OSCILLATIONS OF ROTATING GAS DISKS : *p*-modes, *g*-modes, and *r*-modes

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Short-wavelength adiabatic oscillations of an infinitesimally thin, rotating and self-gravitating gas disk in dynamical equilibrium are studied. The problem is formulated in terms of the theory of oscillations of gas spheres (*i.e.* stars) in order to help galactic and stellar seismologists towards their mutual understanding of their rather similar subjects.

It is natural to expect that the oscillations of a rotating gas disk (*e.g.* a galactic gas disk) share a good deal of physical similarity with those of a rotating gas sphere (*i.e.* a star). Unfortunately, however, density-wave theoreticians and stellar seismologists have not always communicated easily, partly because they haven't shared a common physical language. Schutz and Verdaguer (1983) and Verdaguer (1983) were the first, to the author's knowledge, who tried to fill in this gap. They studied the oscillations of *isentropic* gas disks using the theory of oscillations of stars (*e.g.* Unno et al. 1979).

The present paper gives the outline of a more general discussion of adiabatic oscillations of *non-isentropic* gas disks. Full details will be found in Iye (1983).

The linearized equations of continuity, of motion (in two directions), and of adiabatic perturbation show that there are four modes in general. One can indeed derive a quartic dispersion relation for them (see Iye 1983). These modes can be classified, in the short-wavelength limit, into three types according to their main restoring forces for the perturbation, *i.e.* pressure force, effective gravity (or buoyancy) and Coriolis force. The self-gravity of perturbations makes the oscillations less stable, or even unstable, for certain perturbations of longer wavelengths. This destabilizing force is, however, not significant in the short-wavelength limit.

It is shown that non-isentropic disks have a pair of *pressure* modes ($^{\pm}p$-modes) for which the restoring force is two-dimensional pressure force and a pair of *gravity* modes ($^{\pm}g$-modes) which are restored by

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effective gravity. All of these waves drift with the local rotational flow. The sign of each pair denotes the direction of wave propagation with respect to the local rotating frame. In the short-wavelength limit, $|kr| \gg 1$, the frequency of oscillation ω measured in the corotating frame will be

$$\omega = \pm c\, k \qquad \text{for } {}^{\pm}p\text{-modes} \qquad (1)$$

and

$$\omega = \pm N\, k_t / k \qquad \text{for } {}^{\pm}g\text{-modes} \qquad (2)$$

respectively, where k and $k_t (=m/r)$ are the total and tangential wave number, c is the sound speed, and N is the Brunt-Väisälä frequency defined by $N = (-gA)^{-\frac{1}{2}}$ with g and A being the effective gravity and the Schwarzschild discriminant. The discriminant A for convective stability is defined by $A = d\ln \mu_o/dr - 1/\Gamma_1\, d\ln P_o/dr$, where μ_o is surface density, P_o is two-dimensional pressure of the disk and Γ_1 is the adiabatic exponent.

Since $N = 0$ for isentropic disks, they have no g-modes. In the limit of $A \to 0$, the frequencies of ${}^{\pm}g$-modes merge at $\omega = 0$ in the first order of $(kr)^{-1}$. However, it is found that in the next order of approximation $(kr)^{-2}$, the ${}^{+}g$-mode behaves as another type of oscillation called *Rossby* mode (r-mode) while the ${}^{-}g$-mode remains strictly neutral (ω=0). This r-mode is restored by Coriolis force. Its frequency is

$$\omega = \zeta\, F\, k_t\, / \,k^2 \qquad \text{for } r\text{-mode}, \qquad (3)$$

where ζ is the vorticity, F is defined by $F = d\ln(\zeta/\mu_o)/dr$. F is the inverse of the scale length of a quantity ζ/μ_o, which Lynden-Bell and Katz (1981) identified as an essential conserved quantity for isentropic disks. Isentropic disks, therefore, have a pair of pressure modes and a Rossby mode. One may regard the trivial neutral mode as the fourth mode.

All of these p-, g-, and r-modes are stable in the short-wavelength limit. For perturbations with longer wavelengths, the couplings among these modes become essential and the effect of self-gravity of perturbations may become important.

The standard technique widely used in the study of oscillations of individual stars is proved to be useful for studying the oscillations of gas disks.

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