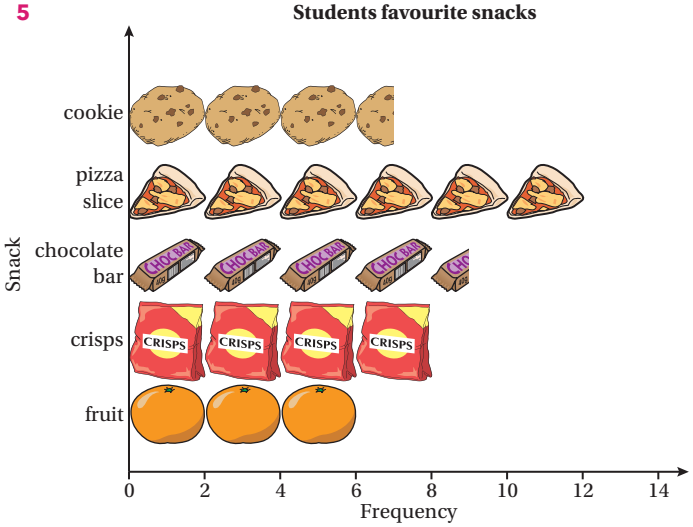
3 a

Visits	Frequency
0	5
1	9
2	11
3	4
4	0
5	1

b

4

5



6 a April **b** 110 mm **c** February
d Approx 220 mm **e** Wetter; there was over 775 mm of rain

7 a She could have started with a blank table.

b

Item	Frequency
Chocobar	10
Apple	3
NRG drink	10
Juicebar	7
Crisps	10
Cheese puffs	9
Gum	6
Fruit chews	8

c

Healthy	Apple	3
	Juicebar	7
Less healthy	Cheese puffs	9
	NRG drink	10
	Gum	6
	Crisps	10
	Fruit chews	8
	Chocobar	10

EXERCISE 37C

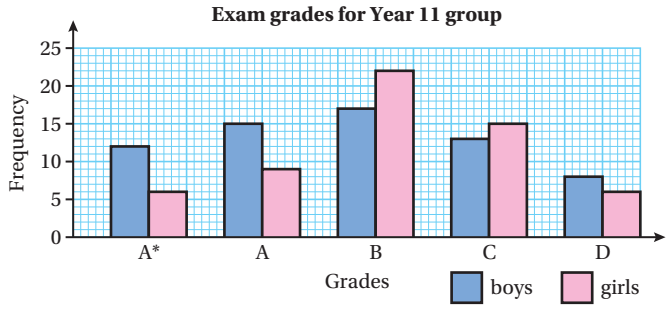
1 a The average time spent watching television and the average time spent doing homework for students in years 7–11.
b The students tend to spend less time watching television.
c The students tend to spend more time doing homework.
d i 90 minutes **ii** 45 minutes

2 a TrueIQ **b** Datalink/G-Commerce
c TrueIQ **d** G-Commerce
e G-Commerce
f Speedlink, costs were greater than income
g £35 000

3 a No, because the graph shows proportions not quantities.
b Company C **c** Company C
d Company C **e** $\frac{3}{20}$
f 30% Shop, 35% Mail Order, 15% Internet sales, 20% Agent

4 a 13% **b** 5.5%
c No, as there is no red 'probable and confirmed cases' component of the bar.
d Africa
e Yes, include the date, so that the audience knows when it is from. Although not up to date may still indicate where the problem is.

5

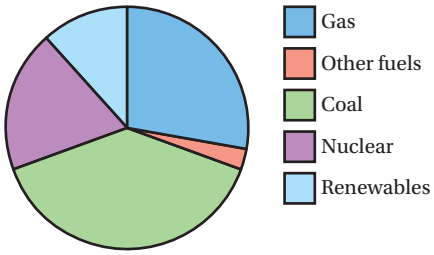


WORK IT OUT 37.3

Graph B is the best as frequency is shown. Graph A is hard to read (too much information) and C is incorrectly labelled.

EXERCISE 37D

1 **Electricity generation**

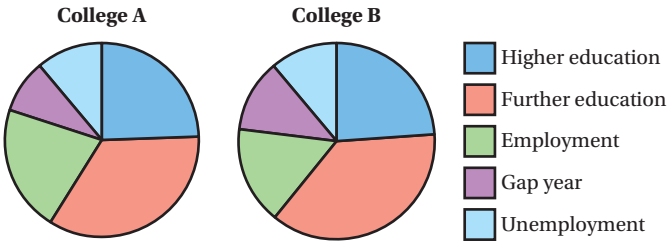


2 **a** Any two proportional responses, such as Ireland has a bigger proportion of under 15s than Greece
b They have the same proportion
c We cannot tell – we don't know the numbers only the proportions

3 **a** 100°; 20 students **b** TV **c** 30°; 6 students

4 **a** Proportion of light goods vehicles has increased. Proportion of motorbikes has increased. Proportion of heavy good vehicles has slightly decreased. Proportion of buses and coaches has slightly decreased.
b 3.54 million (3 sf) **c** 19.4%

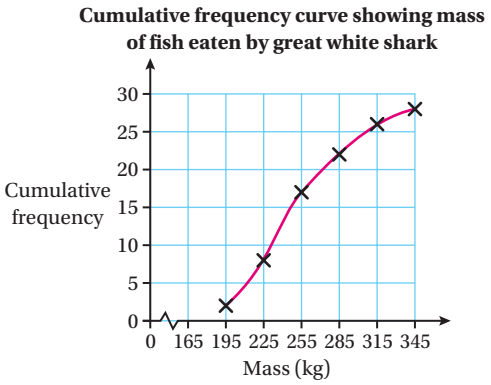
5 College A has a larger proportion of students who go on to further education. A larger proportion of College A students go on to employment. Similar proportions go on to higher education.



EXERCISE 37E

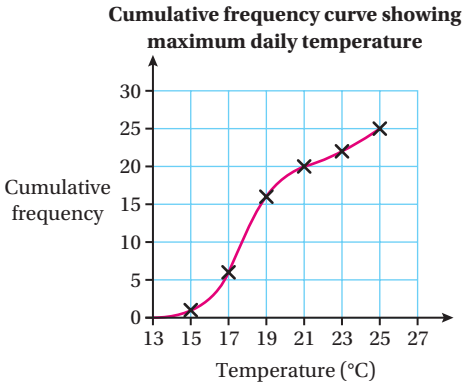
1

Class interval	Cumulative frequency
$165 \leqslant kg < 195$	2
$195 \leqslant kg < 225$	8
$225 \leqslant kg < 255$	17
$255 \leqslant kg < 285$	22
$285 \leqslant kg < 315$	26
$315 \leqslant kg < 345$	28



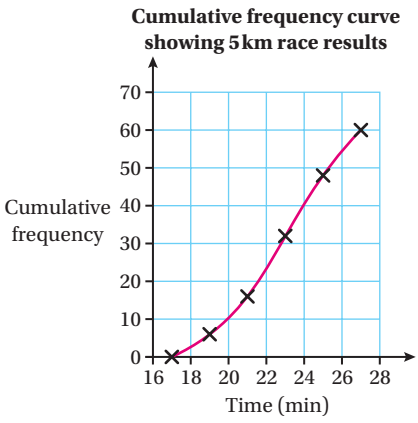
a 9 **b** 3

2



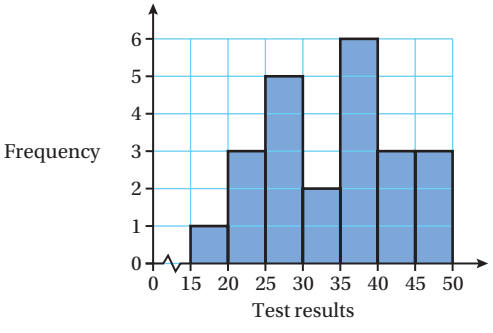
a 19 **b** 4 **c** 2

3

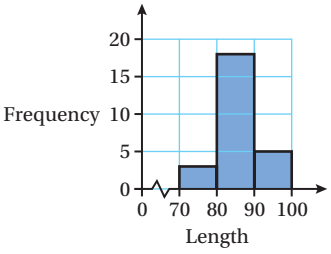


a 12 **b** 9 **c** 6

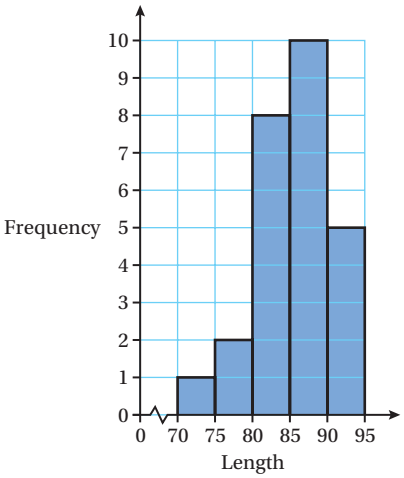
4



5 **a**



b



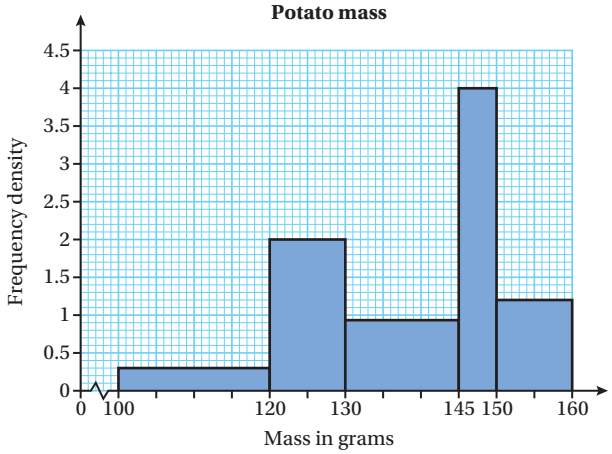
6 **a** 180 **b** 120 **c** $16\frac{2}{3}\%$ **d** $61\frac{1}{9}\%$

7

Group	Frequency
$10 \leqslant \text{temp} < 14$	8
$14 \leqslant \text{temp} < 17$	11
$17 \leqslant \text{temp} < 21$	7
$21 \leqslant \text{temp} < 27$	10
$27 \leqslant \text{temp} < 30$	6

