Reduced magneto-hydrodynamic theory of coherent magnetic chains in the solar wind

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An analytic theory is presented of magnetic structures in collisionless, high- β plasmas. Using a reduced magnetohydrodynamic model, a stationary nonlinear solution is constructed in the form of a Kelvin–Stuart cat’s eyes chain of magnetic islands, on the spatial scale that exceeds the characteristic ion lengths. The solution is imbedded in a background current sheet and possesses both a significant plasma density perturbation trapped inside the magnetic islands and a compressional magnetic field component that is driven mostly by a current loop located at the separatrix of the islands. This theory may provide an explanation for the magnetic structures observed in the solar wind close to the Earth by the Cluster spacecraft.

Key words: astrophysical plasmas, plasma nonlinear phenomena, space plasma physics

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1. Introduction

Current sheets are ubiquitous in space plasmas, such as the slow and the fast solar winds, planetary magnetosheaths, etc., see . Muñoz *et al.* (2010), Lion, Alexandrova & Zaslavsky (2016), Perrone *et al.* (2016, 2017) and references therein. In and around thin current sheets, energy dissipation and particle heating usually occur. The presence of planar structures (current sheets, rotational discontinuities and shocks) is thought to be the main source of intermittency in the solar wind at ion scales, i.e. of the non-Gaussian and bursty occurrences in the turbulent fluctuations that are inhomogeneous and non-uniform in space. One-dimensional current sheets have been identified in different regimes of the turbulent cascade. The current sheets are aligned with the local magnetic field. Recent analyses of the Wind and Cluster data (Lion *et al.* 2016; Perrone *et al.* 2017) have revealed a variety of magnetic structures in the fast solar wind, most frequently Afvénic vortices with $|\delta\mathbf{B}_\perp| \gg |\delta\mathbf{B}_\parallel|$ (where $\delta\mathbf{B}$ is the perturbation of the magnetic field and the subscripts $\perp$ and $\parallel$ denote the vector

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components that are perpendicular and parallel to the background magnetic field $\mathbf{B}_0$, respectively), but the presence of current sheets aligned with the local magnetic field has also been recorded. While in the fast solar wind they are pressure balanced and almost incompressible, the structures in the slow solar wind are moderately, $|\delta \mathbf{B}_\perp| \sim |\delta \mathbf{B}_\parallel|$, or even strongly compressible, $|\delta \mathbf{B}_\perp| \ll |\delta \mathbf{B}_\parallel|$, implying that the natures of the slow and fast winds are different.

Sheared magnetic fields supported by current sheets contain free energy which makes them prone to the reconnection instability. In resistive magnetohydrodynamic (MHD) turbulence, the absence of characteristic scales produces a well-defined power-law behaviour of the spectrum of turbulent fluctuations, while the scale of the coherent structures responsible for intermittency, notably the thickness of the current sheets, is of the order of the dissipation length ℓ_d . However, in the solar wind the collision mean free path is of the order of the radius of the Earth's orbit and its resistivity and the viscosity are extremely low. This permits the solar wind turbulence to develop all the way down to characteristic plasma scales, allowing the onset of coherent structures such as current sheets, within a broad range of spatial scales. The smallest characteristic size of the structures appears to be a few ion inertial lengths λ_i or ion Larmor radii ρ_i , where $\lambda_i = c/\omega_{pi}$ and $\rho_i = v_{T,i}/\Omega_i$. Here $\Omega_i = eB_0/m_i$ is the ion gyrofrequency and $\omega_{pi} = \sqrt{n_0 e^2/m_i \epsilon_0}$ is the ion plasma frequency, while e , m_i and $v_{T,i}$ are the ion charge, mass and thermal speed, respectively.

