Macroeconomic Dynamics (2024), 1–22
doi:10.1017/S1365100524000099

ARTICLE

Bankruptcy law as an alternative to fiscal policy in a Woodford model with a productivity shock

Aloísio Araújo[1,2] and Vitor Costa[2]

[1]FGV EPGE, Rio de Janeiro, RJ, Brazil
[2]IMPA, Rio de Janeiro, RJ, Brazil
Corresponding author: Vitor C. Costa; Email: vitor.chagas@impa.br

Abstract

We show that trade credit contracts between sectors can provide a useful alternative to fiscal transfers during a major productivity shock. Defaults in credit contracts function as transfers between sectors, which can be implemented through a bankruptcy law or through credit renegotiation. Transfers implemented through defaults allow for a reduction in the size of the fiscal policy that restores the economy to the optimal allocation, constituting a relevant alternative to economies without an available fiscal space to implement the optimal policy.

Keywords: Fiscal policy; Network structure; Bankruptcy law

1. Introduction

The massive drop in aggregate demand and production caused by the COVID-19 pandemic has presented challenges to macroeconomists and policymakers. The standard response to aggregate demand shocks is a reduction in interest rates (when possible) and, most recently, unconventional policies such as quantitative easing and negative interest rates. Those policies have been implemented as a response to the global financial crisis and much of the debate regarding their use has been centered around different ways for policymakers to increase monetary stimulus in a zero lower bound situation.

However, many argue that a proper response to a major shock necessarily demands a fiscal response. Woodford (2022) makes the case that in a multisector model, fiscal policy is the more efficient instrument to deal with large drops in aggregate demand caused by a disruption in the flow of payments and that monetary policy is the wrong tool to address this issue. Monetary stimulus fails to increase spending by those sectors directly hit by the shock and may cause agents in sectors currently on the optimal expenditure level to increase their spending inefficiently. Fiscal instruments, on the other hand, will stimulate the economy in the correct direction, even if properly targeted policies are not implemented.

In this context, the first-best policy alternative is to transfer funds to those sectors most affected from agents that would otherwise spend on goods that are unavailable due to the restriction on supply. This strategy may require too much information by the government to be implemented since it requires targeting both transfers and taxes at the correct sectors. A simpler policy would be to raise indebtedness and make a "helicopter" transfer of funds to all agents. This policy prevents the disruption of the flow of payments, allowing affected sectors to purchase goods, but may cause distortions in the choices made by individuals for future consumption and savings, away from the first-best equilibrium.

In addition, while advanced economies may possess necessary fiscal space to implement any of these policies, the situation is different for emerging economies that lack proper fiscal credibility and may be in a situation characterized by fiscal fragility. More recently, those economies have experienced episodes of higher inflation and there is less confidence in the willingness of policymakers to guarantee a path for public debt that stabilizes the inflation level. Thus, for emerging economies, there is a limit to the increase in indebtedness that can be carried to finance a fiscal stimulus, while raising taxes may be difficult in an adverse scenario.

We argue that private credit instruments may provide an alternative method to implement part of the desired transfer policies that bring the economy to an efficient allocation. This is made possible by the existence of trade credit agreements between agents, in which buyers can purchase goods from other sectors paying the supplier in a later moment. When buyers are hit by a productivity shock that makes agreements unfeasible, they may wish to default on the contract. In this paper, we show that a bankruptcy law or a renegotiation procedure that pardons a fraction of the debt of agents affected by a shock implements part of the transfers that would otherwise be carried by the government, alleviating the fiscal cost of the optimal policy.

We show this result through an extension of the model presented in Woodford (2022) by including private trade credit contracts between sectors, and prove that, when a negative shock affects the economy, a trade credit default policy is equivalent to fiscal transfers from sectors that are less affected to those sectors most affected. This mechanism allows for a reduction in the size of the fiscal policy that restores the economy to the optimal allocation, constituting a relevant alternative to economies without an available fiscal space to implement the optimal policy. The size of the reduction of fiscal policy will depend on the relevance of trade credit agreements between different sectors and on the network structure of the economy.

We also show that when credit agreements specify penalties to agents that default in the form of exclusion from credit markets, a bankruptcy law that lifts penalties temporarily will be welfare improving, not only to the affected sectors but also to their creditors. In this context, in the absence of an exogenous law that lifts penalties for defaults, agents themselves would desire to renegotiate credit agreements in order to avoid the worst scenario of autarky. We propose a mechanism in which agents are allowed to postpone payments of the debt, compensating creditors while not being punished with exclusion from credit markets.

This specification of the model provides a rationale to some temporary changes in bankruptcy law that were implemented in several countries as a response to the COVID-19 crisis. Among the more commonly adopted measures, there were temporary suspension or restrictions on creditors starting insolvency procedures or company directors' duty to file for insolvency, moratoria, or restrictions on debt enforcement, specially directed to small and medium enterprises.[1] Our model suggests that there could have been a greater emphasis in adopting emergency measures aimed at increasing access to bankruptcy procedures and providing debt relief during such a crisis. In the United States, for example, support to businesses was implemented by the *CARES* Act, consisting mostly of direct fiscal support measures and extension of forgivable credit to businesses that maintained employment. Wang et al. (2020) observe that there was a reduction on bankruptcy fillings during the pandemic, which could have been motivated by "financial, physical and technological barriers to accessing the bankruptcy system." Increasing access to the bankruptcy system could have alleviated the need for direct fiscal support, specially for small businesses.

The importance of a bankruptcy law as a policy tool in the COVID-19 crisis was recently shown in Brazil. Initially, a legislative project proposed to create an Insolvency Prevention System that suspended foreclosure measures and encouraged debtors and creditors to engage in a renegotiation of their obligations. While the project ended up not being sanctioned by both legislative houses, a different bankruptcy law reform was approved in the end of 2020 and greatly modernized the institutional framework of the bankruptcy procedure. In response to the pandemic, the Brazilian government chose to support businesses through emergency credit lines, by greatly reducing the basic interest rate and by a tax deferral policy, which had the effect of delaying the impact of the pandemic on several businesses. The access to bankruptcy procedures greatly increased in Brazil after the pandemic as a result of the combination of an easier access to the procedure by the new law with the end of the support policies that were implemented during the pandemic.[2]

This paper relates to the literature that studies economic policy responses to the COVID-19 pandemic, as Woodford (2022), Milne (2020), Bigio et al. (2020), and Guerrieri et al. (2020). We emphasize the importance of bankruptcy and insolvency laws as an additional tool that can help stabilize the economy during a negative shock. This paper is related to the literature on equilibrium with incomplete financial markets, such as Araujo and Páscoa (2002) and Kehoe and Levine (2001), as we consider credit structures that allow for bankruptcy and defaults. We also expand on the literature that focus on systemic risk and contagion on financial markets, such as Eisenberg and Noe (2001) and Acemoglu et al. (2015), by considering a shock to a network of interconnected creditors and debtors, in which a disruption of the flow of payments can cause a sequence of defaults. We emphasize a trade credit structure in which defaults can be optimal by allowing affected sectors to consume when facing a negative shock and endogenize the default decision by considering an exclusion mechanism.

The paper is organized as follows: in Section 2, we characterize the general setup of the model, show the first-best allocation that can be attained in each situation, and characterize the decentralized economy environment and the trade credit structure of the economy. We then establish an equivalence between credit defaults and fiscal transfers and present the optimal policy that implements the first-best allocation in each scenario. We assume a simple credit structure that consists of a constant fraction of credit for each agent. In this formulation, the default parameters are set exogenously, and we show some basic properties of the model. In Section 3, we consider the case where agents determine the default choice endogenously, but are subject to a punishment. In this more realistic case, each sector has suppliers and retailers, and its agents are punished through autarky when they choose to default. We then show that in this case, when there is an unforeseeable shock, it is optimal for creditors and debtors to temporarily lift the punishment for defaulters, so that market relations can be preserved in the following period. We present two policies that implement this solution: a bankruptcy law that is implemented by an authority and a credit renegotiation agreement that postpone payments that were due in the period of the shock. We compare the welfare consequences of both policies.

2. Basic model

In this section, we develop the basic structure of the model, based on the N-sector model of Woodford (2022), and derive the first-best optimal allocation of the economy. Then, we extend the original model by allowing agents to agree each period to private credit contracts that are signed during the previous period. As a response to the shock, some contracts may become unfeasible and a default rule will be implemented, while the government may also make fiscal transfers and monetary expansions to stabilize the economy. We define the equilibrium allocation and establish an equivalence between credit defaults and fiscal transfers in this context.