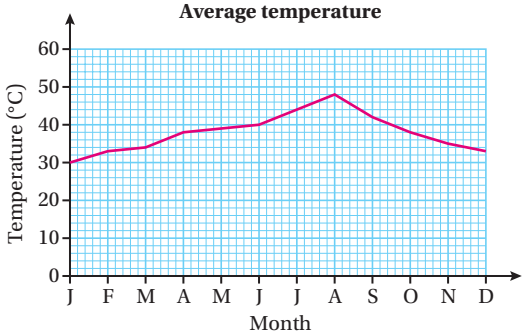
8 **a** 2 **b** $145 \leqslant \text{mass} < 150$ g

c



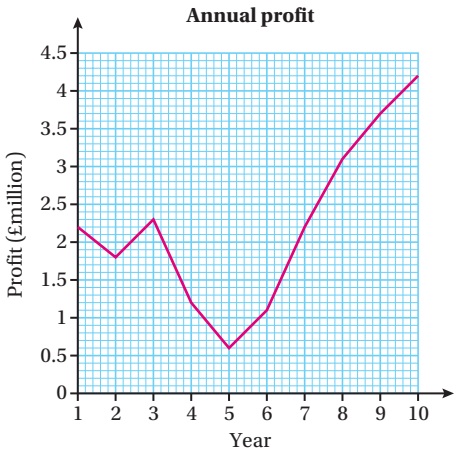
EXERCISE 37F

1 **a**

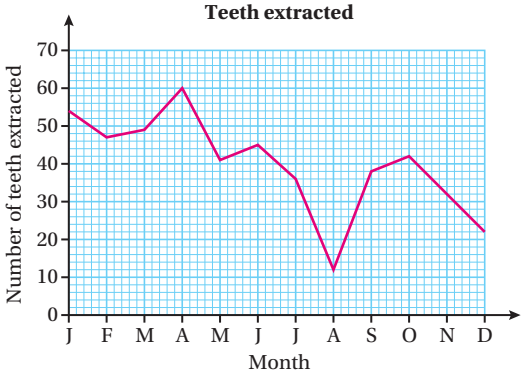


b The temperature gradually rises from a low point in January to a high in August. Temperature then falls more quickly until the end of the year.

2



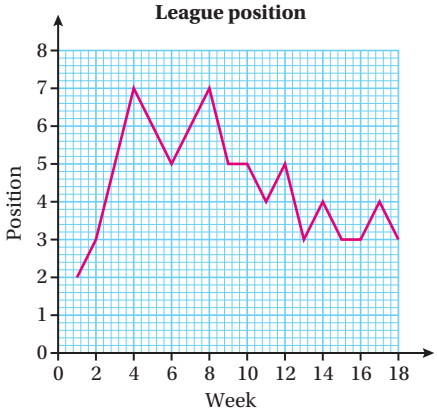
3 **a**



b April was the month with the most teeth extracted, with a sharp decline in August. December was also a quiet month. Otherwise the numbers of teeth extracted do not change much.

c The dentists might have been on holiday for at least part of August.

4 **a**

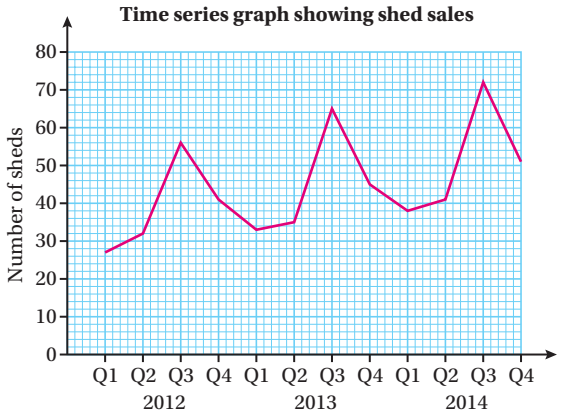


b The team started well in second place but quickly dropped down to seventh place. They were between fifth and seventh for the early part of the season before improving after week ten, when they stayed between fifth and third, where they eventually finished.

5 **a** Quarter 11 **b** Quarter 8

c In Quarter 1 they were £64 000, Quarter 5 £77 000, Quarter 9 £79 000. Given that this is the same quarter in each of three years, it is fair to say that sales are improving.

6 a



b Shed sales have been variable with repeated peaks and troughs. The highest sales were in Q3 of 2014. Generally there seems to be an upward trend each year.

c More sheds are sold in Q3 of each year and less in Q1. This reflects the different seasons.

7 a June and August **b** Water added

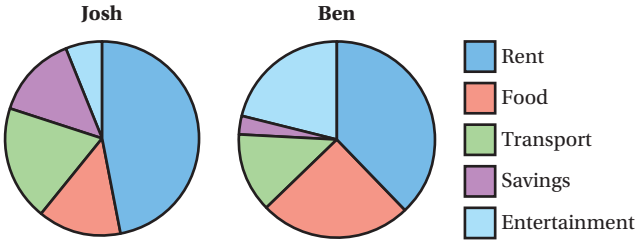
c February to March **d** 35 cm **e** 10 cm

CHAPTER REVIEW

1 a Taking the students whose names begin with a certain letter
Choosing students whose birthday is in a particular month
Any other suitable method

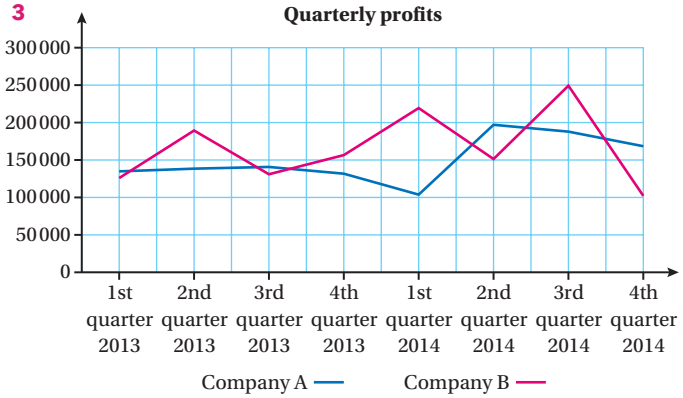
b 204

2 a Pie charts are best for comparison



b Josh spends a larger proportion of his money on rent.
Ben spends a larger proportion of his money on food.
Josh saves a greater proportion of his money.
Ben spends a greater proportion of his money on entertainment.
Josh spends a greater proportion of his money on transport.

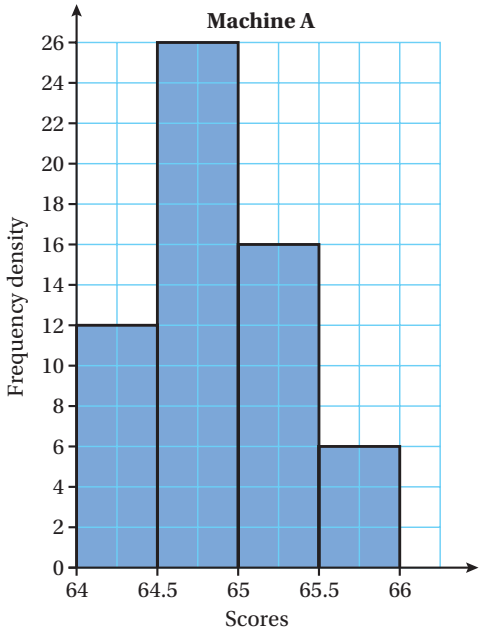
3

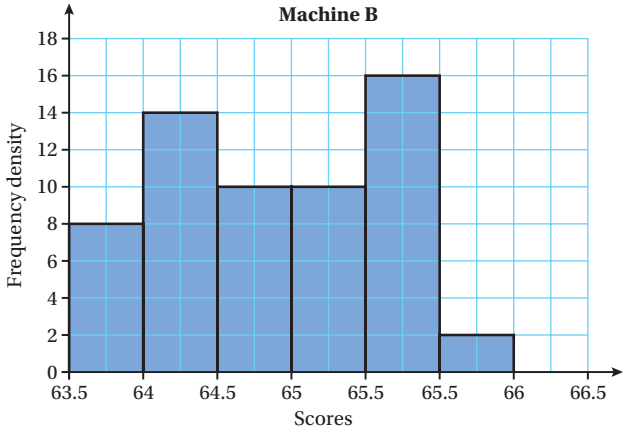


a Generally Company B makes higher profits

b The drop in profits for company B between the 3rd quarter and 4th quarter of 2014

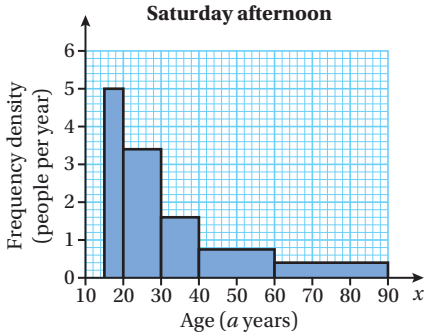
4 a





b Machine A has a smaller range of values. Machine B makes more underweight packets.

5 a



b The highest frequency density in the evening is the $30 \leqslant a < 40$ group, but in the afternoon it is the $15 \leqslant a < 20$ group. They show a similar pattern in the age range from 40 upwards, but it is a bit higher for the evening group. Any other valid comparisons.

38 Analysing data

BEFORE YOU START ...

1 a Mean 3.3, median 3, mode, 2, range 6
b Mean 35, median 35, no mode, range 50.

2 $\left(^{-}1, \frac{1}{2}\right)\left(1, \frac{^{-}1}{2}\right)(2, ^{-}1)$

3 a $\frac{^{-}1}{2}$ **b** $y = \frac{^{-}1}{2}x$

LAUNCHPAD

1 a 0–5 **b** 6–10 **c** 10.95 (2 dp) **d** 21

2 a The median is the same for both sets of data. The range is approximately equal for both months. The interquartile range for March was larger than June, the maximum and minimum scores were both higher in June.
b No, overall performance was worse in June.

