Measurement Uncertainty in Spatial Models: A Bayesian Dynamic Measurement Model

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Abstract
According to spatial models of political competition, parties strategically adjust their ideological positions to movements made by rival parties. Spatial econometric techniques have been proposed to empirically model such interdependencies and to closely convert theoretical expectations into statistical models. Yet, these models often ignore that the parties’ ideological positions are latent variables and, as such, accompanied by a quantifiable amount of uncertainty. As a result, the implausible assumption of perfectly measured covariates impedes a proper evaluation of theoretical propositions. In order to bridge this gap between theory and empirics, the present work combines a spatial econometric model and a Bayesian dynamic item response model. The proposed model accurately accounts for measurement uncertainty and simultaneously estimates the parties’ ideological positions and their spatial interdependencies. To verify the model’s utility, I apply it to recorded votes from the sixteen German state legislatures in the period from 1988 to 2016. While exhibiting a notable degree of ideological mobility, the results indicate only moderate spatial dependencies among parties of the same party family. More importantly, the analysis illustrates how measurement uncertainty can lead to substantively different results which stresses the importance of appropriately incorporating theoretical expectations into statistical models.

Keywords: dynamic ideal point estimation, Bayesian statistics, spatial econometrics, measurement uncertainty, multiparty competition

1 Introduction
Spatial models of party competition within multiparty systems emphasize the parties’ contingency on the strategies of other parties (e.g., Downs 1957; Davis, Hinich, and Ordeshook 1970; Laver 2005; Laver and Sergenti 2012). These models postulate a complex and reciprocal dependence structure among competing parties which condition their strategic behavior. Consequently, the proposition of parties’ interrelatedness is of key interest for a proper test of theoretical models of party competition. To empirically evaluate these interdependencies, political scientists began to adopt techniques from spatial econometrics (e.g., Beck, Gleditsch, and Beardsley 2006; Franzese and Hays 2007, 2008; Williams and Whitten 2015; Böhmelt, Ezrow, Lehrer, and Ward 2016). By explicitly modeling spatial interdependencies, these methods should establish a close match between theoretical predictions and empirical models.

While spatial econometric models are promising to narrow the gap between sophisticated theories and empirics, the latent character of parties’ ideological positions creates problems for a proper evaluation of theoretical expectations. Since latent variables are inherently unobservable, errors in their measurement are inevitable. It is unclear how much variation in party positions can
be attributed to mere measurement error (e.g., Dalton and McAllister 2015). As a result, the implicit assumption of perfectly measured covariates made by spatial econometric models creates a disjuncture between theory and empirics and threatens the validity of the findings. This problem, however, receives almost no attention in empirical studies.

This paper contributes to the literature by investigating how measurement uncertainty affects the substantial inferences about parties’ interrelatedness. To this end, I develop a Bayesian dynamic item response (IRT) model with an evolution function that explicitly fits theoretical propositions. I model the ideological evolution of each party as a spatio-temporal autoregressive process and allow parties to strategically respond to movements made by their political competitors (e.g., Franzese and Hays 2007, 2008; Hays, Kachi, and Franzese 2010). In order to illustrate the effect of neglecting measurement uncertainty in spatial econometric models, I apply it to recorded votes from the sixteen German state legislatures in the period from 1988 to 2016. Previous research finds that parties strategically respond to their competitors in the electoral arena (e.g., Williams 2015; Williams and Whitten 2015; Böhmelt et al. 2016). By asking whether this holds for the legislative arena as well, this application shows how measurement uncertainty directly affects these models’ substantive inferences.

The analysis reveals that there is both a nontrivial amount of measurement uncertainty and mobility in parties’ positions. By disentangling conscious strategic movements from idiosyncratic fluctuations, the results further show that, in contrast to theoretical predictions and previous findings based on election manifestos, parties do not adjust their positions in response to their ideological neighbors in the legislative arena. However, the positions of parties within the same party family affect a party’s ideological position, leading to endogenous dynamics of party competition at the macrolevel. More importantly, this article demonstrates that imperfect measures can severely affect the results. Hence, accounting for measurement uncertainty is not merely a methodological concern but can lead to very different inferences. By presenting a highly flexible model which avoids the assumption of perfectly measured covariates and combines a spatial econometric model with a dynamic measurement model, this paper explicitly links theory and empirics in order to achieve a thorough examination of the proposed mechanisms that cause interrelated party platforms to vary over time.

2 Spatial Econometric Models and Multiparty Competition

Spatial dependence among political parties is a ubiquitous feature of spatial models of party competition, ranging from the classical game-theoretic models following Downs (1957) and Davis, Hinich, and Ordeshook (1970) to the more recent agent-based models introduced to political science by Kollman, Miller, and Page (1992).1 Office-motivated parties do not make decisions about their ideological positions in a political vacuum, isolated from externalities of their competitive environment. Conceptualized as rational actors in the framework of spatial theory, the behavior of competing parties directly affects the parties’ strategic considerations about the vote-maximizing policy position. Yet, empirically evaluating these interdependencies proposed by theoretical models is challenging because ordinary regression analyses require the assumption that the observations are conditionally independent. This assumption inhibits a proper empirical examination of spatial models and causes a mismatch between theoretical propositions and statistical analyses. The observed party positions are not mutually independent but exhibit a spatial autocorrelation which limits the observations’ informational content and introduces endogeneity problems in conventional regression analyses.

In order to overcome this limitation and to exploit the intrinsic spatial nature of the parties’ policy platforms, scholars began to employ a set of statistical techniques from the field of spatial

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1 For an introduction to agent-based models in political science applications, see de Marchi and Page (2014).
econometrics (e.g., Franzese and Hays 2007, 2008; Hays, Kachi, and Franzese 2010; Williams 2015; Williams and Whitten 2015; Böhmelt et al. 2016; Williams, Seki, and Whitten 2016). Importantly, the concept of space is not restricted to geographical spaces but can also be applied to latent conceptualizations of space as well (Beck, Gleditsch, and Beardsley 2006, 28). This feature makes tools from spatial econometrics applicable to data on policy preferences. The posterior means of the parties’ policy platforms at each point in time are the coordinates within a known reference system, represented by a latent $n$-dimensional Euclidean policy space. In the unidimensional case, this latent space and the parties’ positions within it are most often interpreted in terms of left and right.

