Electromagnetic reflecting properties of sub-ice surfaces

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ABSTRACT. The power strength of the radio-echo signal coming from reflecting sub-ice surfaces is used to determine the nature of the reflecting surface, i.e. rock, water or sea water. Electromagnetic analysis shows that the amplitude variations detected by radio-echo sounding are mainly due to the nature of the interface as well as the concave or convex shape of the reflectors. In this paper, some relevant profiles showing the power variations due to the different nature of the interface and the shape of the reflectors are presented. These results are important both for surface shape determination and for subglacial lake detection.

INTRODUCTION

Reflection of radio waves occurs at the interface of two media, and the reflection coefficient, R, provides information on their electromagnetic nature. If the first medium is known (e.g. air (or ice, which in a broad interval of physical conditions maintains certain propagative characteristics nearly constant)), the electromagnetic properties of the second medium can be determined (Tabacco and others, 2002). Ice/water and ice/rock interfaces exhibit different reflection coefficients (Bogorodsky and others, 1985; Bentley and others, 1998). As a consequence of the low gain of the antenna employed by radio-echo sounding (RES) systems, the transmitted radio-wave beam illuminates a relatively large area, and the power of the echo signal greatly depends on the shape of the reflecting surfaces (Tabacco and others, 2000).

In this paper we analyze effects detected in the power variations of the RES signal produced by basal surfaces. When the power of a radio wave reflected along the profile from the sub-ice reflectors is studied, it is usually found to fade and recover according to the nature and the shape of the basal reflector. The power variations of the echo returning from sub-ice reflectors can be due to many properties including the electromagnetic properties of the surface and the shape of the reflector. In a well-characterized RES system under certain constant radio propagative conditions it is possible to evaluate the power variation with sufficient accuracy to study the basal reflectors. In particular, in this paper two types of phenomena are studied: the electromagnetic properties of the ice/water and ice/rock interfaces in the presence of nearly flat reflectors, and the shape of the surface in profiles where a continuous known interface is present.

We evaluate the radio propagative factors which may cause an amplitude variation, observed in the A-scope monitor. One of the greatest difficulties in RES data analysis is to quantify the amplitude signal degradation due to the surface roughness. In fact, the air–ice and basal surface roughness can vary sharply along a profile and it is difficult to distinguish the real signal degradation. In the examined profiles the basal surface is assumed to be smooth, and for the air–ice interface a constant averaged value is distinguished. For the radio-wave scattering, caused by volume inhomogeneity especially in the upper part of the glacier, only a constant average value is given. All the contributions due to other constant losses, such as cable loss, polarization loss and internal and external radio noise, are included in a constant value, C.

ELECTROMAGNETIC ANALYSIS

Starting from the well-known radar equation and expressing all the terms in dB, the received power P_r in an RES system is

$$P_r = P_t - L_g + G_f - L_p , \tag{1}$$

where P_t is the transmitted power, L_g is geometrical loss, G_f a possible source of gain or loss due to the focusing or defocusing effect of the reflector shape (later this effect will be evaluated) and L_p includes all the remaining losses. In aircraft-employed RES systems, the transmitting and receiving antennas are practically equivalent and have the same gain G_a, so the geometrical loss can be written as

$$L_g = 20 \log \left(\frac{8\pi(r_a + r_i/n)}{G_a \lambda} \right) , \tag{2}$$

where n is the index of refraction, λ is the wavelength, r_a is the aircraft height and r_i is the ice thickness. The value r_i can be calculated assuming a propagation velocity of 168 m μs^{-1} (Glen and Paren, 1975). The term L_p can be expressed as the sum of many contributions:

$$L_p = L_a + L_{air-ice} + L_{basal} + C , \tag{3}$$

where the suffix a indicates the loss due to the absorption, air–ice indicates the loss (around 0.36 dB) due to the ice surface, basal indicates the transmission loss at the basal reflector while in the constant C there are the loss contributions from surface roughness, volume scattering, cables, depolarization and internal and external radio noise. Loss contributions due to the roughness of the surfaces and volume scattering can fluctuate widely, especially in the upper layers of the glacier. Depending on the particular profile, an average value around 1–4 dB is chosen. Cable loss, depolarization loss and noise can degrade the signal by <1 dB. The average estimated value for C, obtained after processing the raw data, is 1.5–5 dB.