3 The difference in the two values is only just over 1%, but not starting at zero makes the difference look much bigger.

4 a Negative correlation
b The greater the number of cigarettes smoked per day, the lower the life expectancy.

EXERCISE 38A

1 a

Days absent (d)	Frequency	Midpoint	Midpoint × frequency
$0 \leqslant d < 5$	15	2.5	37.5
$5 \leqslant d < 10$	23	7.5	172.5
$10 \leqslant d < 15$	19	12.5	237.5
$15 \leqslant d < 20$	12	17.5	210
$20 \leqslant d < 25$	6	22.5	135
Total	75		792.5

b $5 \leqslant d < 10$ **c** 10.57 (2 dp), $5 \leqslant d < 10$, 25

2 a

$0 \leqslant s < 20$	8
$20 \leqslant s < 40$	12
$40 \leqslant s < 60$	9
$60 \leqslant s < 80$	7
$80 \leqslant s < 100$	6

b 45.7 (1 dp), $40 \leqslant s < 60$, 100
c Yes, as we have the grouped data and can estimate it
d $20 \leqslant s < 40$

3 a

$1.45 \leqslant h < 1.50$	1
$1.50 \leqslant h < 1.55$	3
$1.55 \leqslant h < 1.60$	11
$1.60 \leqslant h < 1.65$	4
$1.65 \leqslant h < 1.70$	13
$1.70 \leqslant h < 1.75$	2
$1.75 \leqslant h < 1.80$	6
$1.80 \leqslant h < 1.85$	6
$1.85 \leqslant h < 1.90$	3

1.68, $1.65 \leqslant h < 1.70$

b $1.65 \leqslant h < 1.70$

c Mean is 1.67, median is $1.60 \leqslant h < 1.70$

The mean goes down slightly and the median class grows so is a wider value range.

4 a

2h 30 ⩽ t < 3h 00	4
3h 00 ⩽ t < 3h 30	5
3h 30 ⩽ t < 4h 00	6
4h 00 ⩽ t < 4h 30	8
4h 30 ⩽ t < 5h 00	4
5h 00 ⩽ t < 5h 30	3

b Modal class 4h 00 ⩽ t < 4h 30
c Mean = 3h 57, median = 4h 00 ⩽ t < 4h 30, range = 3 hours

5 a 309.6 (1 dp) **b** $300 \leqslant t < 350$ **c** $350 \leqslant t < 400$

EXERCISE 38B

1 Median 45, IQR 20

2 Median 2400, IQR 1100

3 Median 93, IQR 9

EXERCISE 38C

1 A Mean 54, Median 54, Mode 35, Range 34.
B Mean 55.9, median 58.5, no mode, range 40
A's averages are lower than B's and it has a smaller range

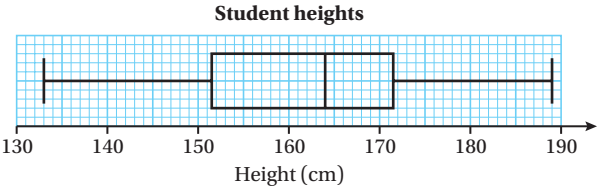
2 a Ahmed Mean 36.7 (1 dp), median 39.5, mode 27, range 48.
Bill mean 26.3 (1 dp), median 28, mode 44, range 46.
b Ahmed has had a better season as his mean and median are higher, as is the total number of runs scored

3 Bus 127 mean 19.9, median 20, no mode, range 6. Bus 362 mean 19.4 (1 dp), median 19, mode 15, range 11. Bus 362 is better on most measures, but it has the greatest range so is potentially less reliable. It does not matter that there is less data as averages used.

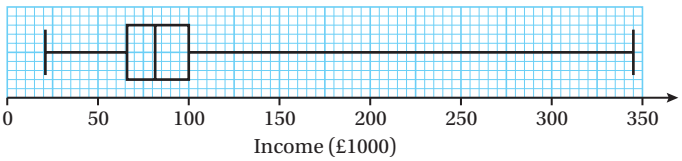
4 Mean machine A = 321.2 (1 dp), machine B 328.9 (1 dp). Both have same range (250), median ($300 \leqslant b < 350$) and modal group ($350 \leqslant b < 400$). Machine B is the sensible choice.

5 a The median negates the impact of extreme values
b The mean is most useful when there are no outliers in a data set. Any example referring to this.
c Scores achieved in a sport, or other categorical data examples.

6 a 151.5, 164, 171.5 **b** 20
c



7 a 67 000, 81 500, 100 000 **b** 33 000
c



8 a 33 **b** 30 **c** Team B
d Team B as it has both a smaller range and IQR than team A
e Team A
f the median and UQ are both higher, as is the highest score

EXERCISE 38D

1 No vertical scale

2 The horizontal scale is not consistent. This is probably to make growth look better than it is.

3 The width of the bars change exaggerating the later sales

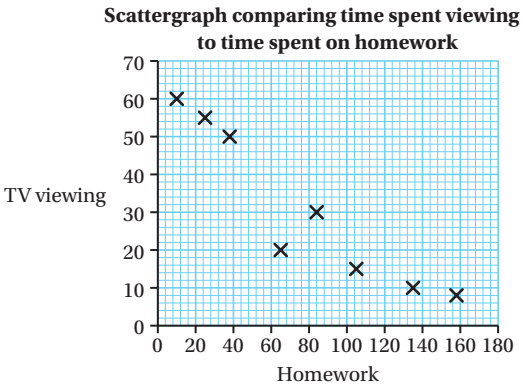
4 a Vertical scale does not start at zero
b To emphasise the changes

5 a They are both the same
b The 3D effect makes the scale difficult to read

6 Either a bar chart or a time series graph would be best.

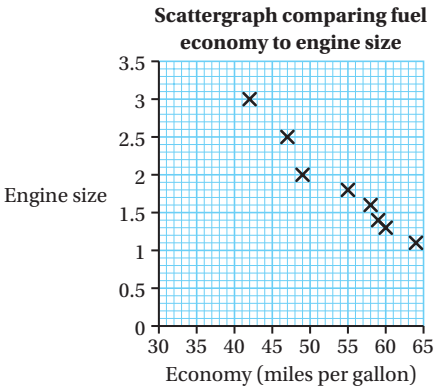
EXERCISE 38E

1

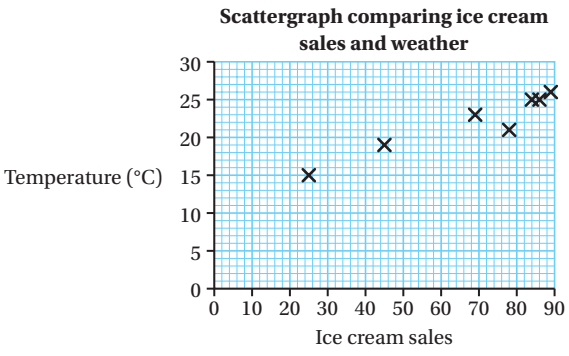


Graph shows a strong negative correlation

2

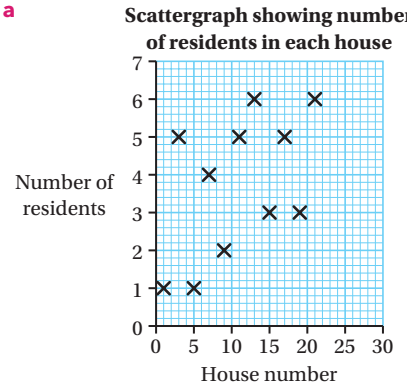


3 a



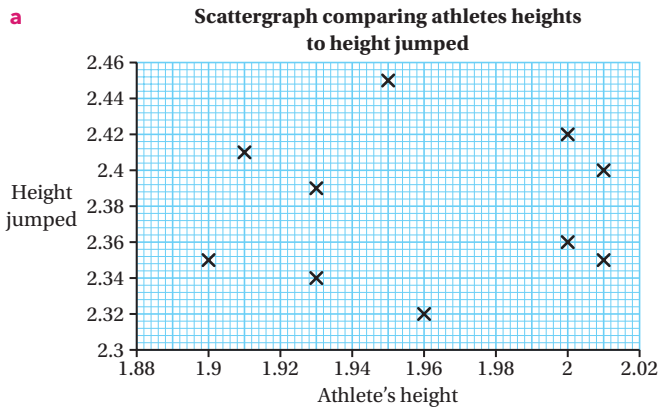
b Sales might also be affected by location, day of the week or time of day

4 a



b Weak positive correlation

5 a

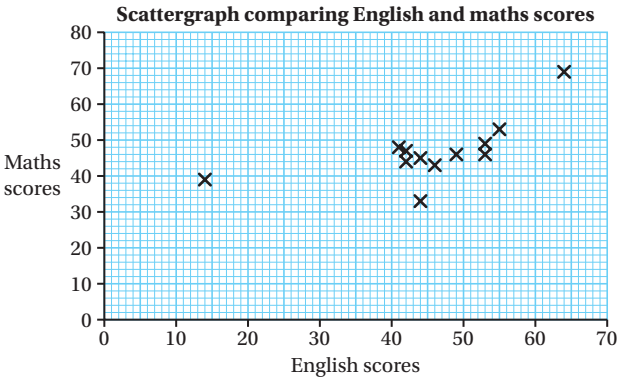


b No correlation

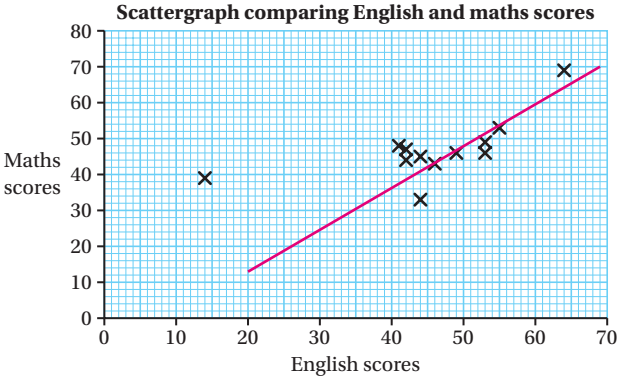
EXERCISE 38F

1 a 48.7 (1 dp) **b** 49 **c** 47
d 46 **e** 46.1 (1 dp)

2 a



b



c The point (14, 39) and the point (64,69)

3 a 112 minutes 54 seconds
b No. The company has calculated without the outlier of 156 minutes and has rounded up to the nearest minute.