Since the parties’ reciprocal relations are of key interest for an evaluation of theoretical models of party competition, spatial autoregressive models became increasingly popular in political science over the past decade. Most empirical studies examine time-series cross-sectional data which requires the modeling of temporal and spatial dependencies. As Franzese and Hays (2008) show, the most direct way of doing so is by specifying a spatio-temporal autoregressive (STAR) model of the following form:

$$ y = \phi M y + \rho W y + X \beta + \epsilon, $$

where $y$ is the $IT \times 1$ dependent variable vector (with $I$ being the number of cross-sections or units and $T$ being the number of time points) stacked by period, $X$ is an $IT \times (m+1)$ dimensional matrix of $m$ exogenous covariates and the intercept, $\beta$ is the corresponding $(m+1) \times 1$ parameter vector, and $\epsilon$ is a $IT \times 1$ vector of independent disturbances. $M$ is a matrix of dimension $IT \times IT$ with ones on the minor diagonal (i.e., at $(I+1, 1), (I+2, 2), \ldots, (IT, IT-I)$), and zeros elsewhere, so that the term $My$ is the first-order temporally lagged dependent variable with the associated coefficient $\phi$.

The remaining part of Equation (1) denotes the spatial lag. It is composed of the spatial coefficient $\rho$, the vector of the dependent variable $y$, and the $IT \times IT$ dimensional connectivity matrix $W$ which has connectivity matrices for each period on its block diagonal and zeros elsewhere. This part of the spatial regression facilitates the proper evaluation of theoretical propositions derived from spatial theory by allowing the researcher to specify a theoretically informed connectivity matrix and include it in the systematic component of the regression model (Neumayer and Plümper 2016). Moreover, the model can be easily extended to include multiple spatial lags which facilitates the simultaneous testing of different relational ties. Williams (2015) explicitly shows the benefits of spatial econometrics. He points out that, as opposed to simple OLS regressions, these models do not assume that the actors are independent. Instead, they allow for interdependencies among parties and, thereby, “narrow the gap between empirical estimation techniques that make unreasonable assumptions (such as independent observations) and rich theory that painstakingly describes the interconnectedness of parties” (Williams 2015, 155).

Most empirical examinations of the spatial theory of party competition utilize election manifestos in order to deduce parties’ policy positions (e.g., Adams and Somer-Topcu 2009; Williams 2015; Williams and Whitten 2015). The dependent variable in these assessments is the ideological position (or the positional change) of parties on a Left–Right dimension and the effect of rival parties is modeled by specifying the spatial lag accordingly. However, office-motivated parties that instrumentally adopt an ideological position in an attempt to receive the support of policy-motivated voters—as argued by spatial theory—have additional means for communicating their position to the electorate. Besides press releases and social media content, an effective way for parties to signal their policy position to voters is by their legislative behavior (e.g., Mayhew 1974). Due to the high salience and the actual legislative consequences associated with it, their parliamentary voting behavior allows parties to credibly signal their strategically adopted policy
position to the electorate (e.g., Theriault, Hickey, and Blass 2011; Hug 2013). Hence, the legislative arena is another valuable channel for office-seeking parties to signal their policy position to the voters. Solely focusing on the electoral arena by analyzing manifesto data does not address the full scope of the proposition of parties’ interdependencies as formulated by spatial theory.

With respect to the parties’ interdependencies, empirical studies based on election manifestos find that the effect of rival parties is not uniform across time and space but contingent on certain characteristics of the parties themselves, including their ideological distances and their membership in a party family (e.g., Williams 2015; Böhmelt et al. 2016; Williams, Seki, and Whitten 2016). For the purpose of this article, I focus on the following well-established hypotheses:

**Party dynamics hypothesis.** The further to the right (left) of the political spectrum the ideologically neighboring parties’ positions, the further to the right (left) a focal party’s position.

Downs (1957) introduces this hypothesis in the context of two-party competition and Adams (2001) expands this theoretical argument to the case of multiparty competition. He shows that the expectation that parties consciously shift their position in the same direction as their ideological neighbors also holds in the context of multiparty competition. The second hypothesis considered here is the **ideological families hypothesis**:

**Ideological families hypothesis.** The further to the right (left) the ideological positions of the other parties within the same party family, the further to the right (left) a focal party’s position.

Both hypotheses have been tested in the electoral arena (e.g., Adams and Somer-Topcu 2009; Williams 2015; Böhmelt et al. 2016). However, due to the direct legislative consequences associated with a parliamentary vote and the ease with which the electorate can monitor the parties’ behavior, the findings about the parties’ interdependencies might differ in the legislative arena. In contrast, when changes in parties’ policy positions are not the result of strategic responses to rival parties’ shifts but caused by other factors or merely random diffusion processes, parties would not be sensitive toward the behavior of either neighboring parties or the parties within the same party family. This random diffusion of a party’s ideological platform also finds empirical support. Dalton and McAllister (2015) report a high degree of ideological stability and conclude that poor measures and random diffusion rather than strategic considerations most frequently account for parties’ ideological movements.

### 3 Ideology as a Latent Variable

Despite the apparent concordance between spatial theory and spatial econometric models, the empirical evaluation of theoretical models of multiparty competition is not as straightforward as previous research might suggest. A key feature of the empirical analyses mentioned above is that the covariates—the parties’ positions—are latent quantities and therefore not directly observable. As a consequence, two major obstacles arise: first, comparing party positions over time is difficult because they need to be located in a common space. Empirical research widely acknowledges this problem and presents various solutions to it (e.g., Poole and Rosenthal 1991; Martin and Quinn 2002; Herron 2004; Bailey 2007; Park 2011; Shor and McCarty 2011; König, Marbach, and Osnabrügge 2013). Second, latent variables are, by definition, unobservable and, as a result, afflicted with a quantifiable amount of uncertainty. Even the most sophisticated measures are subject to random measurement error which can distort the substantive inferences as numerous simulation studies already demonstrate (e.g., Fuller 1987; McAvoy 1998; Blackwell, Honaker, and King 2017; Loken and Gelman 2017). In contrast to the problem of comparing positions over time,
empirical studies predominantly ignore this problem which creates a disjunction between theory and empirics.²

Like all latent constructs, parties’ ideological positions are inherently unobservable. Scholars interested in those abstract concepts need to infer them from observable indicators. By building statistical measurement models which link the indicators to the underlying latent trait, researchers are able to quantify unobservable constructs which are of key interest. Quantities derived in this way, however, necessarily are estimates and accompanied by measurement uncertainty which has direct implications for subsequent analyses. Including latent variables as ordinary regressors in empirical models, however, implicitly assumes that they are perfectly measured which ignores random measurement error and thereby underestimates the parameters’ uncertainty.