2.1. General setup

The economy is populated by N "yeoman farmer" sectors, each containing infinite consumer-producer units. Each unit in any sector produces a single type of good but may consume goods from different sectors. Indices are assumed to respect a modulo-N arithmetic in the space of integers, in the sense that sector 1 is identified with sector $N+1$, sector $N-1$ is identified with sector -1 and so on. Utility of an agent of sector $j = 1, \ldots, N$ is given by the function

$$\sum_{t=0}^{\infty} \beta^j U^j(t)$$

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

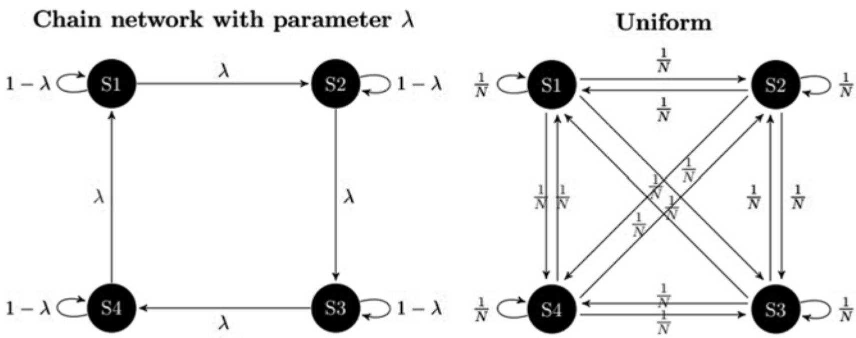

Figure 1. Chain and uniform network.

where $0 < \beta < 1$ is the intertemporal discount factor and

$$U^j(t) = \sum_{k \in K^j(t)} \theta_k(t)\alpha_{k-j} u\left(\frac{c_k^j(t)}{\theta_k(t)\alpha_{k-j}}\right) - v(y^j(t))$$

The function u is continuously differentiable and satisfies the usual Inada conditions: $u'(0) = \infty$, $u'(\infty) = 0$. u is also strictly increasing and strictly concave, such that $u(0) = 0$. The function v is convex, strictly increasing and continuously differentiable, and such that $v(0) = 0$.

$c_k^j(t)$ is the consumption of an agent in sector j of goods produced by sector k in period t. $y^j(t)$ is the production of an agent in sector j at period t. In this extension we assume that from period $t \geq 1$ production is always fixed at the level $\bar{y}$ such that $u'(\bar{y}) = v'(\bar{y})$.[3] However, in period $t = 0$, the economy is subject to a major productivity shock described by a vector $\theta = (\theta_1(0), \ldots, \theta_N(0))$ and production will be determined by demand due to a rigid price assumption that will be explained later on. For periods $t \geq 1$ we assume $\theta_k(t) = 1$ for all sectors k, and we refer to this scenario as the "normal" case.

The coefficients α_k define the symmetric network structure of the economy and are assumed to satisfy $\sum_k \alpha_k = 1$. $K^j(t)$ represents the subset of sectors $1, \ldots, N$, such that $\alpha_k > 0$ for agent j in period t. In a decentralized economy in the normal case when prices are equal for goods produced in different sectors, the coefficients α_{k-j} represent the portion of consumption that an agent in sector j will dedicate to a good produced by sector k. We make a symmetric specification of the model, which implies that this portion will be the same for all sectors $j = 1, \ldots, N$.[4] Two examples of network structures are given in Fig. 1.

Assuming equal prices for all goods, in the chain network each agent in sector j demands goods only from her own sector and sector $j+1$, with a fraction of λ being demanded from the adjacent sector, so that $\alpha_0 = 1 - \lambda$ and $\alpha_1 = \lambda$. In the uniform network, each agent demands the same fraction $1/N$ of consumption from each sector, with $\alpha_j = 1/N$ for every $j = 1, \ldots, N$.

We focus on the effect of a productivity shock that shuts down the production by a single sector j of the economy in period $t = 0$, so that $\theta_j(0) = 0$, due, for example, to health concerns about the consumption of the good produced by this sector, or to a restriction in the production or trade of goods such as a major conflict scenario. We assume an ex ante equal probability for all of the sectors to be hit by the shock. While writing contracts and making consumption decisions, agents cannot condition any decision on the occurrence of the shock. For periods $t \geq 1$, the economy returns to the stationary state so that the shock is completely transitory.

In this model, it is straightforward to characterize the first-best optimal allocation that would be chosen by a social planner. Since ex-ante all sectors are symmetric and subject to the same shock with equal probability, agents agree with the ranking of rotationally symmetric allocations, which are those that depend only on the occurrence of the shock θ, so that the allocation of sector

k when sector j is hit by the negative productivity shock is the same as the allocation of sector $k+1$ when sector $j+1$ is hit by the negative productivity shock instead.

An altruistic social planner maximized the social welfare function

$$\sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{N} U^j(t),$$

which is proportional to the ex-ante expected utility of each agent,[5] subject to the market clearing conditions in the goods market

$$\sum_{k \in K^j(t)} c_j^{j-k}(t) = y^j(t)$$

for all $t \geq 0$ and j. The resulting optimal allocation is such that sectors that are not hit by the negative shock will produce the optimal quantity $\bar{y}$ that satisfies $u'(\bar{y}) = v'(\bar{y})$, and consumption will be distributed such that $c_k^j(0) = \alpha_{k-j}\bar{y}$ if $\theta_k(0) = 1$ and $c_k^j(0) = 0$ if k in the sector that is shutdown, that is, $\theta_k(0) = 0$. In the subsequent periods, all production is restored and all sectors demand the same amount, with $c_k^j(t) = \alpha_{k-j}\bar{y}$ for all k.

For the rest of the paper, we evaluate the welfare consequence of different policies based on whether or not they are able to implement this first-best allocation of resources as an equilibrium. As it will be seen, while there is a fiscal policy that can implement this allocation and restore the first-best outcome, we are interested in whether or not there is a policy framework that can use a bankruptcy law as an alternative to fiscal policy since in practice implementing a complex fiscal policy is unfeasible for a government, specially if it has a restriction on the ability to raise debt or taxes, as it is in most emergent market economies.

In the decentralized economy, agents trade on a competitive market for goods and can buy government bonds that pay an interest rate in the following period that is set by the government. In each period t, $p_k(t)$ is the nominal price of the good produced by sector k. Prices are fixed in the previous period at a level that is assumed to make markets clear. This means that prices do not respond to a shock that is unanticipated, and production is demand determined in this situation. For period $t = 0$, prices will be determined by an initial policy trajectory that may change after the shock is learned by the agents.

Each agent begins period $t = 0$ with an initial nominal wealth of $a^j(0)$ that is the result of fiscal policy. We denote by $a(t) = \sum_j a^j(t)$ the total government debt at period t. Government bonds demanded by an agent in sector j are denoted by $b^j(t)$. Each bond pays an interest $1 + i(t)$ in the following period.

Nominal wealth of agents evolve according to

$$a^j(t+1) = (1 + i(t))b^j(t) - \tau(t+1)$$

where $\tau(t+1)$ is a lump-sum tax collected by the government that is equal for all sectors. In the absence of private credit contracts, the budget constraint of an agent in sector j is then given by

$$\sum_{k \in K^j(t)} p_k(t)c_k^j(t) + b^j(t) = a^j(t) + p_j(t)y^j(t) \tag{1}$$

and the demand for government bonds is assumed to be nonnegative:

$$b^j(t) \geq 0, \tag{2}$$

for all $j = 1, \ldots, N$ and $t \geq 0$, that is, agents cannot borrow from the government.[6]

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

When there are no private credit contracts, an equilibrium for the economy is characterized by a set of prices and allocations given a trajectory for fiscal and monetary policy such that each agent maximizes intertemporal utility and markets clear, that is

$$\sum_{j=1}^{N} c_k^j(t) = y^k(t)$$

for all $t \geq 0$. Market clearing in the goods markets implies that there is also market clearing for government bonds, such that

$$\sum_{j=1}^{N} b^j(t) = a(t)$$

for all $t \geq 0$, which in turn implies a government budget described by

$$a(t+1) = (1 + i(t))a(t) - N\tau(t+1)$$

that must hold in any period. We will append this economy with private credit contracts that will provide an additional tool of transferring resources between agents in a negative scenario.

2.2. An economy with trade credit contracts

We now consider the existence of a credit mechanism through which agents sign contracts in the beginning of the period to purchase goods from other agents and pay for the goods in the end of the period—trade credit. Trade credit agreements are an essential part of the economy, as firms without cash need to purchase inputs for production or supplies to be resold in advance.[7] In our model, trade credit is to be interpreted as a consequence of the scarcity of liquidity by the productive sector that needs financing to purchase goods that are essential to the productive process. For simplicity, we separate the entity that finances goods in each sector and assume that for each sector there exists a continuum of risk-neutral financial institutions that extend credit to consumers of the good produced by that sector before the realization of any uncertainty, collects payments from consumers after the realization of the shock and transfers net gains/losses to the representative consumer of the sector. These financial institutions are to be interpreted as the financial arm of the productive agents in the sector, and not as a separate financial sector. The amount of credit extended can only be used to purchase goods produced by that sector, so as to model a trade credit agreement. Agents may also purchase goods in the retail market after the realization of the shock, at the same price that they could buy the good through a trade credit contract, since prices are assumed to be predetermined before the occurrence of the shock. However, agents will pay an interest rate on credit contracts that will be determined by financial firms in a competitive scenario.

Since credit agreements are signed and payed in the same period, aggregate fluctuations are irrelevant to the mechanism, since they are known beforehand by all agents. For that reason, trade credit is irrelevant for periods $t \geq 1$ when there is no asymmetric imbalance. At period $t = 0$, however, agents may become insolvent due to a shock and not be able to pay for the contracts that were signed.