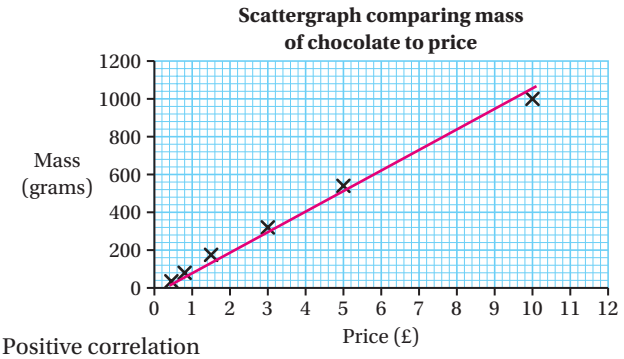
CHAPTER REVIEW

1 First lake: mean 20.14 (2 dp), median 21, range 40, IQR 23
Second lake: mean 16, median 15.5, range 15, IQR 8
first lake would suit experts because all central tendency measures are higher and they could cope with the wider spread of values. Second lake would suit beginners as it is more consistent (lower IQR and range) and has lower wind speeds for all central tendency measures.

2 a Minimum and maximum values, the upper and lower quartiles, the median and the interquartile range
b It is a visual picture of how the middle half of the data relates to the spread of the entire data set, and how representative the median is as a measure of central tendency.

3 a The vertical scales are different.
b The left graph as it shows a steeper climb
c Someone wanting to minimise the appearance of the growth

4 a



b Positive correlation

39 Interpreting graphs

BEFORE YOU START ...

1 A: inverse proportion means that $y = \frac{k}{x}$, where k is a constant. A is the graph of $y = \frac{6}{x}$ therefore represents an inverse proportion.

2 1.5

LAUNCHPAD

1 a Something traveling at constant speed
b Something at rest/stationary
c Something traveling at a constant speed then very rapidly returning to its starting point
d Something traveling very rapidly, stopping for a time, then traveling very rapidly again, then stopping for a time
e Something traveling for a time at a slow speed, then very rapidly, then at a slower speed again but faster then the initial speed
f Something traveling back towards its starting point and then moving back away from it again (or passing it and continuing)

2 Check that students' justifications are sensible.
a A **b** D **c** B **d** B **e** A **f** C

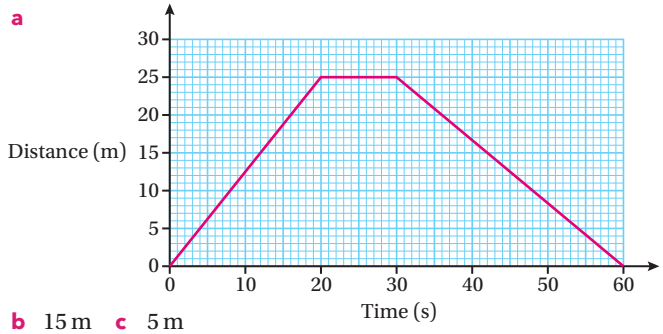
3 B: Area under the graph is the rate of flow × time

EXERCISE 39A

1 a Driving at constant speed; stationary; driving back to base at constant speed; stationary; driving at constant speed ; stationary.
b 50 + 40 + 40 = 130 minutes (or 2 hours 10 minutes) as we presume this is why the taxi is stationary for these periods where the graph is horizontal and no distance is traveled.
c 25 miles
d **i** 12 mph **ii** 10 mph **iii** 6 mph **iv** 6.25 mph

2 a 720 m **b** 7 minutes
c 9:06:40 and 9:21:17 (up to 15 seconds either side is acceptable)
d Going to the supermarket

3 a

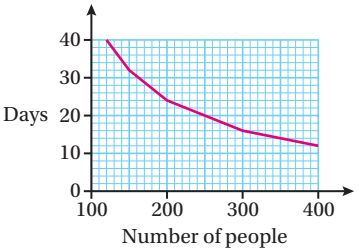


b 15 m **c** 5 m

4 a

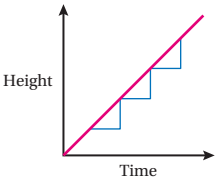
No. of people	120	150	200	300	400
Days the water will last	40	32	24	16	12

b



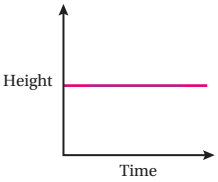
EXERCISE 39B

1 a



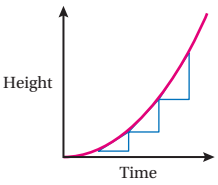
Tank being filled at a constant rate

b



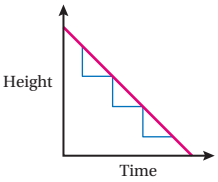
No water going in

c



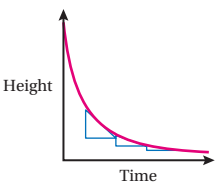
Water flow is increasing

d

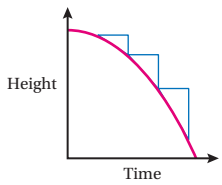


Water draining out of tank at a constant rate

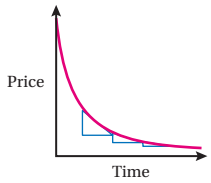
e



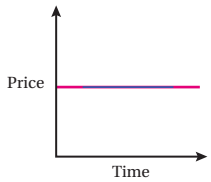
Water draining out of tank, flow is slowing

f 

Water draining out of tank, flow is increasing

2 a 

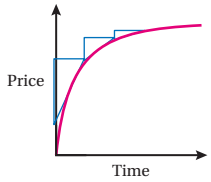
The price is falling at a decreasing rate – the gradient triangles all have negative gradients getting less steep as time goes on

b 

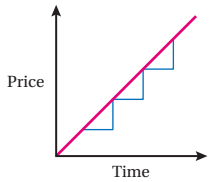
The price is constant – gradient triangle has a gradient of 0

c 

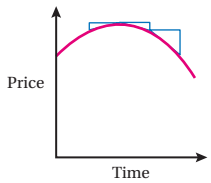
The price is rising at an increasing rate – the gradient triangles all have positive gradients getting more steep as time goes on

d 

The price is rising at a decreasing rate – the gradient triangles all have positive gradients getting less steep as time goes on

e 

The price is rising at a fixed rate – each gradient triangle has the same gradient

f 

The price rises (gradients positive) then decreases (gradients negative)

3 1–2 Speed increasing, acceleration reducing
2–3 Speed increasing, acceleration reducing
3–4–5 constant speed, acceleration zero
5–6 speed decreasing, deceleration reducing
6–7 speed decreasing, deceleration reducing
7–8 constant speed, acceleration zero

EXERCISE 39C

1 a 135 m (allow 5 m either way)
b 1.2 seconds
c 2.5 to 4 seconds
d i Approx. 32 m/s **ii** Approx. 140 m/s
e 100 m/s

2 a 7.15am **b** 10.30am
c i Approx. 0.35 m/h approx. **ii** 1 m per hour
d To holiday makers, the coast guard and anyone using the beach

3 a They accelerate uniformly then travel at a constant speed, then accelerate at a faster rate which peaks and then they decelerate at a constant rate until they stop.
b 5 km/h
c They go from accelerating to decelerating
d 12 km/h^2 **e** 36 km/h^2

4 a i 4 (allow 0.5 either way) **ii** $^-2$ (allow 0.5 either way)
b ($^-1.5$, 2.25)
c No, because the positives and negatives would cancel out

EXERCISE 39D

1 a 80 km **b** 120 km
c 93.75 km **d** Approx 64 km

2 a Area under graph would be the amount of water that had flowed down the river.
b You can't tell as the graph doesn't start at zero
c 210 000 cubic feet
d 180 000 cubic feet
e When it rains there will be more water running into the river upstream

CHAPTER REVIEW

1 a $^-4.65$ (allow 0.5 either way)
b Decreasing population which is becoming more stable (rate of decrease is lessening)

2 a B to C **b** Ben was stationary.
c Slowed down

3 a 32 cm
b 282.74 cm^3/min (to 2 dp)
c 141.37 cm^3/min (to 2 dp)

4 a 90 km/h **b** 0.3 km/min^2 **c** 15 km
d 2 min 30 s **e** 0.3 km/min **f** 17.50 km

40 Algebraic inequalities

BEFORE YOU START ...

1 a 3 **b** 9 **c** $\frac{55}{28}$ **d** $^-8$

2 $x = ^-1$

3 a 3 **b** $y = 3x - 9$

LAUNCHPAD

1 a $p < 0.45$ **b** $x \geqslant ^-4$ **c** $11 < y < 18$

2 a 3, 4 **b** $^-1$, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
c 3, 4, 5

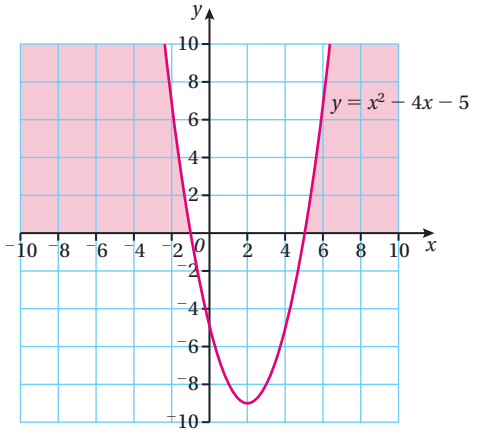
3 a 

b 

c 

4 a True **b** False
5 a $x < 2$ **b** $x \geqslant ^-2$
6 $x = 5$, $x = ^-1$

7



$x^2 - 4x - 5 < 0$ for $x > 5$ and $x < ^-1$

EXERCISE 40A

1 a $4 + 7 > 4 + 3$ **b** $8 - 5 < 13 - 5$
c $^-5 + 3 < ^-1 + 3$ **d** $^-4 - 6 > ^-11 - 6$

2 a $2 \times 7 > 2 \times 3$ **b** $2 \times 8 < 2 \times 13$
c $7 \div 2 > 3 \div 2$ **d** $8 \div 2 < 13 \div 2$

3 a $(^-2) \times 7 < (^-2) \times 3$ **b** $(^-2) \times 8 > (^-2) \times 13$
c $7 \div (^-2) < 3 \div (^-2)$ **d** $8 \div (^-2) > 13 \div (^-2)$

4 a Any four integers greater than 14
b Any four integers greater than or equal to 6
c Any four integers less than or equal to $^-2$
d Any four integers greater than or equal to 4
e Any four integers less than or equal to 9

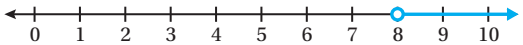
5 Infinitely many numbers all bigger than 6

6 Four values, $x = 4, 5, 6, 7$.
Including decimals and fractions there are an infinite number of values for x that satisfy $3 < x < 8$

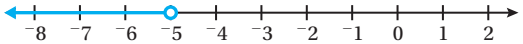
7 Whole number values that satisfy $6 > x > 2$ are 5, 4 and 3.