The idea to account for measurement uncertainty surrounding latent variables in empirical models is not new to political science. The literature widely acknowledges the latent nature of some of their constructs, including (but not limited to) regime type (e.g., Treier and Jackman 2008; Pemstein, Meserve, and Melton 2010), human rights repression levels (e.g., Fariss 2014; Schnakenberg and Fariss 2014; Crabtree and Fariss 2015), and political knowledge (e.g., Jessee 2017). From a conceptual point of view, it seems uncontroversial to add parties’ ideological positions to this list of latent constructs. Empirically, however, scholars interested in parties’ spatial dependencies rarely compensate for the latent character of their constructs and the associated measurement uncertainty (Blackwell, Honaker, and King 2017).

This circumstance constitutes a severe problem because unaccounted random measurement error not only leads to the underestimation of the parameters’ variability. It can also distort the effect estimates. While random measurement error in the dependent variable is generally unproblematic, the same does not hold for independent variables. Indeed, the effect of imperfectly measured covariates can be adverse (e.g. Blackwell, Honaker, and King 2017; Loken and Gelman 2017). Attenuation bias is not the only possible consequence of poorly measured covariates. They can also exaggerate the estimated effect sizes which makes it impossible to assess a priori how the estimates of all covariates change under conditions of measurement error (Benoit, Laver, and Mikhaylov 2009, 506). With respect to the severity of measurement error for statistical inferences and the prevalence of latent quantities in political science, McAvoy (1998, 166) pointedly remarks that “[p]olitical scientists […] do not have the luxury of ignoring measurement error […].” As Equation (1) illustrates, this circumstance has severe implications for spatial econometric models. Measurement uncertainty in the parties’ positions is not only captured by the error term but propagates into the regression’s covariates through the temporally and spatially lagged dependent variable.

In order to circumvent this problem, researchers need to account for this uncertainty by specifying measurement models and integrate them into subsequent analyses. Bakker (2009, 416) already notes that scholars “[…] must choose to either ignore the uncertainty and treat […] latent variable as observed or model the measurement and predictive models simultaneously […].” As I show in Section 5, ignoring measurement uncertainty is not merely a methodological concern but leads to substantively different conclusions. As a solution to this, I combine a predictive spatial econometric model with a dynamic measurement model for the latent quantities—parties’ ideological positions. I illustrate how the simultaneous estimation of party positions and their spatial dependencies avoids the assumption of perfectly measured regressors and presents an opportunity to directly incorporate theoretical predictions about parties’ reciprocal dependencies in empirical models. As a result, the estimates obtained here provide a much more accurate

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² For a notable exception, see Benoit, Laver, and Mikhaylov (2009) who apply an error correction model, termed simulation-extrapolation, to address the problem of random measurement error.
evaluation of theoretical predictions and help to further narrow the gap between sophisticated theories and their empirical assessments.

4 A Bayesian Dynamic Measurement Model

The statistical measurement model I propose is inspired by the dynamic IRT model presented by Martin and Quinn (2002) and applies a scaling technique to recorded votes. It has several desirable advantages for the analysis performed here. Studies based on roll call votes are built on an enormously rich methodological foundation (Shor and McCarty 2011; Theriault, Hickey, and Blass 2011). In contrast to manifesto data, the availability of roll call votes is not limited to election years. This is especially important for a time-series analysis of ideal point dynamics as it enables researchers to investigate temporal changes in much shorter intervals. In addition, the computer-based content analysis of political texts is not suited for time-series estimation and requires the identification of appropriate reference texts (Laver, Benoit, and Garry 2003; Benoit and Laver 2007). Another advantage of the analysis of roll calls, as Poole and Rosenthal (2001) point out, is that scholars do not need to have a prior interpretation of the latent policy space. The only assumption necessary for performing the analysis is the dimensionality of the space. Rather than fitting empirical observations to predefined categories, scholars extract information about the ideological division from the observed voting patterns. Finally, as discussed in Section 2, spatial theory does not imply that manifestos are the only means by which office-seeking parties communicate their instrumentally adopted policy position. They can also use their voting record to credibly signal their policy position to the electorate. After all, recording votes needs to be requested which indicates that at least one party attempts to signal its position. Therefore, this type of data is appropriate for studying dynamics of party competition. In short, relying on data on parliamentary voting behavior constitutes a unique opportunity for empirically testing theoretical predictions about parties’ spatial dependencies in the legislative arena.

For the purpose of this article, I adopt the Bayesian perspective. Krehbiel and Peskowitz (2015, 694) show that this approach for the estimation of ideal points requires fewer assumptions as compared to alternative approaches and is the most accurate measure based on recorded votes. The small number of parties and few votes in a given year do not constitute a problem since the results do not depend on large sample approximations. Furthermore, the Bayesian approach facilitates the interpretation of the results since it treats the parameters as random variables. This is especially important since the interpretation of spatial effects is not a trivial task. Relying on the frequentist hypothesis testing is not necessary and the full posterior distribution can be deployed to learn about the parameters given the data and prior knowledge. Finally, measurement uncertainty is reflected in the parameters’ posterior distributions and directly propagates in the estimation of spatial dependencies. I obtain the full posterior density by the simulation-based approach which facilitates the direct assessment of the effect of measurement uncertainty.³

4.1 Dynamic measurement model

Let $I \subseteq \{1, 2, \ldots, I\}$ be the set of factors—i.e., parties—in the legislature and let $J \subseteq \{1, 2, \ldots, J\}$ denote the set of roll calls voted on. I am interested in modeling the decisions made at time $t = 1, 2, \ldots, T$ on the roll calls $j \in J$ by parties $i \in I$ in order to infer parties’ positions and their interdependencies in a unidimensional Euclidean policy space.⁴ In the simple spatial voting model, actors vote for the alternative closest to their respective policy position—denoted ideal

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³ Even though variational approximation is computationally superior to sampling-based Bayesian inference and a valuable alternative in many applications, it underestimates the variability in the posterior distribution (e.g., Grimmer 2011). Since the focus of this paper is on measurement uncertainty, this limitation makes it inapplicable in this context.