We assume that agents set the amount of credit demanded from suppliers by a fixed rule based on the consumption of the previous period. Given choices $c_k^j(t-1)$ that an agent in sector j consumed at period $t-1$ of the good k, this agent will then sign a contract in the beginning of period t to purchase an amount $s_k^j(t) = s\big(c_k^j(t-1)\big)$, where s is a nonnegative function that specifies the amount of trade credit demand. This means that in period $t = 0$ contracts $s_k^j(0)$ are based on an exogenous initial consumption level that we assume to be the optimal consumption that would be chosen by an altruistic social planner, in a normal scenario in which there is no asymmetric shock, that is, $c_k^j(-1) = \alpha_{j-k}\bar{y}$. We also assume that the rule that specify trade credit contracts is also rotationally symmetric, in the sense that the set $\left\{s_k^{j+1}(t)\right\}_{k=1}^N$ is equal to the rotation of $\left\{s_k^j(t)\right\}_{k=1}^N$, so that there is no asymmetry that arises from the specification of trade credit contracts. This also implies that the total sum of credit contracts extended by each sector is the same for all sectors, that is, $\sum_k s_k^j(t) = \sum_k s_k^{j'}(t)$, for all $j, j' = 1, \ldots, N$.

Since contracts were signed before the productivity shock was known of, some contracts may become unfeasible due to the lack of revenue from production and reduced demand. Additionally, previous trade credit contracts signed with a sector that is hit by the shock are nullified. This means that agents that were extended an amount $s_p^j(0)$ to consume on goods produced by the sector p such that $\theta_p(0) = 0$ are exempted from having to pay these contracts, since they cannot use these resources to spend on goods produced by sector p.

Since contracts may become unfeasibile after the realization of an asymmetric shock, agents may need to default on a fraction of the amount they are required to pay, either by a bankruptcy law or renegotiation with creditors. Let $\psi_k^j(\theta)$ be the fraction of default of an agent in sector j with a contract signed with a supplier from sector k, given a realization of the asymmetric shock θ.[8] Then, total profit of the financial firm in sector j given a realization of the shock and default parameters is given by

$$\Pi^j(\theta) = \sum_{k=1}^N \left[\left(1 - \psi_j^k(\theta)\right)\left(1 + r^j\right) - 1\right] p_j(0) s_j^k(0)$$

where r^j is the interest rate charged by firms in sector j. Firms are risk-neutral and set the real interest rate to maximize expected ex-ante profits, where the expectation refers to the possible occurrences of the asymmetric shock θ. Since there is a continuum of firms in competitive behavior, the equilibrium condition that determines the interest rate r^j is simply $\mathbb{E}\Pi^j(\theta) = 0$, which is solved by setting

$$1 + r^j = \frac{1}{1 - \mathbb{E}\psi_j^k(\theta)\sigma_j^k(0)} \tag{3}$$

where $\sigma_j^k(0) = s_j^k(0)/\sum_k s_j^k(0)$ is the fraction of total credit extended to sector k from sector j. We assume that the default parameters $\psi_k^j(\theta)$ also display a rotational symmetry with respect to the productivity shock θ, so that given a realization of the shock that hits sector p, $\psi_k^j(\theta)$ only depends on the relative position of j and k with respect to p. This assumption is natural since all other aspects of the economy are symmetrical in the same way. This implies that the interest rates r^j are equal for all sectors $j = 1, \ldots, N$.

An agent in sector j in period $t = 0$ chooses consumption levels $c_k^j(0)$ and bond holdings $b^j(0)$, taking as given demand for goods $y^j(0)$ and profits from financial firms Π^j given an initial wealth $a^j(0)$, subject to the constraint

$$\sum_{k \in K^j(0)} p_k(0) c_k^j(0) + \sum_{k \in K^j(0)} \left[\left(1 - \psi_k^j(\theta)\right)\left(1 + r^k\right) - 1\right] p_k(0) s_k^j(0) + b^j(0)$$
$$\leq a^j(0) + \Pi^j(\theta) + p_j(0) y^j(0) \tag{4}$$

and subject to a lower-bound on consumption given the trade credit contract:

$$c_k^j(0) \geq s_k^j(0) \tag{5}$$

for all $k \in K^j(0)$.

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

We are now in a condition to define the equilibrium outcome of the decentralized economy with trade credit.

Definition 1. *An* ***equilibrium with trade credit*** *is a set of price trajectories $\{p_j(t)\}_{j=1}^N$, $t \geq 1$, trade credit interest rate $\{r^j\}_{j=1}^N$ and allocations $\{c_k^j(t)\}_{k,j=1,\ldots,N}$, $\{y^j(t)\}_{j=1}^N$ and $\{b^j(t)\}_{j=1}^N$, for $t \geq 0$ given initial prices $p_j(0)$, policy trajectories $i(t)$, $\tau(t+1)$ and $a^j(0)$ for all j, initial trade credit contracts $\{s_k^j(0)\}_{k,j=1,\ldots,N}$, default parameters $\psi_k^j(\theta)$, for all $k, j = 1, \ldots, N$ and a productivity shock θ, such that*

1. *The goods and bonds demand maximize utility of agents given restrictions (1), (2), (4) and (5) and expected production demand trajectory $\{y^j(t)\}_{j=1}^N$.*
2. *There is market clearing for all goods.*
3. *The interest rate for trade credit contracts satisfies the equilibrium condition (3).*

It is easily seen that there can exist multiple choices for the parameters $\psi_k^j(\theta)$ that are consistent with an equilibrium. For a trivial example, there is always an equilibrium in which all agents default on all trade credit agreements made, that is, $\psi_k^j(\theta) = 1$ for all $j = 1, \ldots, N$, $k \in K^j(0)$.

When there is no penalty to the default, instead of assuming the existence of trade credit as before, we can assume that defaults work as a transfer scheme after the productivity shock is learned of by the agents. This way of writing the problem is more convenient to determine the admissible parameters $\psi_k^j(\theta)$ and allows us to use the analysis of transfers in Woodford (2022) to study the effects of the bankruptcy law in this economy.

Rewrite the budget constraint (4) as

$$\sum_{k \in K^j(0)} p_k(0) c_k^j(0) + b^j(0) = \bar{a}^j(0) + p_j(0) y^j(0) \tag{6}$$

where

$$\bar{a}^j(0) = a^j(0) + \Pi^j(\theta) - \sum_{k \in K^j(0)} \left[\left(1 - \psi_k^j(\theta)\right) \left(1 + r^k\right) - 1 \right] p_k(0) s_k^j(0). \tag{7}$$

The initial nominal wealth $\bar{a}^j(0)$ is now composed of the fiscal transfers made by the government and the equivalent transfers that are made given the default parameters $\psi_k^j(\theta)$. In an equilibrium allocation with default, every agent in sector j will maximize intertemporal utility subject to (6) for period $t = 0$, (1) for periods $t \geq 1$, (2) for all periods with the additional constraint that $c_k^j(0) \geq s_k^j(0)$ for all $k \in K^j(0)$ since the credit good consumption was fixed before the beginning of the period.

We summarize this property in the following proposition:

Proposition 1. *Assume initial trade credit quantities $s_k^j(0)$. Then, an equilibrium with trade credit for an economy with policy trajectories $i(t)$, $\tau(t+1)$ and $a^j(0)$, default parameters $\psi_k^j(\theta)$ for all j and $k \in K^j(0)$, trade credit interest rate r^j that satisfy condition ((3)), and a productivity shock θ is an equilibrium to an economy without private credit contracts and with government policies $i(t)$, $\tau(t+1)$ and $\bar{a}^j(0)$, given by (7), with the added constraint:*

$$c_k^j(0) \geq s_k^j(0) \tag{8}$$

for all $j = 1, \ldots, N$ and $k \in K^j(0)$.

The proposition shows that if condition (8) is met, defaults in credit contracts between sectors will cause part of the large drop in expenditure predicted by the network model in Woodford

(2022) to be accommodated by defaults on private credit contracts. The effect of these defaults can be thought of as transfers from agents in sectors that do not default to agents in sectors that do default. Additionally, agents in sectors in which the impact of the shock on consumption is larger are more likely to have a larger default parameter (in some cases, if initial wealth and fiscal transfers are small, the default parameter will necessarily be high), implying that the transfers are correctly targeted. This result has an important consequence for the optimal fiscal policy since the size of fiscal policy that will restore aggregate demand for the economy can be smaller.

As a simple example, consider the normal case such that $\theta = (1, \ldots, 1)'$ and assume that $\psi^j_k(\theta) = 0$ for all sectors j and goods k. Then if predetermined prices $p_k(0)$ are the same for all goods k, the rotational symmetry hypothesis for credit contracts and implies that $\bar{a}^j(0) = a^j(0)$ for all j, that is, for every sector the profit of financial firms exactly equals the financial burden spent by the agents on trade credit contracts.

2.3. Optimal policy

In the "normal" case in which $\theta = (1, \ldots, 1)'$, we can assume that defaults are set to zero, and we have that $\bar{a}^j(0) = a^j(0)$ for all sectors. We can then analyze this economy as a simple decentralized economy without credit contracts, as in Woodford (2022). In this case, we know that there is a solution to the decentralized economy with equal prices for all goods $p_j(t) = P(t)$ that admits the stationary solution $y(t) = \bar{y}$, where $\bar{y}$ is such that $u'(\bar{y}) = v'(\bar{y})$ and $c^j_k(t) = \alpha_{k-j}\bar{y}$ for all $t \geq 0$, $j = 1, \ldots, N$ and $k \in K$. A necessary condition is that $a^j(0) = a(0)/N$ so that each sector is endowed with the same initial nominal wealth. Another necessary condition for the optimal policy is the Euler condition

$$(1 + i(t))\frac{P(t)}{P(t+1)} = \frac{1}{\beta}$$

to be satisfied for all $t \geq 0$.[9] Additionally, the transversality condition

$$\lim_{t \to \infty} \beta^t \frac{a(t)}{P(t)} = 0$$

is required by the fiscal and monetary policy implemented by the government, where $a(t) = \sum_j a^j(t)$. Finally, the government needs to implement a monetary policy that guarantees a stable path for the price level, such as a Taylor rule around a predetermined price level target trajectory $P^*(t)$.[10]

Equilibrium of the goods market implies that the government budget constraint is satisfied, with

$$a(t+1) = (1 + i(t))a(t) - N\tau(t+1)$$

for all $t \geq 0$. We assume that fiscal policy is such that $a(t) > 0$ for all $t \geq 0$. This condition implies that, in an equilibrium, there always exists a sector that does not have borrowing constraints. Clearing of the bond market requires that

$$\sum_{j=1}^{N} b^j(t) = a(t)$$

for all $t \geq 0$.