EXERCISE 40B

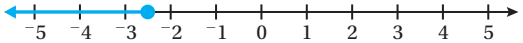
1 a $\{x: x < ^-2\}$ **b** $\{x: x \leqslant ^-2\}$
c $\{x: x > ^-2\}$ **d** $\{x: x \geqslant ^-2\}$

2 a


b

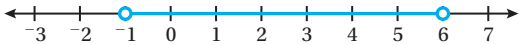

c


d


e


3 a $\{x: 7 < x \leqslant 11\}, \{x: 11 \geqslant x > 7\}$
b $\{x: 7 \leqslant x < 11\}, \{x: 11 > x \geqslant 7\}$
c $\{x: ^-1 < x \leqslant 3\}, \{x: 3 \geqslant x > ^-1\}$
d $\{x: ^-1 \leqslant x < 3\}, \{x: 3 > x \geqslant ^-1\}$

4 a


b


EXERCISE 40C

1 a $x \leqslant 5$ **b** $x \leqslant ^-13$ **c** $x < 3\frac{1}{2}$ **d** $x \geqslant ^-10$
e $x < ^-20$ **f** $x < ^-1$

2 a $h < 19$ **b** $y \leqslant 30$ **c** $x \leqslant ^-1$ **d** $h \geqslant \frac{^-3}{2}$
e $y \geqslant \frac{^-44}{3}$ **f** $n < 48$ **g** $v \leqslant \frac{^-13}{6}$ **h** $z > 62$
i $k > 33$ **j** $e > \frac{31}{28}$

3 $p > 6$

4 $q < 68$

5 $p \leqslant 3$

6 $d > 6$

7 $a > ^-6$

8 a $y \geqslant \frac{150 - x}{2}$ or $y \geqslant 75 - \frac{x}{2}$
b i $y \geqslant 57.5$ **ii** $y \geqslant 50.5$

EXERCISE 40D

1 a Estimates from the graph: $x^2 - 3x - 3 \geqslant 0$ when $x \geqslant$ (accept values between) 3.7 and 3.9 and $x \leqslant$ (accept values between) $^-0.7$ and $^-0.9$
b $x^2 - 3x - 3 < 0$ when $\frac{^-1}{2} < x < 3\frac{1}{2}$

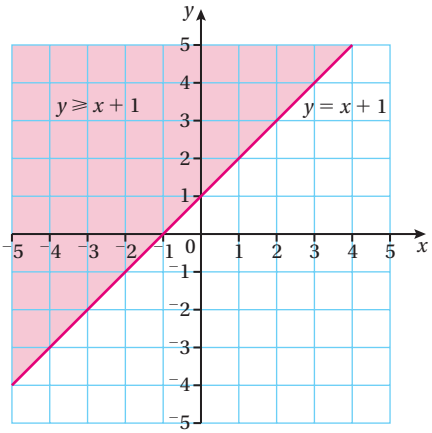
2 a $^-2x^2 + 16x - 24 \geqslant 0$ when $2 \leqslant x \leqslant 6$
b $^-2x^2 + 16x - 24 \leqslant 0$ when $x \geqslant 6$ and $x \leqslant 2$

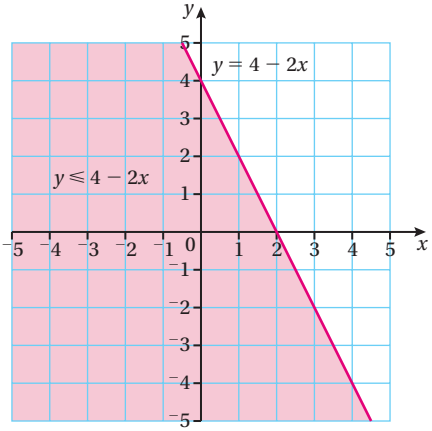
3 a $x > 3$ and $x < ^-2$ **b** $^-4 \leqslant x \leqslant ^-1$
c $x \geqslant 5$ and $x \leqslant 2$ **d** $^-3 < x < 0$

4 a $^-4 \leqslant x \leqslant 1.5$ **b** $0 < x < 5$
c $x \leqslant ^-2$ and $x \geqslant 4$ **d** $x < ^-4$ and $x > 1.5$

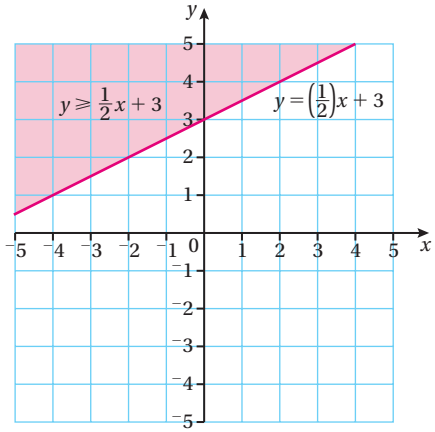
5 a $x^2 - x - 6 > 0$ **b** $x^2 - x - 6 < 0$ **c** $x^2 - x - 6 \leqslant 0$

EXERCISE 40E

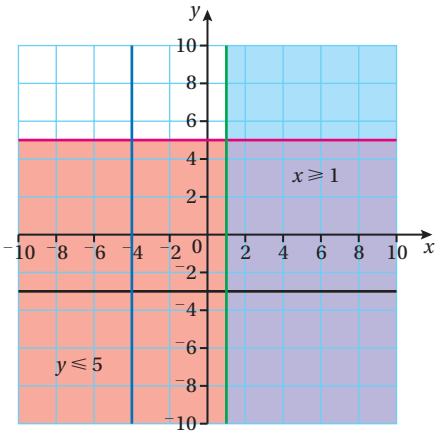
1 a


b


c



2



Any two points with x-coordinate greater than 1 and a y-coordinate less than 5.

3 $y \leqslant \frac{1}{3}x - 2$

Test a point either side of the line.

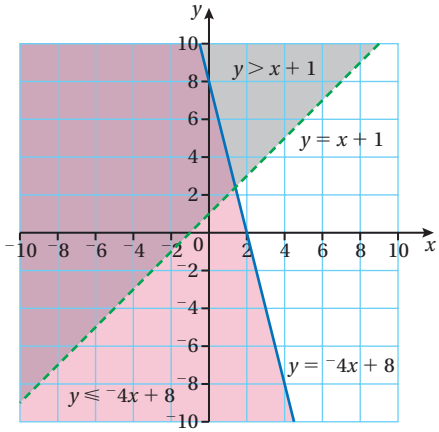
4 a Line is $y = \frac{-5}{4}x + 3$ and shaded region is $y \geqslant \frac{-5}{4}x + 3$

b Line is $y = 2x - 4$ and shaded region is $y \leqslant 2x - 4$

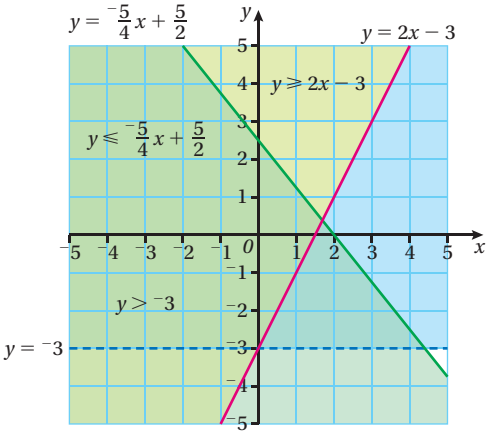
5 $x + 2y < -4$ and $3x + y > 3$

EXERCISE 40F

1



2



3 $y = \frac{-5}{3}x$ $y = \frac{1}{2}x - 4$ Region $y \geqslant \frac{-5}{3}x$ and $y > \frac{1}{2}x - 4$

Test point in the region (3, 3) $3 \geqslant \frac{-5}{3} \times 3$ $3 \geqslant -5$

$3 > \frac{1}{2} \times 3 - 4$ $3 > \frac{-5}{2}$

4 Pink line $y = -3x - 1$ blue line $y = 5x + 1$

purple line $y = \frac{1}{3}x + 4$

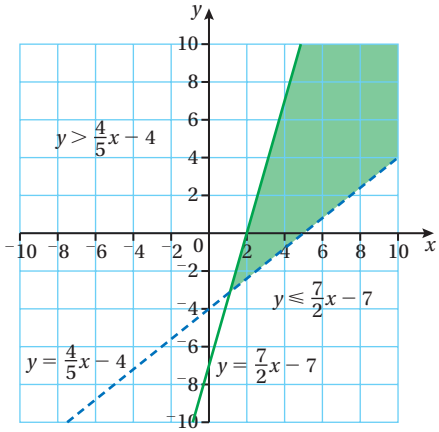
$y \geqslant -3x - 1$, $y \geqslant 5x + 1$, $y \leqslant \frac{1}{3}x + 4$

5 Region is $y \geqslant \frac{-4x}{3}$ and $y \geqslant 2x - 12$ and $y \leqslant x - 6$

Test point in the region (4, -3) $-3 \geqslant \frac{-(4 \times 4)}{3}$ $-3 \geqslant -5.33$