⁴ For simplicity, I assume the Euclidean policy space to be unidimensional. Previous studies find support for this assumption in several parliamentary multiparty systems (e.g., Hix and Noury 2016). Notwithstanding this, the proposed model can easily be expanded to the multidimensional case.
point—plus a random disturbance. Assuming a random utility function with quadratic loss, the utility of voting Yea on roll call $j$ for party $i$ with strategically adopted ideal point $\theta_{i,t} \in \mathbb{R}^1$ at time $t$ is given by $U_{i,t}(Y_j) = -(\theta_{i,t} - y_j)^2 + \xi^y_{i,j,t}$, where $y_j \in \mathbb{R}^1$ represents the location of the Yea vote in the one-dimensional policy space. Similarly, the utility function for voting Nay is $U_{i,t}(N_j) = -(\theta_{i,t} - n_j)^2 + \xi^n_{i,j,t}$, where $n_j \in \mathbb{R}^1$ is the location of the status quo. Both stochastic components $\xi^y_{i,j,t}$ and $\xi^n_{i,j,t}$ are drawn independently from a Gaussian distribution with zero mean and a fixed variance. In this basic model, a party votes in favor of a proposal if $U_{i,t}(Y_j) - U_{i,t}(N_j) > 0$.

Accordingly, the probability that party $i$ votes Yea (i.e., $y_{i,j,t} = 1$) can be written as a standard two-parameter IRT model:

$$
Pr(y_{i,j,t} = 1) = Pr((n_j - \theta_{i,t})^2 - (y_j - \theta_{i,t})^2) > \xi^y_{i,j,t} - \xi^n_{i,j,t} = \alpha_j + \beta_j \theta_{i,t} + \epsilon_{i,j,t},
$$

(2)

where $\alpha_j = -(y_j - n_j)$ is proposal $j$’s difficulty parameter and $\beta_j = 2(y_j - n_j)$ is the corresponding discrimination parameter. Because the variance of $\epsilon_{i,j,t}$ and the other model parameters are not separately identified in the likelihood, I fix it to 1. This common assumption can be found in standard probit models as well.

Building on this basic version of the dynamic model, the underlying behavioral model can be modified in order to incorporate the constitutional features of parliamentary systems. Legislators’ voting behavior in parliamentary systems differs sharply from the behavior of their colleagues in congressional systems. Previous research shows that party (and coalition) unity, for example, tends to be almost perfect in parliamentary systems (Laver 2006; Hug 2013; Stecker 2015; Bräuninger, Müller, and Stecker 2016). The institutional logic of parliamentary systems further results in a complex strategic relationship between the legislative and the executive and imposes specific tactical incentives for legislators which crucially affect their voting behavior on the floor (Laver 2006, 122f). Within this institutional setting, the number of meaningful positions that can be extracted reduces to the number of parliamentary parties. Legislative voting is also determined by the dualism of government and opposition (Bräuninger, Müller, and Stecker 2016; Hix and Noury 2016). It is crucial to consider these particularities. Otherwise, inferences drawn from recorded votes reveal structural conditions rather than policy platforms strategically adopted by office-motivated parties.

In order to account for the constitutional context of parliamentary systems, I adapt the “office model” derived by Bräuninger, Müller, and Stecker (2016, 195) to the dynamic model presented here. The resulting model has the following functional form:

$$
Pr(y_{i,j,t} = 1) = \Phi(\alpha_j + \beta_j \theta_{i,t} + (\delta_1 \chi^G_{i,j,t} - \delta_2 \chi^O_{i,j,t}) (\chi^G_{i,j,t} - \chi^O_{i,j,t})),
$$

(3)

where $\Phi(\cdot)$ is the standard normal cumulative density function, $\chi \in \{0,1\}$ are dichotomous variables indicating whether the proposal $j$ at time $t$ comes from the government ($\chi^G_{i,j,t}$) or the opposition ($\chi^O_{i,j,t}$), and whether party $i$ belongs to the government ($\chi^G_{i,j,t}$) or to the opposition ($\chi^O_{i,j,t}$). Hence, $\delta_1$ captures governmental parties’ gain and opposition parties’ loss of voting in favor of a governmental proposal. The coefficient $\delta_2$ represents governmental parties’ loss and opposition parties’ gain of affirming a proposal moved by the opposition. These terms capture the nonspatial,

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5 While Poole and Rosenthal (1985, 1991, 2001) originally assume that legislators are intrinsically policy-motivated and vote sincerely with respect to their most preferred policy position, this assumption is not necessary. As Martin and Quinn (2002, 138) already emphasize, if actors vote in a nonsincere, strategic fashion, as assumed by spatial theory where actors are solely office-seeking, the estimates can be interpreted as strategically adopted policy positions. These positions are likely to vary over time and rational actors consciously adjust them as a response to other actors’ behaviors as predicted by spatial theory.

6 For a more detailed discussions on this point, see for example Laver (2006).
tactical incentives that account for voting behavior on the floor (Bräuninger, Müller, and Stecker 2016, 195). Since the actors’ institutional environment does not change throughout the period under investigation, tactical incentives for parties occupying a specific position in the strategic setting of multiparty competition are not expected to change over time.