We assume that the government can, as a response to the occurrence of a productivity shock, make additional transfers to agents in period $t = 0$ so that the quantity $a^j(0)$ on the budget constraint of agents represent both the initial wealth of agents and additional fiscal transfers (or taxes).

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

Assume that sector p is the affected sector so that $\theta_p(0) = 0$. Woodford (2022) shows that when the ex ante optimal policy $a^j(0) = a(0)/N$ is such that $a(0) \to 0$, the expenditure equilibrium vector $\boldsymbol{c} = \left(c^1(0), \ldots, c^N(0)\right)'$[11] for this economy will be of the form $\theta\boldsymbol{\pi}$, where $\boldsymbol{\pi}$ is the unique right eigenvector associated with the eigenvalue 1 of the matrix $\mathbf{A}$, with $A_{jk} = \alpha_{j-k}/\left(1 - \alpha_{p-k}\right)$ if $j \neq p$, and $A_{pk} = 0$.[12] It is then seen that a productivity shock to a single sector p can cause major disruptions in the flow of payments, causing large reductions in the expenditure of multiple sectors, depending on the network structure that is specified.

It is also seen in Woodford (2022) that a reduction in the interest rate $i(0)$ is not able to stimulate the affected sector p and restore the first-best optimal equilibrium. Moreover, given any realization of the productivity shock θ, the optimal monetary policy can be shown to be leaving the nominal interest-rate unchanged, since stimulating the economy via an interest rate reduction makes the credit-constrained agents more constrained, while only raising demand from agents that are already on the optimal level of consumption. Therefore, we assume that monetary policy is unchanged when the shock is revealed, and we focus the analysis on fiscal transfers and the effect of credit default.

Now, in an economy without credit, if the fiscal policy is given by

$$a^{*p}(0) = \frac{a(0)}{N} + (1 - \alpha_0)P(0)\bar{y}$$

$$a^{*j}(0) = \frac{a(0)}{N} - \alpha_{p-j}P(0)\bar{y}, \text{ for all } j \neq p$$

then it is shown that the first-best allocation is a transfer equilibrium of this economy.[13] We are interested in investigating whether the trade credit defaults described above are able to reduce the size of transfers that are required to be made since it may be unfeasible for a government to implement this fiscal policy during an adverse event. From here on, we always assume that the government follows an ex ante optimal policy so that $P(0) = \bar{p}$ is fixed with an equal price for every good, and the government follows an ex ante optimal monetary policy that stabilizes the economy in the optimal allocation when there is no shock.

2.4. A simple model with constant fraction of credit

We first illustrate the properties of the model in a simple example in an economy with a constant fraction of credit for each agent and assume that the default parameters are set exogenously.[14] We show the equilibrium outcome and calculate the reduction in the size of the optimal fiscal policy required to bring the economy to the first-best allocation for two network structures (chain and uniform networks) and different trade credit rules, such that $s^j_k(0) = \gamma^j c^j_k(-1)$ for all $k \in K^j(0)$. This setup implies that $s^j_k(0) = \gamma^j \alpha_k \bar{y}$ for every $j = 1, \ldots N$, $k \in K^j(0)$. Assume in both cases that $\bar{p}$ is the equilibrium price for period $t = 0$ when there is no shock and initial fiscal policy $a^j(0) = a(0)/N$ is small. We denote by p the sector such that $\theta_p(0) = 0$ when the economy is hit by a negative productivity shock.

Now, both the distribution of the credit parameters γ^j and the network structure specified will be critical to determine whether there exists default parameters $\{\psi^j_k(\theta)\}_{j=1,..N,\, k\in K^j(0)}$ that can be implemented by a bankruptcy law to reduce the size of fiscal transfers that restore the optimal allocation. We explore two cases: in the first case, each sector has the same quantity of credit, $\gamma^j = \gamma$, while in the second case, a single sector $j \neq p - 1$ does not purchase any goods through credit, that is, $\gamma^j = 0$. In the first case, it is easily seen that there always exists a set of default parameters that reduces the total size of fiscal transfers, exactly by a fraction of γ, regardless of the network structure. In the second case, however, the network structure will be critical for the bankruptcy law to allow for any reduction on fiscal policy.

In the first example, we assume that all sectors have the same amount of consumption financed through trade credit so that $\gamma^j = \gamma$ for every $j = 1, \ldots N$. In this case, the network structure is irrelevant to determine the size of the reduction in the optimal fiscal policy.[15]

Computing the equivalent transfers $\bar{a}^j(0)$ given arbitrary default parameters, we obtain

$$\bar{a}^j(0) = \frac{a(0)}{N} + \gamma\bar{p}\bar{y}\left\{\sum_{k=1}^{N}\left[\left(1-\psi_j^k\right)(1+r)-1\right]\alpha_{k-j} - \sum_{k\in K^j(0)}\left[\left(1-\psi_k^j\right)(1+r)-1\right]\alpha_{j-k}\right\}$$

for $j \neq p$, while

$$\bar{a}^p(0) = \frac{a(0)}{N} - \gamma\bar{p}\bar{y}\sum_{k\in K^p(0)}\left[\left(1-\psi_k^p\right)(1+r)-1\right]\alpha_{p-k}$$

where r is the common interest rate charged by all sector on trade credit contracts. Then, it is easily seen by setting $\psi_k^j = 1$ for every $j = 1, ..N$, $k \in K^j(0)$, that the equivalent transfers can be set as

$$\bar{a}^j(0) = a^j(0) - \gamma\bar{p}\bar{y}\alpha_{p-j}$$

for $j \neq p$, and

$$\bar{a}^p(0) = a^j(0) + \gamma\bar{p}\bar{y}(1-\alpha_0).$$

If the government then implements a reduced fiscal policy of

$$a^j(0) = \frac{a(0)}{N} - (1-\gamma)\bar{p}\bar{y}\alpha_{p-j} \tag{9}$$

for $j \neq p$, and

$$a^p(0) = \frac{a(0)}{N} + (1-\gamma)\bar{p}\bar{y}(1-\alpha_0) \tag{10}$$

then we obtain the equality $\bar{a}^j(0) = a^{*j}(0)$ for all $j = 1, \ldots, N$ and the first-best allocation is restored. In this case, a permissive bankruptcy law that allows all agents to default on their contracts temporarily allows for a reduction of the necessary size of the fiscal policy that restores the optimal allocation. However, setting $\psi_k^j = 1$ for every $j = 1, ..N$ and $k \in K^j(0)$ is not the only possible choice of parameters that allows this same reduced fiscal policy to be implemented. Setting $\psi_j^j = 0$ for every j for instance is immaterial. Additionally, different network structures allow different default parameters to be implemented.

- In the chain network structure with parameter λ, we necessarily have to set $\psi_k^j = 1$ so that each sector defaults on all credit taken with the adjacent sector, with the exception of sector $p-1$. Fig. 2 illustrates transfers made through the bankruptcy law for $N = 4$:
- In the uniform network structure, the same policy can be implemented simply by setting $\psi_k^p = 1$ for every $k \neq p$, while all other parameters can be set to any level, even to zero. This means that in the uniform network, only sector p goes bankrupt and defaults. All other sectors are still solvent and can honor the credit agreements without defaulting.

We summarize this result with a proposition:

Proposition 2. ***(Optimal Bankruptcy Law with Constant Fraction of Credit)*** *Assume that the trade credit structure is given by $s_k^j(t) = \gamma c_k^j(t-1)$ for every $j = 1, \ldots, N$ and $k \in K^j(0)$, for a $\gamma \in [0, 1]$. Then, given a productivity shock to sector p there is always a set of parameters ψ_k^j for $j = 1, \ldots, N$ and $k \in K^j(0)$ such that the reduced fiscal policy given by equations (9) and (10) restores the first-best allocation in the decentralized equilibrium.*

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

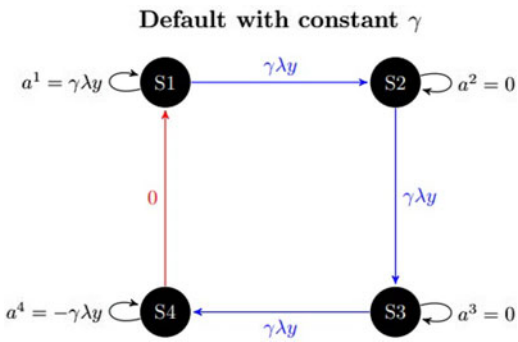

Figure 2. Bankruptcy in the chain network case with constant fraction of credit. Arrows represent the size of default from each sector. The net effect is a transfer from sector 4 to sector $p = 1$.