$-3 \geqslant (2 \times 4) - 12$ $-3 \geqslant -4$ $-3 \leqslant 4 - 6$ $-3 \leqslant -2$

6 $x = \frac{10}{9}$ $y = \frac{-28}{9}$



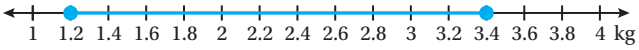
CHAPTER REVIEW

1 a True **b** False **c** False

2 a 6 **b** 8 **c** 12

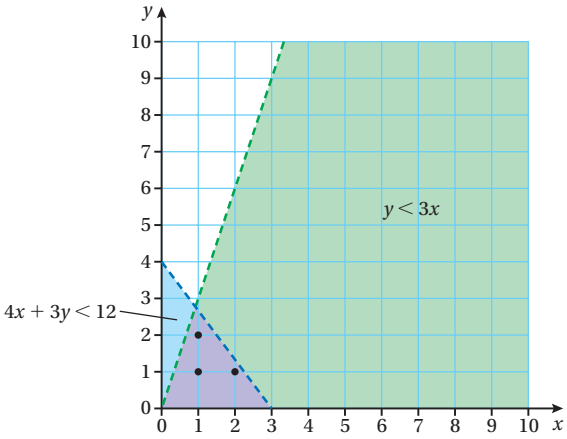
3 a B, C and G **b** $y \leqslant {}^{-}x + 8$

4 a

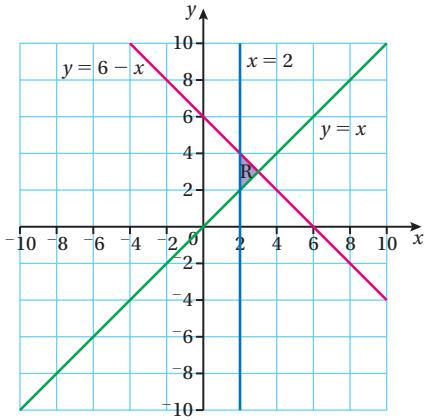


b $1.2 \leqslant m \leqslant 3.4$

5 (1, 1), (1, 2) and (2, 1)



6



7 $y \leqslant 120$ $x \leqslant 150$ $y \leqslant {}^{-}x + 200$

8 $x < {}^{-}3$ and $x > 4$

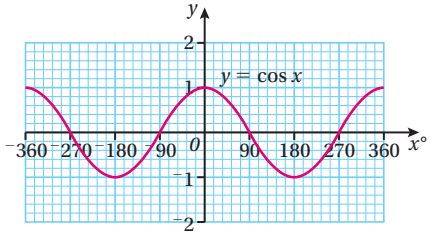
41 Transformations of curves and their equations

BEFORE YOU START ...

1 Function **b** has a straight line graph; the others produce curves
$5y + x = 10$ is a linear graph: sketch is a straight line passing through (0, 2) and (10, 0).
$y = x^2 + 9$ is a parabola with a minimum point (0, 9) and y-axis as the axis of symmetry.
$y = \frac{2}{x}$ is a rectangular hyperbola in the 1st and 3rd quadrants.
$y = (x + 7)^2 - 2$ is a parabola, turning point $({}^{-}7, {}^{-}2)$, line of symmetry $x = {}^{-}7$, y-axis intercept (0, 47).

2 a $y = \sin x$ **b** $y = \tan x$

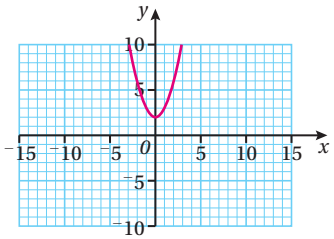
c



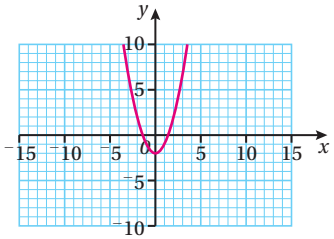
3 $3x^2 + 6x + 7 = 3(x + 1)^2 + 4$

LAUNCHPAD

1 a Axis of symmetry $x = 0$, vertex (0, 2)



b Axis of symmetry $x = 0$, vertex $(0, {}^{-}2)$

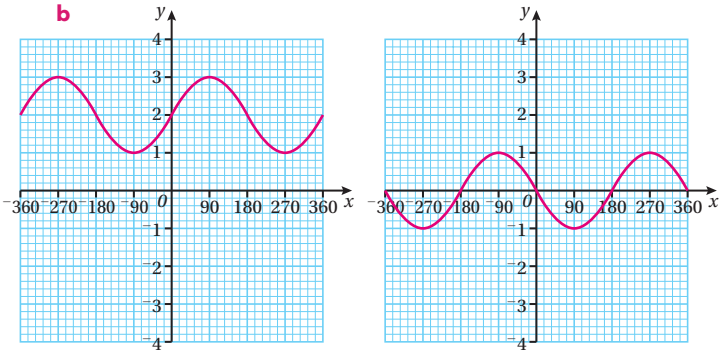


c $y = {}^{-}x^2$

d The graph remains the same since the y-axis is the axis of symmetry for the curve.

2 a $\sin 90° = 1$; $\cos 90° = 0$; $\cos \theta = 1$ when $\theta = 360n$ for integer values of n

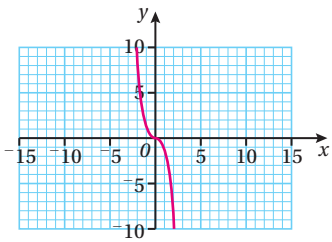
b



3 a $y = x^3$

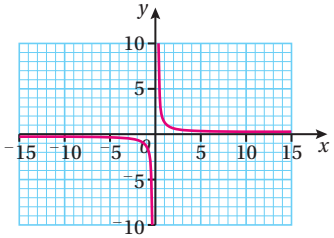
b $y = {}^{-}x^3$

i



ii This is the same as a reflection about the x-axis and it equation is $y = {}^{-}x^3$

c



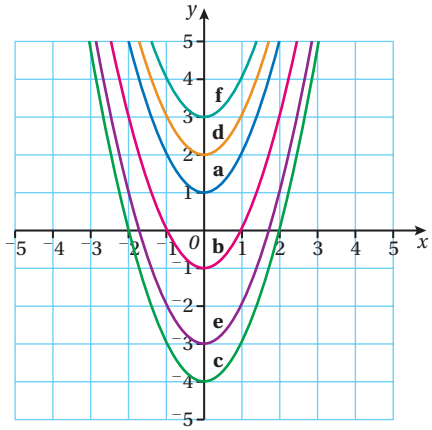
4 A $y = \frac{{}^{-}1}{(x - 3)}$

WORK IT OUT 41.1

Graph A has two solutions.
Graph B has no solution and Graph C has one solution.

EXERCISE 41A

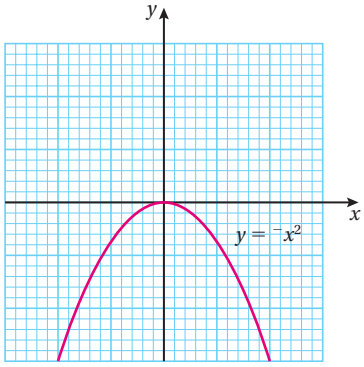
1



Vertices are as follows:

a (0, 1) **b** (0, ⁻1) **c** (0, ⁻4)
d (0, 2) **e** (0, ⁻3) **f** (0, 3)

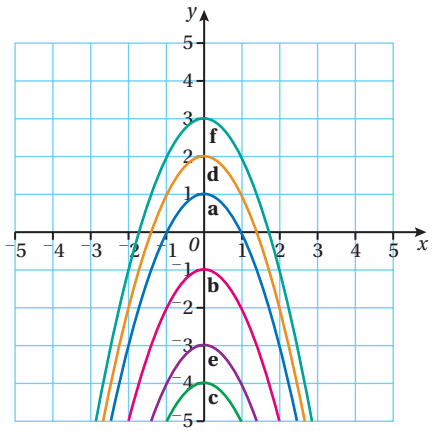
2



Reflection

3 As x increases and decreases in value, y decreases.

4



Reflection (about a horizontal line through the vertex)

EXERCISE 41B

1

	Axis of symmetry	Vertex	y-intercept
a	$x = 5$	(5, 0)	(0, 25)
b	$x = 2$	(2, 9)	(0, 13)
c	$x = ⁻6$	(⁻6, ⁻7)	(0, 29)
d	$x = 3$	(3, ⁻10)	(0, ⁻1)

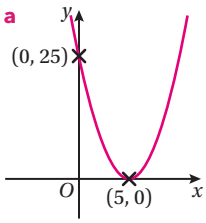
2 $y = x^2 - 4x - 1$

3 All are reflections of the originals about the following lines:

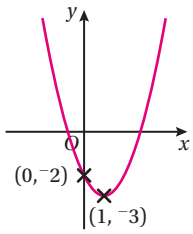
a $y = 0$ **b** $y = 9$ **c** $y = ⁻7$
d $y = ⁻10$

