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Corrigendum

TEST IDEALS IN RINGS WITH FINITELY GENERATED ANTI-CANONICAL ALGEBRAS – CORRIGENDUM

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In the article by Chiecchio *et al.* [1] there was an error. This error is explained and corrected in this notice, in particular, the statement of Lemma 2.10 is incorrect.

The corrected version of the statement, appearing as Lemma 2.10 here, is slightly weaker. Fortunately, all of the applications in the original paper of the original lemma only needed the weaker statement. We outline the setting and state and prove the corrected version of Lemma 2.10. We then give a summary of the effected results from the original paper and how they only depend on this weaker version.

Suppose that X is an F-finite normal integral Noetherian scheme and Γ is a $\mathbb{Q}$-divisor on X. One can form

$$S = \mathscr{R}(X, \Gamma) = \bigoplus_{i \in \mathbb{Z}_{\geqslant 0}} \mathcal{O}_X(\lfloor i\Gamma \rfloor).$$

Note that there is a canonical map $\mathcal{O}_X \to S$ and dually a map $\mathbf{Spec}\, S \xrightarrow{\kappa} X$ of schemes (note S may not be Noetherian). If S (or equivalently $Y = \mathbf{Spec}\, S$) is Noetherian, then we also have a canonical map $\mathbf{Proj}\, S \xrightarrow{\mu} X$.

Lemma 2.10. *Assuming that S is finitely generated, then $K_{\mathbf{Proj}\, S} \sim \mu^* K_X$. If additionally, Γ is a Weil divisor then, locally on the base, we have that $K_{\mathbf{Spec}\, S} \sim \kappa^* K_X + \kappa^*\Gamma$. In particular, if $\Gamma = -K_X - B$, then $K_{\mathbf{Spec}\, S} \sim \kappa^*(-B)$ so that $K_{\mathbf{Spec}\, S} + \kappa^* B \sim 0$. Thus if $\Gamma = -K_X$ then S is quasi-Gorenstein.*

Proof. Recall that κ is an $\mathbb{A}^1$-bundle outside a set of codimension 2 and hence κ^* makes sense. The computation of $K_{\mathbf{Spec}\, S}$ can be found in [2, Theorem 4.5]. The initial statement that $K_{\mathbf{Proj}\, S} \sim \mu^* K_X$ is obvious since μ is small. □

The error in the original lemma was in the formula for $K_{\mathbf{Spec}\, S}$. We utilized the incorrect formulation in exactly two spots in the original article. First, in Corollary 3.4 we assert that $K_S + h^* B$ is Cartier. However, $K_S \sim h^* K_R + h^*(-K_R - B)$ and so $K_S \sim h^*(-B)$ and hence $K_S + h^* B \sim 0$ and it is still Cartier. Second, we use Lemma 2.10 in Theorem 3.5. Here we need that $K_S \sim h^* K_R + h^*(-K_R - \Delta)$, which gives the utilized statement

$K_S + h^*\Delta \sim 0$. Thus, no corrections are needed to the statements of Corollary 3.4 and Theorem 3.5 in the original paper.

Reference

1. A. Chiecchio, F. Enescu, L. E. Miller and K. Schwede, Test ideals in rings with finitely generated anti-canonical algebras, *J. Inst. Math. Jessieu* First published online 13 January 2016, doi:10.1017/S1474748015000456.
2. S. Goto, M. Herrmann, K. Nishida and O. Villamayor, On the structure of Noetherian symbolic Rees algebras, *Manuscripta Math.* **67**(2) (1990), 197–225. MR 1042238 (91a:13006).