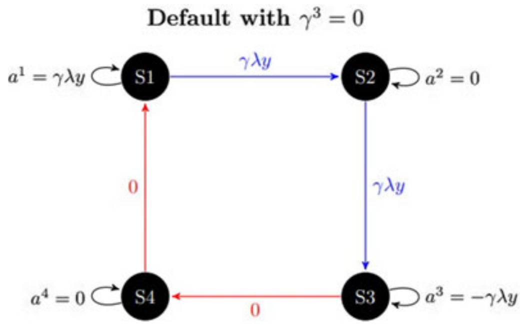

Figure 3. In the chain network, when a single sector does not finance consumption by credit, transfers implemented by the bankruptcy procedure may not be correctly targeted.

In a second formulation, assume now that there is a single sector l such that $\gamma^l = 0$, that is, agents in this sector do not finance their consumption through trade credit, while they extend credit to their customers. In this case, the specified network structure now deeply influences the attainable transfers through credit defaults. To illustrate this idea, we study the two networks considered previously: the chain network and the uniform network.

In the chain network structure, assume that $l \neq p - 1$. Computing the equivalent transfers to sector l we obtain

$$\bar{a}^l(0) = a^l(0) - \psi_l^{l-1}\gamma^{l-1}\lambda\bar{p}\bar{y}$$

which implies that if $\psi_l^{l-1} > 0$, the only way that this sector's demand can be restored is if the government transfers additional resources to sector l to outweigh the reduction caused by the sector $l-1$ default. Since in the optimal transfer policy, the only sector that should be taxed is sector $p-1$, we see that a bankruptcy law in this case will be ineffective to restore the optimal allocation. We illustrate in Fig. 3 this case with $N = 4$ and all other default parameters $\psi_{j+1}^j = 1$:

In the uniform network case, there is a different outcome to this situation. As we have seen before, we can always implement a fraction of the optimal fiscal policy through credit defaults in the uniform case by setting sector p parameters $\psi_k^p = 1$ and all other parameters to zero. In this case, the only credit parameter of importance is γ^p, and setting the credit parameter $\gamma^l = 0$ of any sector $l \neq p$ is inconsequential to this result. The connectedness of the uniform network implies that it is easier to implement an adequate bankruptcy law that transfers resources to the affected sector.

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

3. Bankruptcy law and credit renegotiation

In this section, we complete the model by developing a formulation in which agents can choose to default on the trade credit agreements but are excluded from credit markets if they choose to do so, as in Kehoe and Levine (2001). In this type of credit structure, we consider that each sector purchases some goods only through credit markets (suppliers), while other goods are purchased only through cash markets (retailers). The penalty for default is the exclusion from markets of supply goods. We show for a particular credit structure that individuals will not default when they are not subject to shocks, but if the agent belongs to a sector that is hit by the productivity shock, defaulting may be inevitable. In this case, it may be optimal for creditors not to exclude individuals from markets when they default since doing so will also exclude creditors from access to the goods produced by the sector in autarky.

In this situation, we evaluate two alternative policies. The first is a bankruptcy law that allows sectors affected by the shock to default without suffering the consequences of exclusion from markets. This rule can be thought of a one-time-only policy implemented in response to the shock that temporarily lifts the consequences of default on credit contracts.

The second alternative is a credit renegotiation between creditors and debtors, such that the creditors are still paid in future periods, but allowing debtors to postpone debt expenditure during a stress. While inferior to the exogenous bankruptcy law in terms of welfare, this more *laissez-faire* alternative is easily implementable and becomes closer to the previous policy as we assume that $\beta \to 1$.

3.1. Credit markets and autarky

In the general setup as in Section 2, assume that for every sector j we can decompose the sets $K^j(0)$ in two sets $\mathcal{S}^j$ and $\mathcal{R}^j = K^j(0) \backslash \mathcal{S}^j$. The set $\mathcal{S}^j$ denotes the *supplier* sectors of sector j so that all consumption from these sectors must be credit-financed. The set $\mathcal{R}^j$ are the *retailers*,[16] and all consumption from these sectors are cash-financed. We do not assume *a priori* that the sets of suppliers and retailers are symmetric.

As before, given a policy trajectory from the government, each agent plans consumption and asset accumulation and makes trade credit contracts $s_k^j(t) = c_k^j(t-1)$ for $k \in \mathcal{S}^j$, with $s_k^j(0) = c_k^j(-1)$ given by the ex ante optimal consumption, and $s_k^j(t) = 0$ for $k \in \mathcal{R}^j$. An agent can choose to default on credit agreements made before regardless of the occurrence of a productivity shock, but this agent is excluded from purchasing from supply markets ever again.[17] For an equilibrium to exist, we also assume that if a sector j is such that $\mathcal{R}^j = \{j\}$, then in autarky, other sectors $k \neq j$ are also excluded from purchasing goods from sector j.[18] The optimization problem for the agent in sector j in autarky for periods $t \geq 1$ is maximizing

$$\sum_{t=0}^{\infty} \beta^t U^{j,aut}(t)$$

where

$$U^{j,aut}(t) = \sum_{k \in \mathcal{R}^j} \theta_k(0) \alpha_{k-j} u \left(\frac{c_k^j(t)}{\alpha_{k-j}\theta_k(0)} \right)$$

for $t \geq 1$, with the usual utility function for period $t = 0$ with $c_k^j(0) = s_k^j(0)$ for $k \in \mathcal{S}^j$, and an agent in autarky in period $t = 0$ is subject to the budget constraint

$$\sum_{k \in \mathcal{R}^j} p_k(0) c_k^j(0) + b^j(0) = a^j(0) + \Pi^j + p_j(0)\bar{y}$$

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

and

$$\sum_{k \in \mathcal{R}^j} p_k(t) c_k^j(t) + b^j(t) = a^j(t) + p_j(t)\bar{y}$$

for periods $t \geq 1$.

The individual rationality condition for agent j not to choose to default when there is no negative shock is simply that

$$\sum_{t=0}^{\infty} \beta^t U^j(t) \geq \sum_{t=0}^{\infty} \beta^t U^{j,aut}(t).$$

It is straightforward to obtain the following proposition:[19]

Proposition 3. *(**Individual Rationality Condition**) If the government policy is given by the ex ante optimal policy, then for any network structure there is always a β sufficiently close to* 1 *such that the individual rationality condition is satisfied when there is no productivity shock.*

This proposition states that when there is no adverse shock, agents will not default on the credit taken with suppliers, for a value of β close enough to 1.[20] This means that the penalty of excluding agents from markets is sufficient, in this model specification, to guarantee that credit markets function as in a complete market model, and the penalty is never actually enforced. However, when there is a productivity shock, agents in affected sectors will have no alternative but to default.

3.2. Optimal bankruptcy law

We assume that the government follows an optimal initial policy such that $a(0) \to 0$ before the productivity shock is known of. As a consequence, agents hit by an adverse shock will have no alternative but to default and be penalized by being excluded from credit markets. This situation will be suboptimal both for the sector that defaults and its creditors, in the case of the two network structures that were presented above.

In an economy with a chain network structure with parameter λ, assume that $\mathcal{S}^j = \{j+1\}$ and $\mathcal{R}^j = \{j\}$. The optimal initial policy guarantees that if there is no default in the economy, each agent consumes $\lambda\bar{y}$ of the good produced by the supplier and $(1-\lambda)\bar{y}$ of the own sector's goods.

In this case, when the economy is hit by a negative productivity shock to a sector p, then this sector defaults in period $t = 0$, and the only possible equilibrium is for every sector to also default, except for sector $p-1$, since it will not have sufficient funds to cover its debt.[21] Since all sectors default, they are penalized with autarky so that for $t \geq 1$ each sector $j = 1, \ldots, N$ will consume only its own goods. The utility from this autarky in any period $t \geq 1$ is at most $(1-\lambda)u(\bar{y}/(1-\lambda))$ which is strictly dominated by the utility of $u(\bar{y})$ that is attained when there is no default, by the strict concavity of the utility function.

Therefore, a bankruptcy law that pardons the trade credit debt in period $t = 0$ when there is a negative shock will be optimal not only for the affected sector but also for all agents, including creditors. By the equivalence proposition of the previous subsection, the full pardon of the debt of size $\lambda\bar{p}\bar{y}$ in period $t = 0$ will be equivalent to a fiscal transfer that is financed by sector $p-1$, with multiplier λ^{-1} to sector $p+1$.[22] Sector $p+1$ will then spend $\bar{p}\bar{y}$ in period $t = 0$, and so will sector $p+2$ and so on, so that every sector of the economy is able to consume exactly the first-best optimal allocation of this economy. In this case, no additional fiscal policy is needed to support the first-best equilibrium.

In an uniform network structure, regardless of the sets $\mathcal{R}^j$ and $\mathcal{S}^j$ specified, when sector p is hit by a productivity shock all agents of this sector necessarily default as in the chain network case, but the budget constraint of agents in other sectors still allows them not to default, as long as the

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

supplier and retailer specifications $\mathcal{R}^j$ and $\mathcal{S}^j$ are the same for all sectors $j = 1, \ldots, N$. In this case, only sector p will be in autarky in periods $t \geq 1$.