4 Equation a has one root, b has no real roots, c and d each have two roots.

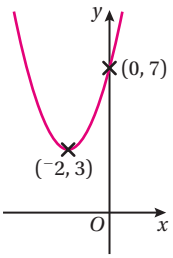
5 a



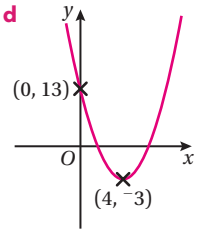
b



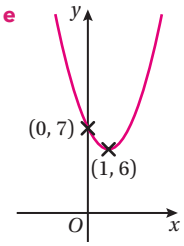
c



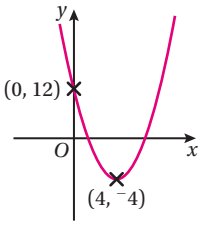
d



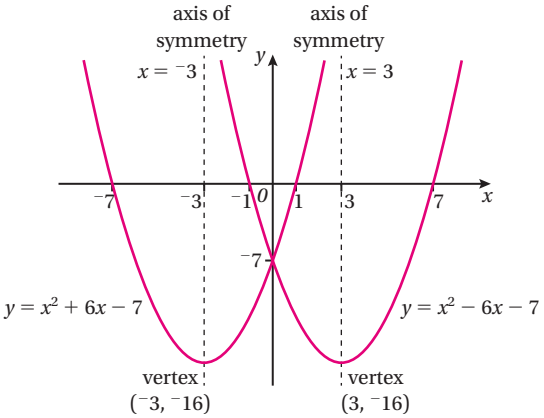
e



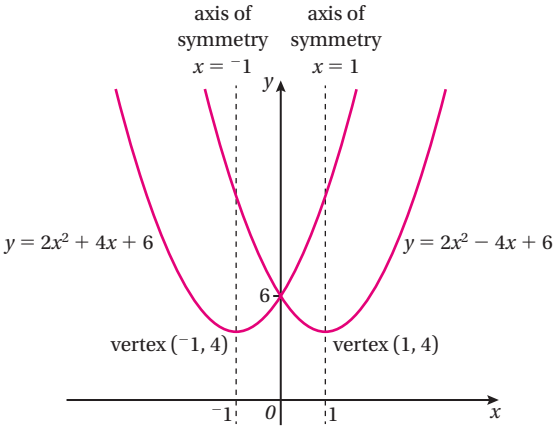
f



6 a i Axis of symmetry $x = ⁻3$, vertex (⁻3, ⁻16), y-intercept (0, ⁻7)
ii Axis of symmetry $x = 3$, vertex (3, ⁻16), y-intercept (0, ⁻7)
c $y = x^2 - 6x - 7$
d 6 units right
e



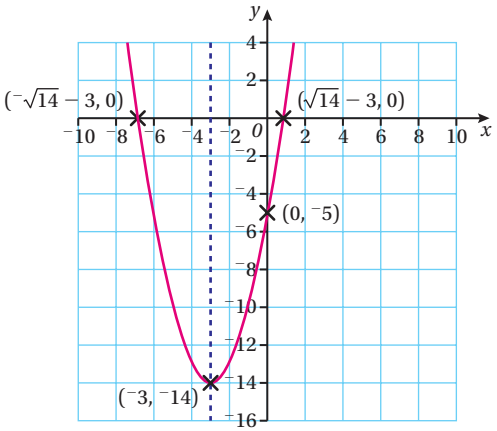
7 a i Axis of symmetry $x = ⁻1$, vertex (⁻1, 4), y-intercept (0, 6)
ii Axis of symmetry $x = 1$, vertex (1, 4), y-intercept (0, 6)
c $y = 2x^2 - 4x + 6$
d 2 units to the right.
e



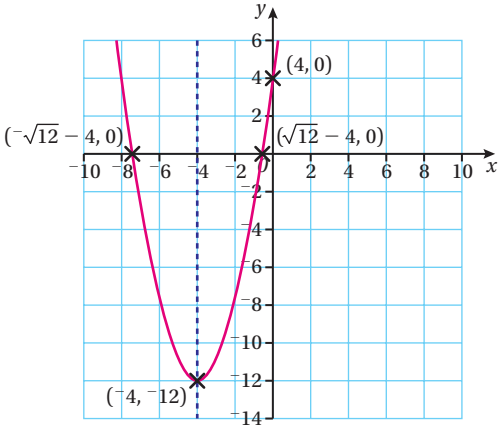
8 a $y = 3x^2 - 8x - 2$ **b** $y = x^2 - 3x - 2$

EXERCISE 41C

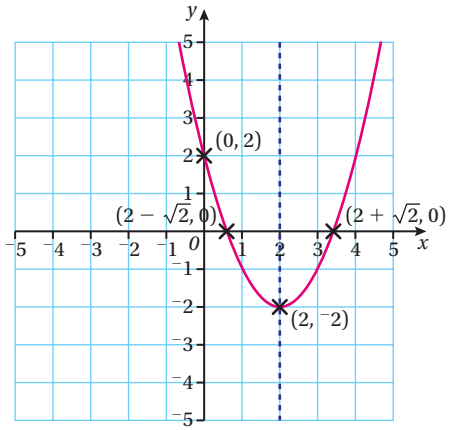
1 a $y = (x + 3)^2 - 14$



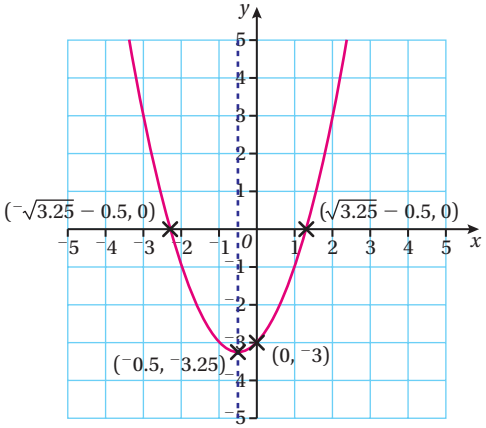
b $y = (x + 4)^2 - 12$



c $y = (x - 2)^2 - 2$

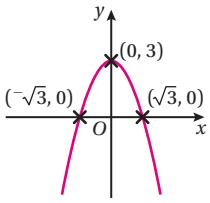


d $y = (x + 0.5)^2 - 3.25$

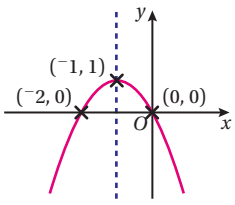


2

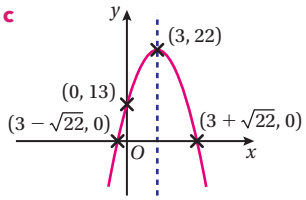
a



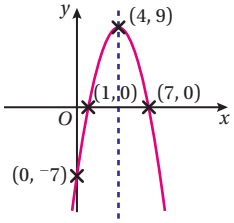
b



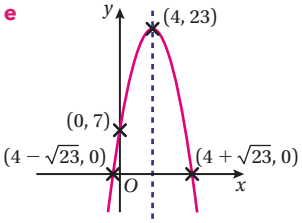
c



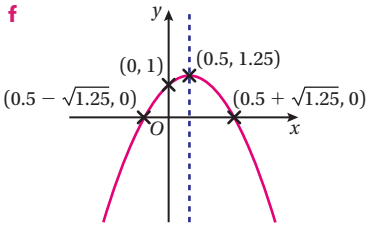
d



e



f



EXERCISE 41D

1 a $y = \sin x + 2$ and $y = \sin x - 2$ are vertical translations. The sine waves move up 2 units and down 2 units.

b $y = \cos x + 2$ and $y = \cos x - 2$ are vertical translations. The cosine waves move up and down 2 units.

2 a $y = \sin(x + 90°)$ and $y = \sin(x - 90°)$ are horizontal translations to the left and right by 90°

b $y = \cos(x + 90°)$ and $y = \cos(x - 90°)$ are horizontal translations to the left and right by 90°

3 a $y = \sin x + 1$: a vertical translation up of 1 unit

b $y = \sin(x + 45°)$: a horizontal translation left of 45°

c $y = \cos(x - 45°)$: a horizontal translation right of 45°

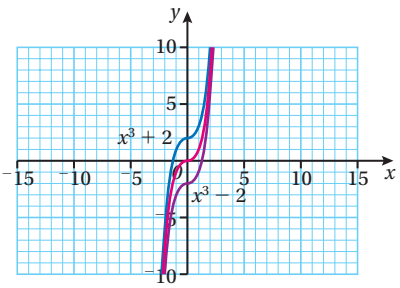
d $y = \cos x - 1$: a vertical translation down 1 unit

4 $y = -\sin x$ is a reflection of $y = \sin x$ in the x-axis

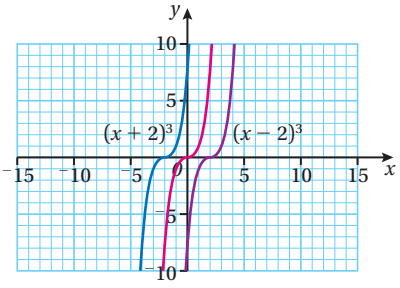
5 $y = -\cos x$ is a reflection of $y = \cos x$ in the x-axis

