A Further Note on the Validity of Weinberg's Differential Rule

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Let p be the probability that a dizygotic (DZ) twin birth is male. Then if p were equal and independent in all pairs of DZ twins, the frequencies of pairs with 2, 1 and 0 males would be in proportion to the binomial terms $p^2 : 2pq : q^2$, where $q = 1 - p$. And if p is close to the value 0.5, the expected numbers of same-sexed and opposite-sexed pairs of DZ twins will be approximately equal. This is the reasoning behind Weinberg's Differential Rule (Weinberg, 1901). If the rule were valid, it may be used to estimate the proportion of DZ and monozygotic pairs in a sample of twin pairs in respect of which the only available genetic information is their sex combinations.

However, across the intervening years, evidence has accumulated to suggest that (in singletons at any rate), p shows appreciable Poisson variation within, and Lexis variation across, sibships. Since this would nullify one of Weinberg's premises, a problem is posed by the finding that, to a close approximation, Weinberg's Rule is nevertheless valid (Fellman & Eriksson, 2007). I proposed a solution by noting evidence that in singleton births, the measures of Lexis variation and Poisson variation are roughly equal. If, in DZ twins too, p were similarly subject to roughly equal measures of Lexis and Poisson variation, then the distribution of DZ twin pairs containing 0, 1 and 2 males would be roughly binomial (James, 2007). Thus, the problem of the validity of Weinberg's Rule would be potentially solved. In the present note, I note that for practical purposes however, the point may be irrelevant because the Poisson and Lexis variation are probably too small to have much effect anyway.

The variance, V, of the binomial is npq. In the Lexis binomial, this value is augmented by $n(n-1)\sigma_b^2$, while in the Poisson binomial, this value is diminished by $n\sigma_w^2$, where n is the sample size (2 in the case of DZ twins), σ_b^2 is the Lexis variance of p between samples, and σ_w^2 is the Poisson variance of p within samples (e.g., Weatherburn, 1949, p. 115).

So the variance of a distribution containing both Lexis and Poisson processes, is V where

$$V = npq + n(n-1)\sigma_b^2 - n\sigma_w^2.$$

In my previous note (James, 2007), I cited three different studies all based on different estimation procedures and all concluding that (in singleton births) the magnitude of σ_b^2 is of the order of 0.0025 (Edwards, 1958; James, 1975; Pickles et al., 1982). It is not entirely clear whether this agreement is due to a concordance in direction and magnitude of bias, or whether it is a reasonable approximation to the truth. Let us provisionally choose the latter alternative. In the case of twin pairs, $n = 2$. So it will be seen from the above equation that V = npq, or approximately 0.5. It may provisionally be assumed that the two sigmas are very approximately equal (James, 2007). So, in the equation above, each of the terms containing a sigma takes a value of the order of 0.005. In other words, each of these terms takes a value that is of the order of one hundredth of the value of npq. So, unless

1. the three estimates of σ_b^2 are all seriously biased to roughly the same extent and in the same direction, and
2. the two sigmas are not roughly equal,

it would follow that any plausible Poisson and/or Lexis variation do not have the power appreciably to affect the accuracy of estimates based on Weinberg's Differential Rule.

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