### 4.2 Spatio-temporal autoregressive evolution model

To model the process by which the strategically adopted ideal points evolve over time, I specify a theoretically informed evolution function. The innovation is that the observed party positions are not modeled to be mutually independent but rather are allowed to exhibit a spatial patterning in addition to a first-order temporal autocorrelation. The evolution equation captures this dependence structure by the specification of a STAR model (Franzese and Hays 2007, 2008; Hays, Kachi, and Franzese 2010). The ideal points’ evolution at time \( t \in \{2, 3, \ldots, T\} \) in the \( I \times 1 \) ideal point vector is given by

\[
\theta_t \sim N(\mu_t, \sigma^2_\theta).
\]

If a party did not cast a vote in any period, this equation serves as a predictive model and imputes the missing values for the ideal points. At \( t = 1 \), I set the evolution variance parameter to \( \sigma^2_\theta = 1 \) in order to set the scale of the latent space.\(^7\) At \( t > 1 \), this parameter determines the amount of smoothing that can take place from one period to another and is constant over time and for all parties. Since this parameter only depicts the upper bound of ideological shifts between two successive periods, this assumption does neither upwardly bias the ideological flexibility of parties with relatively stable preferences nor does it dampen the overall validity of the findings. I test different specifications of the evolution equations’ deterministic part in order to evaluate the effect of measurement uncertainty and to validate the findings. The deterministic part of the full model is given by the STAR model:

\[
\mu_t = \gamma + \phi_{t-1} + \rho_1 W^N_{t-1} \theta_{t-1} + \rho_2 W^F_{t-1} \theta_{t-1} \quad \forall t \in \{2, 3, \ldots, T\},
\]

where \( \theta_{t-1} \) is the \( I \times 1 \) dimensional first-order temporally lagged ideal point vector with its corresponding coefficient \( \phi \).

The remaining part of Equation (4) denotes the two spatial lags that are of central interest for the hypotheses tested here. Both are composed of temporally lagged and \( I \times I \) dimensional connectivity matrixes \((W^N_{t-1} \text{ and } W^F_{t-1} \forall t \in \{2, 3, \ldots, T\})\) in each period which have nonzero entrances for spatially connected parties and zeros elsewhere, the temporally lagged ideal point vector \( (\theta_{t-1}) \), and spatial coefficients \( \rho_1 \) and \( \rho_2 \). More formally, one spatial lag for party \( i \) at period \( t \) is given by:

\[
W_i \theta_{t,i} = \sum_{k \neq i}^{K} w_{i,k,t} \times \theta_{k,t}.
\]

where \( W_i \) denotes the spatial connectivity matrix (either \( W^N \) or \( W^F \)) at period \( t \) and \( \theta_{k,t} \) is the ideal point of party \( k \neq i \) at time \( t \). A crucial step in the analysis is the specification of connectivity matrixes. These matrixes specify (i) which observations spatially depend on each other and (ii) how they do so. This part of the spatial lag gives scholars full leverage to closely translate theoretical expectations into an empirical model and to test the expected spatial dependencies by specifying \( W \) (Franzese and Hays 2008; Neumayer and Plümper 2016).

The party dynamics hypothesis states that ideological neighbors are spatially connected. Thus, the elements of the connectivity matrix \( W^N_i \) are zero for all nonneighbors and a value larger than zero for spatial neighbors in each period. The ideological family hypothesis declares that parties within the same ideological family follow similar strategies and are more responsive to

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\(^7\) For the Socialist party, however, I set the evolution variance at \( t = 1 \) to 0.1 in order to solve the problem of reflection invariance (see Section 5).
ideological shifts made by other parties within the same ideological family. The connectivity matrix $W^F_t$ therefore includes zeros for all parties that are not in the same ideological family and nonzero values for family members. I further lag the spatially lagged dependent variable by one unit in time to account for delayed reactions to rival parties’ ideological movements (Williams 2015, 151).

The next question is how strongly the parties are spatially connected. Ideally, the theory would ultimately determine the exact specification of a connectivity matrix. Yet, as noted elsewhere (e.g., Plümper and Neumayer 2010; Williams 2015; Neumayer and Plümper 2016), there is no clear theoretical justification for any specific form of connectivity weights and the arbitrariness in the choice of a specific weighting scheme requires empirical justification through the implementation of robustness checks. Theoretical models of party competition predict that parties’ spatial interdependencies declines with relative proximity. Parties that are spatially closer have a stronger influence than parties that are further apart. Thus, I follow Williams (2015, 150f) and use the absolute distance between party $i$’s ideal point and all other parties $k \forall k \neq i \in I$ ideal points at period $t$ and subtract it from the maximum distance of all party dyads in each period. More formally, the nonzero cells of the connectivity matrices are given by

$$w_{i,k,t} = (\max_t - \bar{\theta}_{i,t} - \theta_{k,t})^x. \text{ The } x \text{ determines the functional form. This specification ensures that larger positive values indicate stark spatial interdependence whereas small positive values indicate little spatial interdependence. I test several different functional forms like a simple binary dummy } (x = 0), \text{ linear } (x = 1), \text{ and quadratic } (x = 2) \text{ in order to assess the effect of measurement uncertainty for different specifications of the spatial lag and to ensure that the results are not sensitive toward minor changes in the functional form. I do not row-standardize the connectivity matrices since this imposes the assumption of homogeneous exposure to spatial signals (Plümper and Neumayer 2010; Neumayer and Plümper 2016). If one party has fewer neighbors, for example, the weights of their neighbors’ distances will be higher which I consider to be hardly justifiable from a theoretical point of few.}

Due to the small number of parties, the varying number of votes in each period, and the number of parameters, the Bayesian framework is especially valuable in this context. It also facilitates the identifiability of the model (Jackman 2001; Martin and Quinn 2002; Clinton, Jackman, and Rivers 2004). I solve the problem of additive and multiplicative aliasing by normalizing the ideal points at time $t = 1$. The normalized ideal points are given by $\theta_{i,1}^{adj} = (\bar{\theta}_{i,1} - \bar{\theta}_1)/s_{\theta_i} \forall i \in I$, where $\bar{\theta}_1$ is the ideal points’ mean and $s_{\theta_i}$ denotes their variance at $t = 1$. In order to retain a common scale, I transform the difficulty and discrimination parameters as well. The adjusted parameters are $\alpha_j^{adj} = \alpha_j + \beta_j \times \bar{\theta}_1$ and $\beta_j^{adj} = \beta_j \times s_{\theta_j} \forall j \in J$. This transformation has the additional advantage that it reduces the correlation in posterior densities which leads to a faster convergence of the Markov chains and a more efficient estimation of the model (Bafumi et al. 2005, 176f). The prior distributions specified in Section 5 solve the identification problem of reflection invariance.