EXERCISE 41E

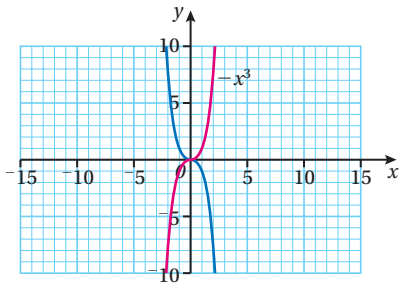
1 a



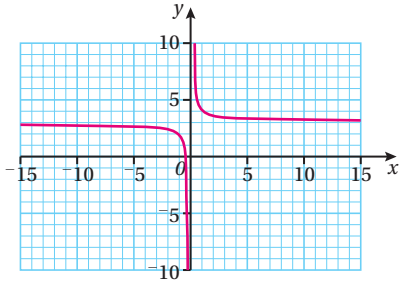
b



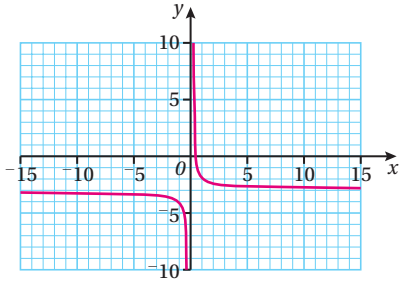
c Line of reflection is $y = 0$



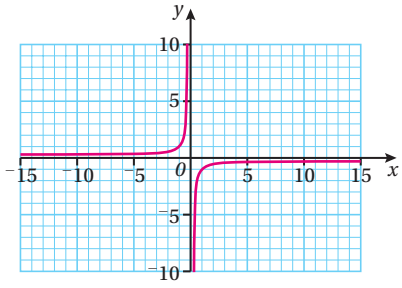
2 a



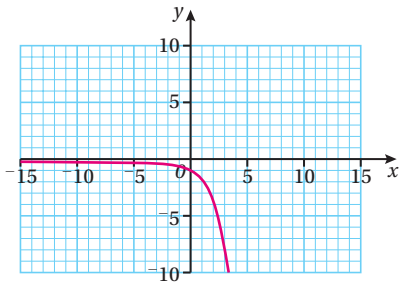
b



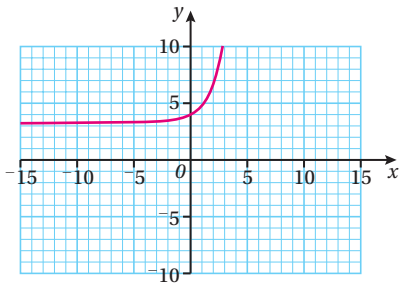
c



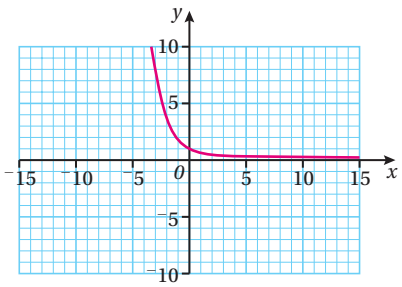
3 a



b

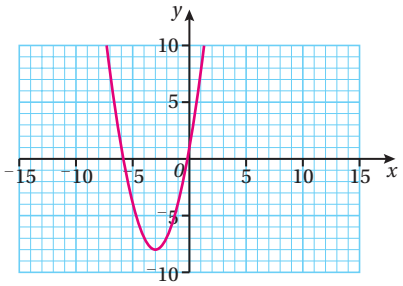


c



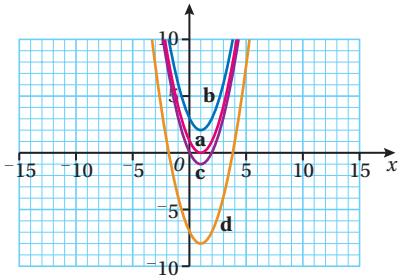
EXERCISE 41F

1

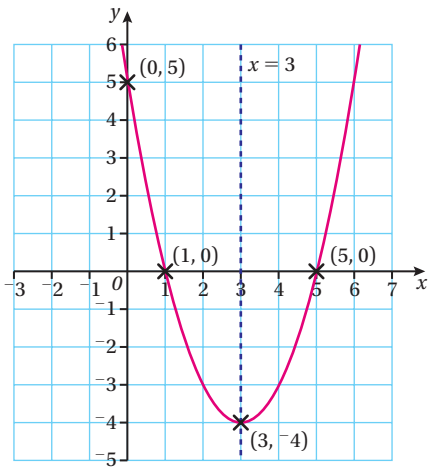


a $y = x^2$ **b** $y = (x + 5)^2 - 11$

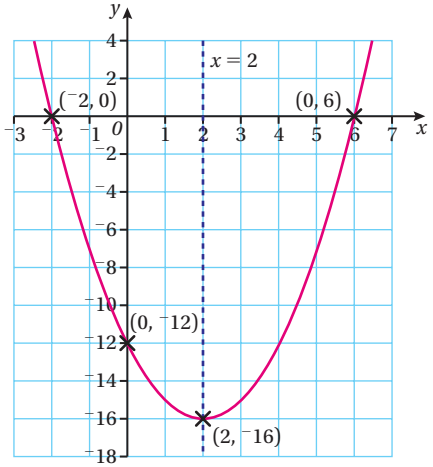
2 a $c = 0$ **b** $c = 2$ **c** $c = -1$ **d** $c = -8$



3 a

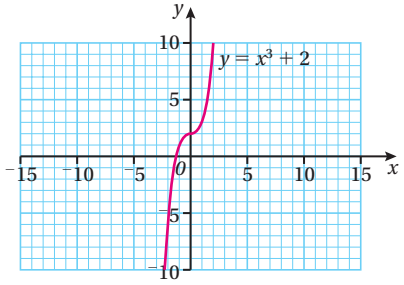


b

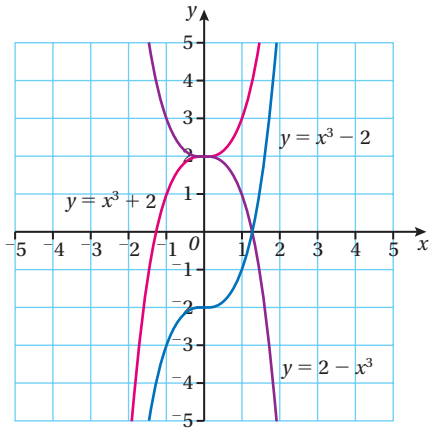


4 $y = \sin(x + 90°)$

5 a



b



6 $y = 2^{(x-3)}$

7 a A translation of 60° to the right and vertical translation down of $^-1$

$y = \cos(x - 60°) - 1$ $\quad (150°, {}^-1)$

$^-1 = \cos(150° - 60°) - 1 = 0 - 1 = {}^-1$

b A translation 1 unit to the right and 1 unit down

$y = x^2 - 2x$ $\quad (^-1, 3)$ $\quad 3 = (^-1)^2 - 2(^-1) = 3$

c Translation of 1 unit to the left

$y = \dfrac{1}{(x+1)}$ $\quad (^-2, {}^-1)$ $\quad {}^-1 = \dfrac{1}{(-2+1)} = \dfrac{1}{(-1)} = {}^-1$

$\left(1, \frac{1}{2}\right)$ $\quad \frac{1}{2} = \dfrac{1}{(1+1)} = \frac{1}{2}$

d A translation 1 unit left followed by a reflection in the line $y = 0$ followed by a translation 1 unit up

$y = \dfrac{-1}{(x+1)} + 1$ $\quad \left(1, \frac{1}{2}\right)$ $\quad \frac{1}{2} = \dfrac{-1}{(1+1)} + 1 = {}^-\frac{1}{2} + 1 = \frac{1}{2}$

e Vertical translation of 1 unit down

$y = 2^x - 1$ $\quad (1, 1)$ $\quad 1 = 2^1 - 1 = 2 - 1 = 1$

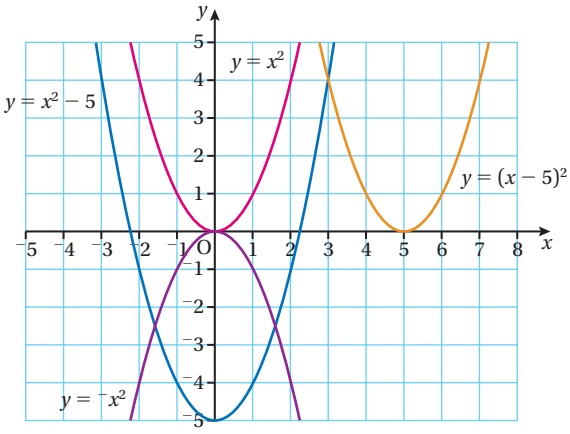
f Vertical translation of 2 units down

$y = \tan x - 2$ $\quad (0, {}^-2)$ $\quad {}^-2 = \tan 0 - 2 = 0 - 2 = {}^-2$

8 $y = \sin(x\ {}^-40°) - 2$

CHAPTER REVIEW

1 a

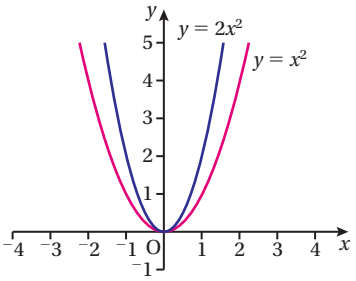


b Translation 5 units down

c Reflection in the x-axis ($y = 0$)

d Translation of 5 units to the right

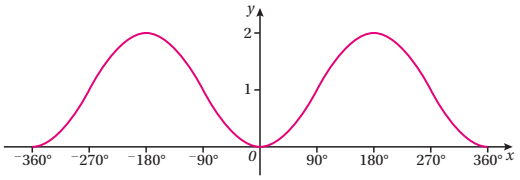
2 a



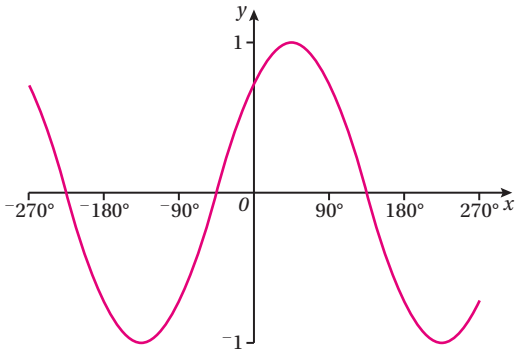
b Translation of 3 units down

3 a $y = \sin(x - 45°)$

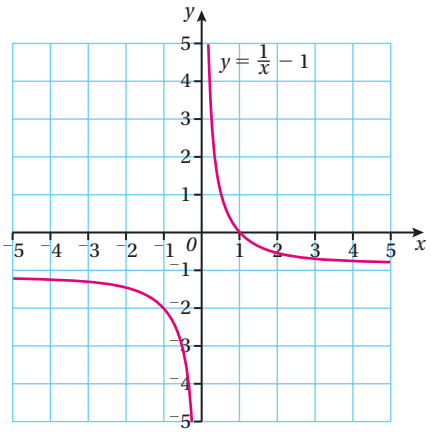
b i $y = \sin(x - 90°) + 1$



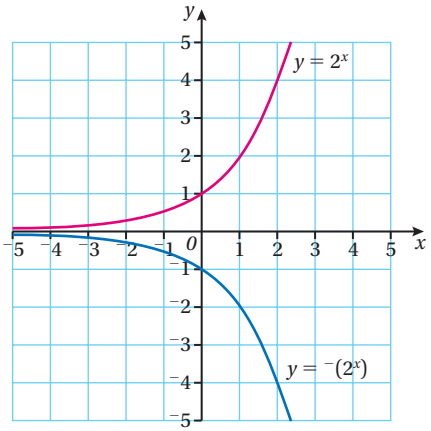
ii $y = \cos(x - 45°)$



4 a



b



5 It would move one unit to the left.