In autarky, agents in sector p can only purchase goods from sectors $k \in \mathcal{R}^p$. It can be shown[23] that any equilibrium in this scenario is such that there are two stationary relative prices $q_S < 1 < q_R$, where $q_k = p_k(t)/P(t)$ and $q_k = q_R$ if $k \in \mathcal{R}^p$ and $q_k = q_S$ otherwise. Reduced demand from sector p implies that p-supplier sectors will have a smaller equilibrium relative price from $t = 1$ forward. This implies that they will purchase a smaller total quantity of goods in equilibrium since they will still desire to consume goods from p-retailer sectors that are now relatively more expensive. This means that intertemporal utility of p-suppliers is smaller when sector p is in autarky, and it will be optimal for these agents to lift the consequences of the default.

The previous examples show that lifting consequences applied to agents that default during a productivity shock constitutes not only an alternative to a targeted fiscal policy to sectors affected but also improves the welfare of their creditors. This happens because the usual bankruptcy procedure that excludes agents that default from markets, while being effective in preventing agents from deliberately choosing to default in normal times, also punishes creditors by reducing their access to goods produced by defaulters.

An important advantage of this bankruptcy procedure is that it only requires the government to understand which sectors are directly affected by the shock, so that their penalties are lifted ex-post. Since these sectors will default regardless of the existence of penalties, the affected sectors will be exactly those that default in the trade credit contracts. Solvent sectors will not wish to default, as they will expect to suffer the exclusion from credit markets as a consequence.[24] For this reason, bankruptcy policies can aid policymakers in countries subject to extreme unexpected events, by alleviating the consequences of these shocks in those most affected sectors, while requiring less information of the structure of the economy from the government than direct fiscal transfers.

We conclude this subsection with a general proposition:

Proposition 4. *Let an economy with trade credit be characterized by a symmetric credit structure so that $\mathcal{S}^{j+1} = R(\mathcal{S}^j)$ for every $j = 1, \ldots, N$, where R is the rotation operation. Given a productivity shock to sector p, an ex ante optimal fiscal and monetary policy and initial constant price $\bar{p}$, there always exists an exogenous bankruptcy law that guarantees that the modified initial nominal wealth is given by*

$$\bar{a}^p(0) = a^p(0) + \alpha_{\mathcal{S}^p}\bar{p}\bar{y}$$

where $\alpha_{\mathcal{S}^p} = \sum_{k \in \mathcal{S}^p} \alpha_k$ and

$$\bar{a}^j(0) = a^j(0) - \alpha_{p-j}\bar{p}\bar{y}$$

for every j that is a p-supplier.

In addition, a fiscal policy given by

$$a^p(0) = \frac{a(0)}{N} + (1 - \alpha_{\mathcal{S}^p} - \alpha_0)\bar{p}\bar{y}$$

and

$$a^j(0) = \frac{a(0)}{N} - \alpha_{p-j}\bar{p}\bar{y}$$

for every $j \neq p$ that is a p-retailer, restores the optimal allocation. In particular, if $R^p = \{p\}$, no additional fiscal policy is needed.

This proposition is straightforward from the definition of the modified initial nominal wealth $\bar{a}^j(0)$ by setting $\psi_k^j = 1$ for every $j = 1, \ldots, N$ and $k \in \mathcal{S}^j$. However, this choice of default parameters is not unique, and other choices of default parameters can be implemented through the bankruptcy law to obtain the transfers described in the proposition.

3.3. Credit renegotiation

We now extend the example with penalties to the debtor by autarky previously analyzed and consider the case in which creditors accept to renegotiate debt from agents that are insolvent due to the adverse shock. Given initial credit contracts $s_k^j(0)$ and a productivity shock to sector p, creditors and insolvent debtors agree to an alternative payment scheme in future periods $t \geq 1$ so that creditors are compensated and debtors are allowed to default in period $t = 0$ and postpone the payment of credit contracts.

Assume that agents either pay or default in full their credit commitments. Denote by $D \subset \{1, \ldots, N\}$ the subset of sectors that default in period $t = 0$. An agent that defaults is punished by autarky, which is undesirable both to the debtor and the creditor. Assume that agents negotiate a delayed payment scheme, represented by a sequence $\{h^j(t)\}_{t \geq 1}$ of nonnegative amounts that extends to infinity, satisfying

$$\sum_{t \geq 1} Q_{1,t} h^j(t) = (1 + i(0)) \sum_{k \in S^j} p_k(0) s_k^j(0)$$

where $Q_{1,t} = \prod_{s=1}^{t-1} \frac{1}{1+i(s)}$ and $Q_{1,1} = 1$ is the discount factor for future cash flows. In this sense, the creditor is paid out in full as the present value of future payments equals the amount that was agreed to be paid at period $t = 0$.

Assuming that the government follows an ex ante optimal policy so that initial prices $\bar{p}$ are equal, the sequence $a(t)$ is also assumed to satisfy a condition that guarantees that the borrowing constraint never binds.[25] In this case, the existence of equilibrium is guaranteed, with a default parameter of $\psi^j = 1$ for all $j \in D$, and a different borrowing constraint that consider the existence of future period payments:

$$\sum_{k \in K^j(0)} p_k(t) c_k^j(t) + b^j(t) + \mathbb{1}_{j \in D} h^j(t) = a^j(t) + \sum_{k \in D} h^k(t) + p_j(t) y^j(t).$$

where $\mathbb{1}_{j \in D} = 1$ if $j \in D$ and 0 otherwise. It can then be shown[26] that if we assume as in Woodford (2022) that $\beta \to 1$, then the existence of future payments will not disrupt the equilibrium efficient allocation, as these payments will become more insignificant to determine consumption choices as the agent becomes more patient. We can then conclude that a mutually beneficial alternative of credit renegotiation can be agreed to by creditors and debtors that allows agents in period $t = 0$ to consume in this period without generating a penalty for defaulters. This more *laissez-faire* alternative is equivalent to a government policy that taxes creditors in period $t = 0$ to transfer to debtors and then taxes debtors in future periods to compensate creditors.

To compare with other bankruptcy law alternatives, assume that the government also implements a residual fiscal policy in period $t = 0$ that is optimal, in the sense that it taxes cash consumers of the affected sector p and transfers to sector p, as dictated by the optimal policy rule. In this case, if insolvent sectors in period $t = 0$ are allowed to default on credit contracts, then demand will be restored to the first-best level considering a negative shock. There are then three cases: (i) if sector p defaults and is punished by autarky, there is a suboptimal allocation in all periods $t \geq 1$ for all sectors, including creditors; (ii) if a bankruptcy law allows for insolvent sectors to default freely, then the first-best level is restored for all periods; and (iii) if there is an endogenous credit renegotiation that establishes a future compensation for creditors that suffer default, then there is a distortion of future allocations in favor of creditors that vanish as $\beta \to 1$.

A different type of credit policy is considered in Woodford (2022), in which the government is able to relax the borrowing constraint of different sectors, allowing agents to borrow from the government during the occurrence of a negative shock. While this policy can stimulate those sectors most affected by the shock, it cannot replicate the optimal allocation, with the exception of the limiting case when $\beta \to 1$, since it distorts future consumption when agents need to pay for the credit. While similar to the credit renegotiation scenario analyzed in this section, the relaxation of borrowing constraints requires full information by the government of the sectors that need to be stimulated. An advantage of bankruptcy procedures such as the trade credit defaults postulated in this paper is that they depend less on the information available by the government, since agents themselves will choose to default or renegotiate credit contracts. The role of the government is to temporarily allow for this mechanisms to function by changing the prevailing bankruptcy law and provide incentives to credit renegotiation.

4. Concluding remarks

In this paper, we demonstrated how the credit relation between agents in an economy with multiple sectors can be exploited during an adverse scenario to implement transfers directed to the sectors that are hit by a productivity shock. A bankruptcy mechanism that allows insolvent agents to default during stress situations that are unforeseen, such as a pandemic or a major conflict, is an important tool that alleviates the impact of a shock and the fiscal burden of transfers that restore the optimal allocation of the economy.

Future research on the topic is necessary to better understand the equilibrium effects of bankruptcy and default on more general network structures, including endogenous structures that emerge from production functions as in Baqaee and Farhi (2020). Another characteristic observed in trade credit markets that is not approached in this paper is the fact that credit suppliers are usually endowed with better access to bank credit and are from sectors with higher market concentration, which could imply an asymmetry on the supply of trade credit in the market.

Financial support. This study was financed in part by the Coordenação deAperfeiçoamento de Pessoal de Nível Superior—Brasil (CAPES) - Finance Code 001. This study was financed in part by Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (Faperj) and Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

Notes

1 For a detailed list of actions taken during the COVID-19 crisis see the INSOL International and World Bank Group report "Global Guide: Measures Adopted to Support Distressed Business Through the COVID-19 Crisis," available on https://insol.azureedge.net/cmsstorage/insol/media/documents_files/covidguide/30aprilupdates/2-covid-map-17-may.pdf.

2 This fact was given a great deal of attention by the local specialized economic and financial media: https://valor.globo.com/impresso/noticia/2023/11/16/2023-deve-terminar-com-recorde-de-pedidos-de-recuperacao-judicial.ghtml (in Portuguese).

3 We use this simplifying assumption to focus on equilibria such that the production is always at the optimal *a priori* level, which is unchanged due to any shock that may happen in period $t = 0$.

4 The coefficients α_k are also assumed to be irreducible, in the sense that the network cannot be split into two separate networks that do not communicate, and that $\alpha_0 > 0$ whatever the network structure may be.

5 See Woodford (2022) for a detail proof of the equivalence above.

6 See Woodford (2022) for an analysis of an economy in which agents can borrow from the government during a negative scenario in order to finance consumption. Borrowing is suboptimal in comparison with the appropriate fiscal policy, since future debt repayments distort the equilibrium for the following periods after the negative shock hits the economy.