5 Application: Ideological Dynamics and Spatial Dependencies in the German State Parliaments

In order to assess the effect of measurement uncertainty in spatial econometric models and to emphasize the necessity of the proposed model for substantive research, I estimate the parties’ ideological positions using roll call votes from the sixteen German state parliaments. The German subnational party systems are an ideal case for several reasons. Although not perfectly indistinguishable, previous empirical research suggests that regional parties within the sixteen party systems at the German subnational level exhibit a homogeneous alignment of positions across states which permits the pooling of votes (Bräuninger and Debus 2012; Debus and Müller 2013). This cross-sectional pooling facilitates the estimation of a dynamic model with tactical voting incentives (see Equation (3)) since it yields multiple coalition patterns and
government-opposition configurations in each period. Similar cameral rules for requesting and selecting recorded votes as well as for agenda formation minimize the risk of a selection bias (Stecker 2015, 793).

The analysis is based on a dataset compiled by Bräuninger, Müller, and Stecker (2016). The dataset contains information about the outcome of recorded votes, derived from minutes of plenary proceedings in the German state legislatures, aggregated by parliamentary parties. I update the dataset by manually identifying missing information in the plenary protocols and by scraping an NGO’s website that lists all recorded votes within several German state parliaments. The final dataset comprises six parties with a total of 7,799 votes on 2,254 roll calls in the years from 1988 to 2016.

The Bayesian approach facilitates the model’s identifiability through the specification of priors (Jackman 2001; Clinton, Jackman, and Rivers 2004). The prior distributions for the parameters are based on the estimates from the static version of the model in Equation (3). The superscript “stat” indicates that the parameters are estimated by the static model. I solve the problem of reflection invariance by specifying a weakly informative prior for the ideal point of the Socialist party PDS at the initial period $t = 1$. Whereas the prior distribution for all other ideal points is given by $\theta_{i,1} \sim \mathcal{N}(\theta_{stat}^i, 1)$ for $i \neq PDS$, the prior for the ideal point of the PDS is $\theta_{PDS,1} \sim \mathcal{N}(\theta_{stat}^{PDS}, 0.1)$. Accordingly, the prior distribution for the other model parameters are given by $\alpha_j \sim \mathcal{N}(\alpha_{stat}, 1)$, $\beta_j \sim \mathcal{N}(\beta_{stat}, 1)$, $\delta_1 \sim \mathcal{N}(\delta_{stat}, 1)$, and $\delta_2 \sim \mathcal{N}(\delta_{stat}^2, 1)$. I further define a prior for the evolution variance parameter such that $\sigma^2 \sim IG(1, 0.1)$.

I use MCMC simulation to sample from the posterior density and to obtain inferences for the model parameters. After an initial adaption and burn-in phase of 30,000 iterations, three Markov chains draw 100,000 samples, thinned to every fifth observation in order to handle autocorrelation. I evaluate the convergence of the Markov chains via visual inspection of trace and autocorrelation plots and the potential scale reduction factor. In all models, the parameters’ potential scale reduction factors differ from 1 by at most 0.001 which suggests that the chains successfully converged to the posterior distribution.

5.1 Dynamic party positions
Before analyzing the two hypotheses about spatial dependencies within the German state parliaments explicated in Section 2, it is important to investigate parties’ ideological evolution. If party platforms are relatively stable, as Dalton and McAllister (2015) find, studying dynamics becomes technically difficult—and substantively meaningless—because there is only little variation that can be explained by any statistical model.

This section further evaluates the amount of uncertainty associated with the estimated positions. Scholars recognize the difficulties in inferring the true party positions from observable

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8 Replication materials are available at the Political Analysis Dataverse (Juhl 2018).
9 The information is publicly available at www.abgeordnetenwatch.de. Since the NGO is funded by donations, it does not include all German state parliaments. Parliaments that are included are the state parliaments of Baden-Wuerttemberg, Bavaria, Hessia, Lower Saxony, North Rhine-Westphalia, Rhineland-Palatinate, and Schleswig-Holstein.
10 The parties included in the analysis are the Christian Democrats (CDU), Christian Social Union (CSU), Liberals (FDP), Social Democrats (SPD), Greens (GRU), and the Socialists (PDS). However, I treat the CDU and the CSU as one single party since they form a political alliance and share a common parliamentary group in the Bundestag. At the state level, the CSU only contests elections in Bavaria while the CDU engage in electoral competition in the remaining 15 German states.
11 Supplementary materials A displays the ideal point estimates and the estimates for the tactical incentives ($\delta_1$ and $\delta_2$ in Equation (3)) derived from the static model as presented by Bräuninger, Müller, and Stecker (2016).
12 In June 2007, the party Partei des Demokratischen Sozialismus (PDS) and the party Arbeit & Soziale Gerechtigkeit – Die Wahlalternative (WASG) amalgamated and formed the party Die Linke. For the sake of clarity, I refer to this party as PDS throughout the paper.
13 The results are robust toward changes in the ideal points’ prior variances. Supplementary Materials B shows the spatial coefficients across all models when the prior variance at $t = 1$ equals the ideal points’ posterior spread from the static model.
14 I perform the analysis with JAGS version 4.2.0 (Plummer 2016) and the package runjags (Denwood 2016) in R (R Core Team 2016).
indicators such as party manifestos (e.g., Laver, Benoit, and Garry 2003; Benoit and Laver 2007; Busch 2016). While Laver and Sergenti (2012, 3f) already state that some variation in the positions might be due to measurement error, Dalton and McAllister (2015, 777f) conclude that, at least with regard to the popular estimates from the Comparative Manifesto Project, the largest part of this variation can be ascribed to imperfect measures. In the context of this paper, the amount of uncertainty accompanying the estimates is of central importance. If the uncertainty is high, subsequent analyses underestimate the estimates’ uncertainty and might therefore be severely biased.