From the arrival time we can evaluate r_i and, if we assume the conductivity σ of the medium depending on the temperature of the glacier, the absorption L_a of the radio wave can be evaluated using the relation

$$L_a = k\sigma r_i , \tag{4}$$

where k is a constant equal to 913 and σ the conductivity.

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Table 1. Electromagnetic properties of the considered media

Medium	Real part of relative permittivity ε'	Conductivity σ S m^{-1}
Air	1.0	0
Ice (–20°C)	3.18	$\approx 1.6\times10^{-5}$
Ice (–50°C)	3.18	$\approx 1\times10^{-6}$
Sea water	84.4	≈ 3
Water	81	$\approx 10^{-3}$
Bedrock	10–11	$\approx 10^{-7}$–10^{-3}
Sea ice	3.44	$\approx 10^{-2}$–0

Table 2. Power reflection coefficient R^2 and approximate power lost at the indicated interfaces

Interface	Reflected power ratio R^2	Loss dB
Air/ice	0.08	–11
Air/sea water	0.9	–0.4
Ice/sea water	0.83	–0.8
Ice/water	0.446	–3.5
Ice/rock	0.083	–11
Ice/sea ice	4.10^{-4}	–34

Above a lake, on the hypothesis of thermal steady state and neglecting horizontal advection, the temperature profile can be estimated with sufficient accuracy by exploiting a thermal flux model of the glacier (Robin, 1955; Siegert, 2000). If the chemical composition of ice is assumed to be constant in the whole surveyed area, the electromagnetic properties can be calculated along the entire profile. For the Antarctic Plateau using an average value for the conductivity, the absorption value $L_a \approx 8$ dB km^{-1} was adopted. For a floating ice tongue, an approximate value of 1.6×10^{-5} S m^{-1} for σ was chosen considering the temperature of the glacier close to –20°C. Though the estimation for a floating ice tongue of the absorption ($L_a \approx 15$ dB km^{-1}) is affected by an error due to the uncertainty of the values of conductivity, temperature and radio-wave path, the variation of the received power is not strictly dependent on the exact value of this absorption. Others (Shabtaie and others, 1987; Bentley and others, 1998) have given similar values for L_a. Once the above quantities have been calculated or estimated, the analysis can provide the contributions related to L_{basal} and G_f. In the next two sections the contributions of L_{basal} and G_f are analyzed separately for the following two cases: (i) a flat profile (G_f approximately constant) in which the nature of the interfaces is considered, and (ii) a continuous known ice–water interface (L_{basal} approximately constant) in which the surface shape contributions are examined.

A FLAT (SUBGLACIAL LAKE) PROFILE

About 80 subglacial lakes have been identified in East Antarctica (Siegert and others, 1996). These were identified by means of RES data analysis utilizing three main characteristics; (i) smooth basal surface, (ii) flat basal surface (mirror-like interface) and (iii) strong amplitude signal (Gorman and others 1999). The value of the reflection coefficient R at an ice/water interface with respect to an ice/rock interface changes the power of the reflected echo by several dB. The reflection coefficient is given by

$$R = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}}, \tag{5}$$

where ε_{r1} and ε_{r2} are the complex relative permittivities of the two media, water or rock, and R is the percentage of the amplitude of the echo signal. RES techniques are based on the analysis of the power of the signal, and $10\log_{10}(R^2)$ is the quantity that takes into account the percentage of reflected power in dB. Hence, the strength of the echo signal depends on the particular interface involved in the wave-propagation path. The complex relative permittivity can be written as, $\varepsilon_r = \varepsilon' - i\varepsilon''$, where ε' is the real part, $\varepsilon'' = \sigma/\varepsilon_0\omega$ is the imaginary part, σ (S m^{-1}) is the conductivity, i is the imaginary unit, ε_0 (F m^{-1}) is the vacuum dielectric permittivity and ω (rad s^{-1}) is the angular pulsation of the radio wave. A knowledge of the real part of the relative permittivity ε' and conductivity σ is sufficient to calculate R. Table 1 reports the above-mentioned quantities at the operating frequency of the RES system and for the indicated media. Table 2 lists the power reflection coefficient R^2 for the described interfaces, indicating the percentage of power reflected by the interface as well as the loss in dB.