7 If one sector represents a financial sector of the economy, then the credit agreement can be thought of an unsecured credit contract, paid in the end of the period, that generates utility to the consumer by providing a financial service.

8 We first analyze the structure of the model assuming that the parameters ψ_k^j are set exogenously. In the following section, we complete the structure of the model and allows these parameters to be decided optimally be the agents, assuming a penalty for agents that default.

9 In a more general setting where prices $p_j(t)$ are not equal, this Euler equation will be a necessary optimal condition with $P(t) = (1/N)\sum_j p_j(t)$.

10 See Woodford (2022) for details of the implementation of this policy and a proof that it is able to attain the first-best allocation in a normal scenario.

11 Here, $c^j(t) = \sum_k c^j_{j+k}(t)$ is the total expenditure by sector j in period t.

12 The matrix $\mathbf{A}$ with elements A_{jk} that represents the fraction of expenditure of an agent in sector j on the good produced by sector k. Since sector p is shut down, the fraction of expenditure from agents in all sectors in the good produced by sector p is null.

13 Intuitively, each agent in the affected sector is endowed with the amount of wealth necessary to purchase the optimal bundle of goods in period $t = 0$, while the agents in other sectors are taxed exactly by the amount that they would spend in sector p. See Woodford (2022) for a demonstration of this result.
14 Endogenous parameters are considered in the following section, where agents are subject to a penalty when defaulting in the form of exclusion from credit markets.
15 To be precise, considering that the optimal fiscal policy transfer to the shutdown sector is equal to $(1-\alpha_0)\bar{p}\bar{y}$, the fiscal policy size depends on the coefficient α_0, but for a fixed α_0, the network structure (determined by the other parameters $\alpha_k, k \neq 0$) is irrelevant
16 We call j-suppliers and j-retailers the supplier and retailer sectors of sector j, respectively.
17 We call this situation autarky, even if the agent can still purchase goods from the retailer sectors in subsequent periods.
18 If this was not the case, there could be no equilibrium in autarky since an agent in sector j in autarky would demand the whole amount $\bar{y}$ produced by this sector, while other sectors k such that $\alpha_{j-k} > 0$ would also demand a positive amount of goods from sector j and demand would never be equal to supply.
19 See Appendix A for a proof.
20 The reason that a condition over this parameter is necessary is that if β is small, an agent will choose to default and obtain a higher consumption in $t = 0$ and accept the smaller utility from future periods.
21 This is straightforward from the budget constraint since $y^j(0) = c_j^j(0)$ for all j so that agents can only use resources from the initial wealth allocation (which is arbitrarily small) or from the debt payment of sector $j-1$ (which is zero by induction) to pay for their own debt. This is not the case for sector $p-1$ since this sector has no debt.
22 See Woodford (2022) for an analysis of the fiscal multipliers of fiscal transfers in the chain and uniform networks.
23 Details in the Appendix B.
24 This analysis depends on the shock being understood as a non-anticipated event, so that agents cannot default expecting to be bailed out by the government.
25 See Appendix C for the condition and the sufficiency of this condition to guarantee the existence of an equilibrium in periods $t \geq 1$.
26 See details in the Appendix.
27 We solve the equilibrium assuming $a(0) = 0$ and consider an adequate approximation for a situation where $a(0) \to 0$.
28 See Appendix A.

References

Acemoglu, D., A. Ozdaglar and A. Tahbaz-Salehi (2015) Systemic risk and stability in financial networks. *American Economic Review* 105(2), 564–608.

Araujo, A. P. and M. R. Páscoa (2002) Bancruptcy in a model of unsecured claims. *Economic Theory* 20(3), 455–481.

Baqaee, D. and E. Farhi (2020) Supply and demand in disaggregated keynesian economies with an application to the covid-19 crisis.

Bigio, S., M. Zhang and E. Zilberman (2020) Transfers vs Credit Policy: Macroeconomic Policy Trade-Offs During Covid-19. National Bureau of Economic Research, Working Paper: No. 27118.

Eisenberg, L. and T. H. Noe (2001) Systemic risk in financial systems. *Management Science* 47(2), 236–249.

Guerrieri, V., G. Lorenzoni, L. Straub and I. Werning (2020) Macroeconomic Implications of Covid-19: Can Negative Supply Shocks Cause Demand Shortages? National Bureau of Economic Research, Working Paper: No. 26918.

Kehoe, T. J. and D. K. Levine (2001) Liquidity constrained markets versus debt constrained markets. *Econometrica* 69(3), 575–598.

Milne, A. K. L. (2020) A critical covid-19 economic policy tool: Retrospective insurance.

Wang, J., J. Yang, B. Iverson and R. Kluender (2020) Bankruptcy and the Covid-19 Crisis. Harvard Business School, Working Paper.

Woodford, M. (2022) Effective demand failures and the limits of monetary stabilization policy. *American Economic Review* 112(5), 1475–1521.

A. Proof of Proposition 3

Proof. Assume that the government follows the optimal ex ante policy so that in equilibrium every good has the same price and every sector wishes to consume an amount $c_k^j(t) = \alpha_{k-j}\bar{y}$ in every period. The initial fiscal policy is such that $a^j(0) = a(0)/N$. This specification also means that trade credit contracts in period $t = 0$ are $s_k^j(0) = \alpha_{k-j}\bar{y}$ for all $j = 1, \ldots, N$ and $k \in \mathcal{S}^j$.

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

We prove that given that no other sector defaults, it is optimal for a single agent in a sector j not to default either. Let $\alpha_{\mathcal{S}^j} = \sum_{k \in \mathcal{S}^j} \alpha_k$ and $\alpha_{\mathcal{R}^j} = \sum_{k \in \mathcal{R}^j} \alpha_k$ so that $\alpha_{\mathcal{S}^j} + \alpha_{\mathcal{R}^j} = 1$. The agent's budget constraint in period $t \geq 0$ after he defaults is

$$P(t) \sum_{k \in \mathcal{R}^j} c_k^j(t) + b^j(t) = a^j(t) + P(t)\bar{y}$$

since he can only purchase goods from retail sectors and takes product demand $\bar{y}$ as given. Let $c^{\mathcal{R}^j}(t) = \sum_{k \in \mathcal{R}^j} c_k^j(t)$. The first-order condition for the problem given constant prices implies that

$$c_k^j(t) = \frac{\alpha_{k-j}}{1 - \alpha_{\mathcal{R}^j}} c^{\mathcal{R}^j}(t).$$

If the agent's borrowing is not constrained in any period $t \geq 0$, then he will choose a constant quantity $c^{\mathcal{R}^j}$ that maximizes the intertemporal utility

$$\alpha_{\mathcal{R}^j} u \left(\frac{c^{\mathcal{R}^j}}{\alpha_{\mathcal{R}^j}} \right)$$

with the intertemporal budget constraint considering that the period $t = 0$ present value of the tax burden to an agent in sector j in the optimal policy is given by $a(0)/N$ is given by

$$\bar{p} c^{\mathcal{R}^j} \leq \bar{p}\bar{y}$$

which is solved simply by $c^{\mathcal{R}^j} = \bar{y}$. However, this allocation can be financed with $a^j(t) \geq 0$ for all $t \geq 0$ for any initial quantity $a^j(0) = a(0)/N$, by choosing exactly the same amounts $b^j(t)$ that the agent chooses when he does not default, which proves that the agent that defaults is never borrowing-constrained.

The solution to this problem gives the intertemporal utility of

$$\alpha_{\mathcal{S}^j} u(\bar{y}) + \frac{1}{1-\beta} \left[\alpha_{\mathcal{R}^j} u \left(\frac{\bar{y}}{\alpha_{\mathcal{R}^j}} \right) - v(\bar{y}) \right]$$

which is dominated by the strict concavity of u by the intertemporal utility obtained when not defaulting, which is given by

$$\frac{1}{1-\beta} \left[u(\bar{y}) - v(\bar{y}) \right]$$

for a value of β sufficiently close to 1. □

B. Equilibrium in autarky in an uniform network

In this section, we show an equilibrium in an uniform network setting in which a sector p is hit by a productivity shock in period $t = 0$ and is obligated to default on trade credit agreements made with suppliers in $j \in \mathcal{S}^p \neq \emptyset$. We assume that the specification of the supplier sets $\mathcal{S}^j$ is equivalent up to a rotation for all sectors, in the sense that $\mathcal{S}^{j+1} = R(\mathcal{S}^{j+1})$, where R is the rotation operator. Assume that the government implements the optimal initial policy, which implies an equal price $\bar{p}$ for all goods in period $t = 0$, and that $a(0) \to 0$ initially so that an agent in an affected sector will have no choice but to default by the budget constraint (4). However, it still is feasible for other sectors to honor their commitments to suppliers (except to sector p that is shutdown) and to purchase the ex ante desired amount of goods from retail sectors (also not equal to p).

Since all agents in sector p default on their commitments, they are penalized by autarky in periods $t \geq 1$, that is, they can no longer purchase from sectors $j \in \mathcal{S}^p$. We now have to compute an equilibrium for this situation. We focus on stationary equilibrium so that consumption c_k^j of good k by sector j is constant in every period $t \geq 1$ since there are no further shocks to the economy in this specification. Let $q_j = p_j(t)/P(t)$ the relative price for the good j, which is also assumed to be stationary. It will be the case that there will be only two different relative prices $q_S < 1 < q_R$ so that $q_j = q_S$ if sector j is a supplier of sector p and $q_j = q_R$ otherwise.