Figure 1 visualizes the parties’ ideological evolution. It shows the parties’ strategically adopted ideal points (solid lines) and their 95% highest posterior density (HPD) intervals (dashed lines). The asterisks at the bottom of the figure depict the years in which no votes are recorded for the respective party. The estimates are obtained by the dynamic IRT model with a simple first-order Markov evolution function which serves as baseline model.\textsuperscript{15} Even after a brief inspection, the graphs clearly show that the ideal points of all parties exhibit a certain amount of mobility. This intuition finds further support by the estimated evolution variance parameter. This parameter illustrates the relative importance of within-party versus between-party variance since the observation equation variance is fixed at 1. Values close to zero would indicate that there is no ideological mobility while values approximating 1 indicate that the within-party variance is almost as large as the between-party variance and that parties are practically not restricted by their previous position. The estimate for this parameter is $\sigma^2 = 0.365$ with 95% of the posterior

\textsuperscript{15} The ideal point estimates do not change when I specify the evolution function according to the multiparametric STAR model in Equation (4). Supplementary materials C shows the ideal point estimates based on this full specification.
distribution’s mass between 0.202 and 0.601. In contrast to the findings presented by Dalton and McAllister (2015), there is a certain amount of ideological mobility.

Compared to the estimates from the static model ($\delta_{\text{stat}}^1 = 1.538$ 95% HPD interval of [1.412; 1.668] and $\delta_{\text{stat}}^2 = 2.355$ with 95% HPD in [2.24; 2.477]), both tactical incentives $\delta_1$ and $\delta_2$ in Equation (3) are substantially higher. The parameter of $\delta_1 = 1.879$ with a 95% HPD interval between 1.718 and 2.044 captures the governmental parties’ gain and the opposition parties’ loss to vote for a governmental proposal. Similarly, $\delta_2 = 2.799$ (95% HPD bounded by [2.665; 2.936]) is the governmental parties’ loss and opposition parties’ gain to vote yea on an opposition proposal. This suggests that the importance of the institutional setting as analyzed by Bräuninger, Müller, and Stecker (2016) increases when accurately accounting for dynamics in party positions.

Concerning measurement uncertainty, Figure 1 suggests that for some party years, identifying the true position based on the indicators is difficult and the point estimates are only rough approximations. Measurement uncertainty is directly reflected in the posterior distributions’ dispersion. Compared to the estimates for the other parties, the estimates for the Christian Democrats and the Greens are more precise with a maximum posterior standard deviation of 0.657 and 0.589, respectively, in 2016. The maximum standard deviations over the years for the Liberals (1.407 in 2016), the Social Democrats (0.994 in 2016), and the Socialists (0.855 in 1997) are higher, reflecting more uncertainty. In addition, measurement uncertainty also varies over time with an increase in more recent years. A potential cause of this increase is that the influence of the parties’ platforms on their voting behavior decreases. As a result, the observation equation (see Equation (3)) might not explain the behavior as adequately as in previous years. Figure 1 also illustrates the potential problem caused by the ideal points’ measurement uncertainty for subsequent spatial econometric analyses. The probability that the Conservatives’ ideal point is to the left of the ideal point of the Liberals in 1997, for example, is 65.78%. Hence, given the data and the priors, there is a huge amount of uncertainty about the parties’ relative positions. This causes problems when calculating the spatial lags, since there is uncertainty about where the parties are located within the policy space and who the neighboring parties are.

Taken together, these results suggest that there is both ideological mobility and a nontrivial amount of uncertainty in party positions. This uncertainty causes problems for specifying and calculating the spatial lag which has substantive implications for the empirical analysis of spatial dependencies. At worst, measurement uncertainty causes a disjunction between theory and empirics which impedes a proper evaluation of theoretical propositions.

5.2 Parties’ spatial dependencies
In line with the hypotheses considered here, I estimate three different specifications of the STAR model defined in Equation (4). The first model is the neighbor model, which includes only the temporally lagged dependent variable and the spatial lag with the connectivity matrix $W^N$ defined in Equation (5). The second model, the family model, includes a spatial lag with the connectivity matrix $W^F$ instead of $W^N$. The third model is the full model. It estimates both spatial lags simultaneously. As described earlier, the connectivity matrixes’ cells of the nonzero elements are given by $w_{i,k,t} = (\max_t |\theta_i,t - \theta_k,t|)^x$, where $x = 1$. The results presented here are robust toward the specification of connectivity matrixes with different functional forms.16

In order to test the severity of measurement uncertainty in spatial econometric models, I compare the results from a model that does not account for measurement uncertainty with the results obtained by the proposed model that combines a measurement model with the

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16 Since the raw coefficients of STAR models are not straightforward to interpret due to the theoretically informed decision not to row-standardize $W$ which causes spatial effect heterogeneity and the temporal dependence structure, I only provide them in Supplementary Materials D along with robustness checks for different specifications of the connectivity matrixes.
STAR models. To this end, I separately estimate the parties’ ideal points using the dynamic IRT model with a first-order Markov evolution and the three STAR models. I refer to these models as the *no error models*. I contrast these models’ parameter estimates with the estimates derived from the *Bayesian dynamic measurement model* that includes the corresponding evolution function. Figure 2 illustrates the marginal posterior densities of the spatial coefficients $\rho_1$ and $\rho_2$. The top row shows the densities derived from the *full model* with both spatial lags and the bottom row displays the densities derived from the *neighbor model* and the *family model*.

In contrast to theoretical predictions and unlike empirical findings obtained with manifesto data, the results show that the positions of the neighboring parties, if at all, negatively affect a focal party’s ideal point. Based on the posterior belief from the *full model*, the probability that the parameter estimate for neighboring parties is below zero is about 77.09% in the *no error model* and about 83.53% in the *Bayesian dynamic measurement model*. This negative estimate vanishes by only looking at the *neighbor model* (see the graph at the bottom left of Figure 2) which suggests that, as opposed to the electoral arena, the position of ideologically neighboring parties is of no substantive importance in the legislative arena. The *party dynamics hypothesis* finds no support here.

In line with the *ideological family hypothesis*, and with what Williams (2015, 152) reports, the positions of other parties within the same party family, however, do matter. While the probability of a positive spatial coefficient in the *no error model* is about 91.7%, the *Bayesian dynamic measurement model* estimates this probability at about 97.76%. Moreover, the spatial coefficient estimated while accounting for measurement error in parties’ ideological positions substantively increases from 0.011 with a 95% HPD interval between −0.005 and 0.027 to 0.043 (95% HPD in [0.001; 0.01]). Thus, by appropriately accounting for measurement uncertainty, the estimated spatial coefficient of the family members’ positions increases by a factor of almost 4. Based on the *no error model*, the probability that this coefficient is at least as high as the mean of the estimate from the *Bayesian dynamic measurement model* is only about $8.33 \times 10^{-3}%$. Hence, neglecting measurement uncertainty leads to substantively different conclusions about the strength of the parties’ interdependencies.
Figure 2 further illustrates how the larger standard deviations of the posterior distributions mirror the ideal points’ uncertainty. While the dispersion of the posterior distributions in all models are larger when accounting for measurement uncertainty, the point estimates of the no error models are closer to zero. The assumption of perfectly measured covariates spuriously decreases the posterior uncertainty. Researchers feel more certain about their results than they actually can be, given that latent constructs cannot be measured without error. Even more, the analysis performed here shows how the negligence of measurement uncertainty can lead to substantively different conclusions. Measurement uncertainty, thus, is not merely a methodological issue but can have profound implications for the substantive inferences drawn from spatial econometric models.