In the ideal case of smooth, flat reflecting surfaces and neglecting all the other radio propagative parameters (assumed constant), for ice/rock, ice/water and ice/sea-water interfaces we would expect the power losses indicated in Table 2. In a variety of analyzed profiles when it is possible to exclude other causes, the differences in the value of the return intensity allow us to distinguish interfaces, especially ice/rock and ice/water interfaces. In particular, due to the different reflecting properties in the case of ice/rock and ice/water interfaces, in continental glaciers >7–8 dB variation in power is expected. Of course some intermediate cases such as ice/wet-ice and ice/wet-rock are possible, but other morphological interpretations, such as the extension of the supposed lake-surface reflector in comparison with whole profile contours and the internal layering of the glacier, can further sustain this interpretation. Ice/rock and ice/water interfaces introduce losses of about –11 and –3.5 dB, as can be seen in Table 2. For example, Figure 1a shows a profile of a suspected subglacial lake, and its electromagnetic analysis is reported in Figure 1b. The flat segment of the reflector shows a power variation in the return signal of about 6.5 dB with respect to the adjacent profile. This fact confirms that the flat reflector is a subglacial lake or a wet, flat surface.

A CURVED INTERFACE

Assuming that ice/rock or ice/water interfaces are always smoothed surfaces of the same nature, we can analyze the second case. When the investigated bottom surfaces are concave or convex, the radio signal is respectively focused or defocused, producing gain or loss in the received power of the signal (Davies, 1990). Depending on the shape of the surface and other geometrical conditions, the

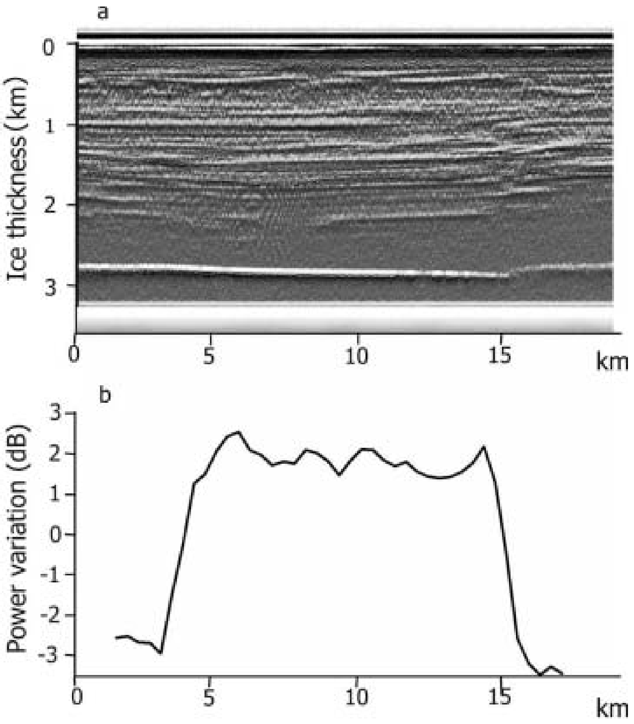

Fig. 1. (a) The profile of a subglacial lake and (b) the variation in the return intensity (dB). The power of the return signal from the subglacial lake (bottom flat reflector) is about 6.5 dB greater than that of the ice/rock interface.

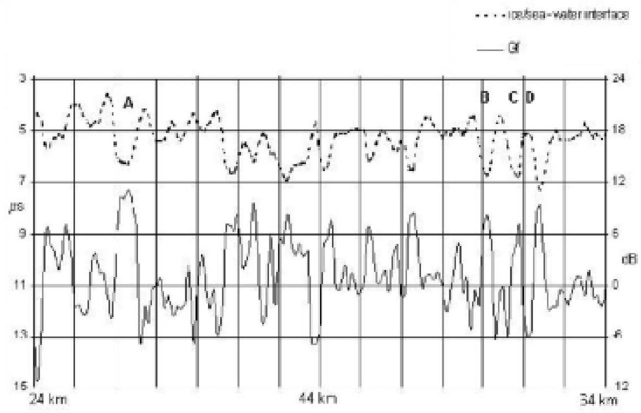

Fig. 2. Smooth bottom surface profile (distance vs ice thickness) and relative amplitude variation (dB). The quadratic regression shows that the circular arc approximates concave reflectors when these coincide with the greatest recorded signal amplitude (A–C). It approximates convex reflectors when these coincide with the lowest level of the signal (D). The radii of curvature are shown in Table 3.