We solve the continuation problem for an agent in sector j for $t \geq 1$. Let

$$c_R^j = \sum_{k \text{ is a retailer of sector } p} c_k^j$$
$$c_S^j = \sum_{k \text{ is a supplier of sector } p} c_k^j$$

The intertemporal budget constraint of a sector $j \neq p$ from period $t \geq 1$ is

$$q_R c_R^j + q_S c_S^j \leq \frac{1-\beta}{\beta \bar{p}} \left[b^j(0) - \frac{a(0)}{N} \right] + q_j \bar{y}$$

and the assumption that $a(0) \to 0$ implies that the right-hand side of the inequality becomes simply $q_j \bar{y}$.[27]

Likewise, an agent in sector p is subject simply to the condition that $q_R c_R^p \leq q_R \bar{y}$, again assuming $a(0) \to 0$. Let $N_R = \#\mathcal{R}^p$ be the number of retailers and $N_S = \#\mathcal{S}^p$ the number of suppliers. Given an equal relative price for retailers and suppliers, and the uniform network structure, agents will wish to purchase an equal amount of each good so that $c_k^j = c_R^j/N_R$ when k is a retailer of sector p and $c_k^j = c_S^j/N_S$ otherwise. The solution to the continuation problem for periods $t \geq 1$ of an agent in sector $j \neq p$ is the maximum of the stationary utility

$$\frac{N_R}{N} u \left(\frac{N c_R^j}{N_R} \right) + \frac{N_S}{N} u \left(\frac{N c_S^j}{N_S} \right)$$

subject to the intertemporal budget constraints, assuming $a(0) \to 0$. For an agent in sector p that is in autarky, stationary utility is simply

$$\frac{N_R}{N} u \left(\frac{N c_R^p}{N_R} \right).$$

Sector p problem is simply solved by $c_R^p = \bar{y}$ by the monotonicity of the utility function, and an agent in sector p will purchase an amount of $\bar{y}/N_R$ from each retail sector.

It is easy to see that all p-supplier sectors are solving the same maximization problem subject to the same constraints, and the same is true for the p-retailers. This means that the solution to this continuation problem will be the same for every p-supplier and p-retailer. Let $c_k^j = c_k^R$ be the solution for the retailer and $c_k^j = c_k^S$ for the supplier, where $k = \{R, S\}$. A stationary equilibrium for this economy will be a set $\{c_R^R, c_S^R, c_R^S, c_S^S, q_R, q_S\}$ that satisfy the following equations:

The budget constraints:

$$q_R c_R^R + q_S c_S^R = q_R \bar{y}$$
$$q_R c_R^S + q_S c_S^S = q_S \bar{y}$$

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

Market clearing:

$$\frac{\bar{y}}{N_R} + \frac{N_R - 1}{N_R} c_R^R + \frac{N_S}{N_R} c_R^S = \bar{y}$$

$$\frac{N_R - 1}{N_S} c_S^R + c_S^S = \bar{y}$$

First-order conditions:

$$\frac{u'\left(\frac{N}{N_R} c_R^R\right)}{q_R} = \frac{u'\left(\frac{N}{N_S} c_S^R\right)}{q_S}$$

$$\frac{u'\left(\frac{N}{N_R} c_R^S\right)}{q_R} = \frac{u'\left(\frac{N}{N_S} c_S^S\right)}{q_S}$$

The definition of the price index:

$$\frac{N_R q_R + N_S q_S}{N} = 1$$

We can drop one of the budget constraints since if one of them is valid and there is market clearing in both markets the second budget constraint will be satisfied. We are then left with 6 equations for 6 variables, and the strict monotonicity of u' guarantees that there will always be a solution that satisfies these equations simultaneously.

Now, since u is strictly increasing in every good, the market clearing condition and the first-order conditions imply that the p-retail goods are relatively more scarce, and the price index definition implies that $q_S < 1 < q_R$. This condition along with the budget constraint for an p-supplier implies that $c^S = c_R^S + c_S^S < \bar{y}$. However, then, strict concavity of u implies

$$\frac{N_R}{N} u\left(\frac{N c_R^S}{N_R}\right) + \frac{N_S}{N} u\left(\frac{N c_S^S}{N_S}\right) < u(c^S) < u(\bar{y}),$$

where the term on the left denotes the stationary utility in equilibrium and the term on the right denotes the stationary utility when no sector is in autarky. This condition shows that it is suboptimal for a supplier to penalize debtors and a bankruptcy law that pardons the debt when there is a productivity shock will restore optimality in periods $t \geq 1$.

C. Equilibrium in a credit renegotiation

We consider the framework of Section 3, in which insolvent debtors default in full on credit contracts and compensate their creditors with a future trajectory of payments $\{h^j(t)\}_{t\geq 1}$. Represent by $h_k^j(t)$ the amount of the payment from sector j that is destined to sector k at time t. Denote by $D \subset \{1, \ldots, N\}$ the set of sectors that are insolvent and therefore are required to default in full on credit agreements in period $t = 0$. The specified trajectories $\{h^j(t)\}_{t\geq 1}$ are assumed to satisfy

$$\sum_{t\geq 1} Q_{1,t} h^j(t) = (1 + i(0)) \sum_{k\in S^j} p_{j+k}(0) s_{j+k}^j(0) \tag{C1}$$

where $Q_{1,t} = \prod_{s=1}^{t-1} \frac{1}{1+i(s)}$ and $Q_{1,1} = 1$ is the discount factor for future cash flows.

https://doi.org/10.1017/S1365100524000099 Published online by Cambridge University Press

To solve for equilibrium in this specification, we consider the continuation problem of an agent in sector j from period $t = 1$ forward, and using the hypothesis of rational expectations and the fact that $y^j(t) = \bar{y}$ for all $t \geq 1$ to conclude that any agent in this sector will choose a stationary sequence of consumption c^j_{j+k}, for $k \in K$, as long as relative prices $q_k = p_k(t)/P(t)$ are also stationary and the agent is not borrowing-constrained in future periods. We also assume that the government follows the optimal ex ante policy by fixing prices $p_j(0)$ to an equal amount $\bar{p}$ before the adverse shock is known of and that the government follows the optimal monetary policy trajectory which remains unchanged after the shock is known of, so $1 + i(t) = P(t+1)/(\beta P(t))$ for all $t \geq 1$.

If these conditions are met, then the intertemporal budget constraint for an agent in sector j is given by

$$\frac{1}{1-\beta} \sum_{k \in K^j} q_k c^j_k + \tau^{PV} + \leq \frac{1}{\beta} \frac{b^j(0)}{\bar{p}} + \frac{1}{1-\beta} q_j \bar{y} + \frac{1}{\beta} \omega^j \tag{C2}$$

where

$$\omega^j = \sum_{k \in D} \frac{s^k(0)}{\bar{p}} - \mathbb{1}_{\{j \in D\}} \frac{s^j(0)}{\bar{p}}$$

is the total net amount of real credit of an agent in sector j, $\mathbb{1}_{j \in D}$ is the indicator function, and τ^{PV} is the real present value of future tax collections so that $\tau^{PV} = a(0)/(N\beta\bar{p})$.

Additionally, at a given period t, it follows by induction from the budget constraint that the real asset position of an agent in sector j is given by

$$\beta^{t-1} \frac{b^j(t)}{P(t)} = \tilde{a}^j(1) + \frac{1-\beta^t}{1-\beta} \left(q_j \bar{y} - \sum_{k \in K^j} q_k c^j_k \right) - \left(\tau^{PV} - \beta^{t-1} \frac{a(t)}{NP(t)} \right) + \sum_{r=0}^{t} \beta^{r-1} \frac{s^j(r)}{P(r)}$$

where $\tilde{a}^j(1) = (1 + i(0))b^j(0)$ is pretax initial wealth in period $t = 1$ and

$$s^j(t) = \sum_{k \in D} h^k_j(t) - h^j(t)$$

is the net payment received from debtors, discounted the amount paid to creditors, by an agent in sector j. Now, using equations (C1) and (C2) and the fact that $\tilde{a}^j(1) \geq 0$, we conclude that the non-borrowing-constrained condition $b^j(t) \geq 0$ is equivalent to the inequality

$$\frac{a(t)}{NP(t)} \geq \beta\tau^{PV} - \left(\omega^j + \sum_{r=t+1}^{\infty} \beta^{r-t} \frac{P(1)}{P(r)} s^j(r) \right)$$

so that by imposing the condition

$$\frac{a(t)}{NP(t)} \geq \beta\tau^{PV} - \min_j \left\{ \omega^j + \sum_{r=t+1}^{\infty} \beta^{r-t} \frac{P(1)}{P(r)} s^j(r) \right\}$$

we can assume that no agent is borrowing constrained and a solution for the continuation problem exists for every agent in every sector $j = 1, \ldots, N$. This solution will be the choice of consumption bundles that maximize stationary utility subject to condition (C2). Now, using the same approximation method as Woodford (2022),[28] we see that by setting $\beta \to 1$, there is a unique equilibrium for the continuation problem with equal relative prices q_j and such that each agent consumes the first-best allocation in every period $t \geq 1$.

Cite this article: Araújo A and Costa V (2024). "Bankruptcy law as an alternative to fiscal policy in a Woodford model with a productivity shock." *Macroeconomic Dynamics*. https://doi.org/10.1017/S1365100524000099