However, unlike standard linear models, parameter estimates from spatial econometric models cannot be interpreted like effect estimates (see also Franzese and Hays 2008, 760). Although the model estimates a common spatial coefficient, differences in the parties’ spatial configuration cause heterogeneous spatial effects because each party has a different neighboring scheme as captured by the connectivity matrix. In order to provide a substantive interpretation in terms of effect sizes, I simulate a counterfactual party system which shows how the ideological change of one focal party propagates through the whole party family. It further illustrates the effect of relative distances between parties. Consider a party system with five parties: Party A, Party B, Party C, Party D, and a focal party. The only party family within this system consists of Party A, Party B, and the focal party. Since this is a counterfactual party system, the positions of the parties are determined by design and, therefore, exactly known which would never be the case in any real-world party system. The horizontal axis in Figure 3 depicts the five parties and their ideological positions within a unidimensional policy space. This figure shows the scenario where the focal party shifts its ideal point from −0.5 to 0.5.

The vertical axis shows the predicted instantaneous shifts of the four other parties at time $t$, caused by the focal party’s shift at $t - 1$. It contrasts the estimates from the full specification of the no error model with the estimates derived from the Bayesian dynamic measurement model. The left part depicts the predicted shifts for the neighboring parties. Clearly, the model which assumes perfectly measured covariates underestimates both the spatial effect and the uncertainty associated with it. Zero is a credible value for the predicted shift of neighboring parties which indicates that parties are not systematically affected by their neighboring parties’ ideological shifts in the legislative arena.

Figure 3. Predicted shifts in a counterfactual party system based on the full model.
The right part of Figure 3 shows the predicted shifts for the parties within the same party family. All else being equal, the focal party’s shift affects Party B stronger than Party A while it does not affect Party C and Party D because they do not belong to the party family. Based on the model which accounts for measurement uncertainty, the predicted shift of Party A is 0.087 (95% HPD bounded by the interval [0.001; 0.2]) while Party B shifts its position by 0.13 with 95% of the posterior mass between 0.002 and 0.299. This example illustrates how the predictions change with relative distances, leading to heterogeneous spatial effects. However, these changes are moderate in strength. Even if Party B respond to the focal party’s movement by changing its position from −1 by the highest credible value (0.299) to the new position of −0.701, this movement is only about 7.48% of the latent policy space. Nevertheless, this figure illustrates that the no error specification would not only greatly underestimate the spatial coefficient but also the size of the spatial effect.

6 Discussion and Conclusion

Spatial econometric models provide an excellent opportunity to empirically model theoretical predictions about parties’ interrelatedness derived from spatial theory. Yet, the direct translation of theoretical expectations into these models is not as straightforward as previous research suggests. This article demonstrates how the latent character of parties’ ideological positions violates the implicit assumption of perfectly measured covariates and causes a mismatch between theory and empirics. The model presented here combines a spatio-temporal autoregressive model with a measurement model in order to avoid this assumption.

By applying the model to data on recorded votes from the German state parliaments and comparing a model which assumes no measurement error to the model presented here, this paper reveals how imperfectly measured covariates can jeopardize statistical inferences. Neglecting random measurement error in spatial econometric models greatly underestimates the uncertainty surrounding the estimates and the substantive effect sizes. However, it is important to stress the fact that measurement uncertainty can have adverse effects on the estimates and it is impossible to know a priori whether the estimated effect sizes are exaggerated or attenuated (Blackwell, Honaker, and King 2017; Loken and Gelman 2017). As a result, measurement uncertainty is not merely a methodological concern but has profound implications for the conclusions researchers draw based on their models.

Substantively, the application presents evidence for both a varying amount of uncertainty surrounding ideal point estimates and positional dynamics. Ideological mobility is not just the result of poor measures as Dalton and McAllister (2015) suggest but empirical reality. By properly accounting for these dynamics, the importance of the institutional environment in which parties are embedded increases. Tactical incentives are even more important for the behavior of parties than Bräuninger, Müller, and Stecker (2016) conclude. The results further point toward important differences between parties’ competitive behavior in the electoral and the legislative arena. While parties are sensitive toward ideological movements made by their family members in both contexts, the behavior of their ideological neighbors has no effect in the legislative arena. Hence, the present study shows how the parties’ strategic calculations are contingent on the context they act in. While differences in the partisan composition of the government facilitates the simultaneous estimation of tactical incentives and dynamic ideal points, there is no reason to believe that these findings are restricted to the German system and subsequent analyses may investigate other parliamentary systems as well.

These results point toward a promising direction for future research on party competition. While previous research primarily focuses on party positions derived from manifestos, this study explicitly looks at legislative voting behavior. Within the legislative context, parties are more responsive to institutional incentives than to their ideological neighbors. Yet, this does not necessarily hold for competition in the electoral arena. Thus, asking if parties consciously...
respond to movements made by rival parties obscures the more nuanced characteristics of party competition. Asking when parties strategically respond to their political opponents provides a more promising way to study the endogenous dynamics of party competition.

The model presented here is applicable to a broad range of empirical phenomena. Since researchers can arbitrarily specify the evolution function, they can exploit this Bayesian dynamic measurement model in many substantive fields where latent quantities are of central interest. Political scientists are just beginning to utilize the powerful tools of spatial econometric techniques. This study contributes to a further methodological development in this direction by fostering the adaptation of these techniques to contexts which are of key interest to political scientists. Thereby, it paves the way for a sound integration of theoretical expectations into empirical models.

Supplementary material
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References