power variation can be several dB (Tabacco and others, 2000). Variations of about –6.6 to +9.0 dB were recorded along a profile of the Drygalski Ice Tongue and are reported in the last column in Table 3. To study and quantify G_f a rippled floating ice tongue was chosen. In this case a constant ice/sea-water interface is considered and a power loss of 0.8 dB due to a reflection coefficient of 0.830 is found (Table 2). A segment of the profile is shown in the upper trace of Figure 2, while the lower trace in the same figure shows power variations due to concave and convex reflecting surfaces. Numerical modelling was used to create two-dimensional (2-D) and three-dimensional (3-D) representations of the bottom ice surface. These models can explain the variations in amplitude observed during measurements. The bottom reflecting surfaces were divided into different segments corresponding to concave and convex faces, and individual reflectors were studied. The best-fit arc and its radius of curvature were thus identified. If we have similar surface shape with different constant interface (i.e. ice/rock) we expect the same power variation depending only on the shape of the basal reflector.

For a convex and concave spherical cap, adopting 2-D and 3-D models and exploiting a simple law of the optical geometry, the reflector gain can be determined. Referring to Figure 3, the RES system is placed at a distance r from the concave-up reflector, and the antenna beam illuminates a relatively large area (say L). The power of the echo signal is significantly determined by the shape of the reflecting surfaces. In fact, if the reflector is approximately an arc (2-D model) with a radius of curvature ρ_1 and a point-image is at a distance q from the arc, the receiving antenna, linear dimension l, captures power proportional to $1/l$ (Whitehead, 1956). From Figure 3, we can write directly from the optical geometry,

$$(1/q) + (1/r) = 2/\rho_1,$$

and

$$1/l = q/L(q - r).$$

Eliminating q we obtain

$$1/l = 1/(2L) \cdot [1/(1 - r/\rho_1)].$$

Apart from the first factor, the amplitude is proportional to the second term. This focusing effect produces a gain G_f expressed by the formula

$$G_f = +\frac{1}{1 - \frac{r}{\rho_1}}. \tag{6}$$

In the case of a convex-up arc reflector, there is a power loss

Table 3. The selected reflectors in Figure 2 and the respective G_f due to shape

Reflector as in Fig. 2	Range r	Radius of curvature ρ_1 or ρ_0	G_f (2-D model)	G_f (3-D model)	Power variation
	m	m		dB	dB
A	650	1190	+3.39	+ 6.86	+ 8.79
B	732	1200	+ 4.01	+ 8.17	+ 6.67
C	720	1300	+ 3.5	+ 7.01	+ 5.47
D	570	1000	–4.05	–8.09	–6.61

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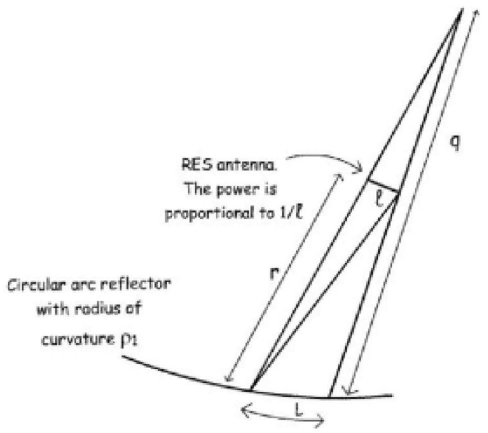

Fig. 3. The optical geometry considered for a 2-D model in the case of an arc.

of the same amount. If spherical reflectors are considered (3-D model), the gain is

$$G_f = \pm \frac{1}{(1 - \frac{r}{\rho_0})^2}, \tag{7}$$

where ρ_0 is the radius of curvature of the spherical reflector. Four reflectors with their geometrical characteristics and the relative gain/loss are reported in Table 3.

CONCLUSIONS

During radio-echo surveys, the observed power variations of the signal may be affected by the kind of reflector interface and by focusing or defocusing effects due to the shape of sub-ice reflectors. We have shown some examples of how these effects can be estimated. By means of a simple electromagnetic analysis, it is possible to distinguish the interfaces involved in the reflection phenomenon. In the case of continental glaciers, two possible interfaces (ice/water and ice/rock) are studied. Considering only the power contribution from the bottom interface, the power of the return signal varies by 7–10 dB.

An RES longitudinal profile over the Drygalski Ice Tongue was used to develop an electromagnetic analysis that has shown a correspondence between the shape of the bottom reflecting surface and the power variation of the received signal. Power variations along the examined profile are, in general, compatible with the two proposed models based on the optical reflection laws. The power variations are well explained by a circular arc, when we use a 2-D model, or by spherical reflectors for the 3-D model that presents concave or convex faces towards the sounding apparatus.

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