Particle in cell (PIC) simulations (Cerri & Califano 2017) have shown that large-scale turbulent motions establish a $-5/3$ spectrum at scales bigger than λ_i and simultaneously feed the formation of current sheets at places where magnetic reconnection turns out. As a result, coherent magnetic structures are generated that together with the associated small-scale non-ideal electric field mediate the transition between the inertial and the subproton-scale spectra. The signature of a collisionless reconnection event has been recognized in the pair of large-scale currents sheets with characteristic features of the magnetic reconnection exhausts, observed by the Cluster mission in the solar wind upstream of the Earth's bow shock (Muñoz *et al.* 2010). Likewise, the chains of magnetic vortices (or islands) with the spatial scale of several ion lengths observed by Cluster, see figure 13 of Perrone *et al.* (2017) (chain of three adjacent vortices) and figure 5 of Perrone *et al.* (2016) (linearly polarized compressive chain of magnetic holes), could be the results of past reconnection events, but they might have been created also in the turbulence cascade simultaneously with the current sheets in which they are imbedded, since such configuration is energetically favourable compared to the current sheet alone.

Two decades ago, a simple hydrodynamic theory (Jovanovic & Vranjes 1994; Jovanović & Pegoraro 1998) was developed that recovered the general features of the magnetic structures (chains) in low- β plasma, $\beta \ll m_e/m_i$ [where β is the ratio of the thermodynamic and magnetic pressures, $\beta \equiv 2(n_e T_e + n_i T_i)/(c^2 \epsilon_0 B^2)$] and on the electron scale, $\lambda_e = c/\omega_{pe}$, created in the fast reconnection, as found in simulations (Cafaro *et al.* 1998; del Sarto, Califano & Pegoraro 2003; Porcelli *et al.* 2004). However, the reconnection is an inherently multi-scale process that involves both kinetic phenomena, on electron scales, and magnetohydrodynamic phenomena. The energy conversion and dissipation processes (anomalous resistivity, Hall effect, gyro-viscosity, resonant electrons) that lead to the magnetic field line break occurs on microscopic (electron) scales, while the energy is stored and the plasma responds on global (bigger than ion) scales (Hesse *et al.* 2011; Zweibel & Yamada 2016). In this paper, we develop an analytic theory of nonlinear magnetic structures in large- β plasmas, $\beta \sim 1$, using the reduced MHD model (Kadomtsev & Pogutse 1973,

1974; Strauss 1976, 1977) and construct a vortex chain solution that possesses a compressional magnetic field and a finite perturbation of plasma density. In contrast to the low- β shear-Alfvén vortex chain (Jovanovic & Vranjes 1994; Jovanović & Pegoraro 1998) that is governed by the nonlinearity coming from the equilibrium sheared field profile, our solution is of the ‘cat’s eyes’-type and determined by the nonlinearity associated with the trapping of the plasma inside the magnetic islands. Its scale length is (much) bigger than the ion scale and its macroscopic profile is in qualitative agreement with the coherent structures observed by the Cluster spacecraft in the solar wind, close to Earth.

2. Reduced MHD model and stationary states

We study electromagnetic perturbations of a warm plasma with the unperturbed density n_0 , immersed in a background magnetic field that consists of a strong homogeneous guide field $\mathbf{B}_0 = B_0 \mathbf{e}_z$ and a small sheared component $\delta \mathbf{B}_\perp(x, y)$ with $|\delta \mathbf{B}_\perp| \ll B_0$. Perturbations that are slow compared to the ion gyrofrequency $d/dt \ll \Omega_i$, weakly z -dependent $\partial/\partial z \ll \nabla_\perp$ and whose perpendicular characteristic length exceeds the ion length and the ion Larmor radius $\nabla_\perp^{-1} \gg \max(c/\omega_{pi}, v_{Ti}/\Omega_i)$, are described by the standard Kadomtsev–Pogutse–Strauss equations of reduced MHD (Kadomtsev & Pogutse 1973, 1974; Strauss 1976, 1977).

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0} (\mathbf{e}_z \times \nabla \phi) \cdot \nabla \right] \nabla_\perp^2 \phi + c_A^2 \left[\frac{\partial}{\partial z} - \frac{1}{B_0} (\mathbf{e}_z \times \nabla A_z) \cdot \nabla \right] \nabla_\perp^2 A_z = 0, \quad (2.1)$$

$$\frac{\partial A_z}{\partial t} + \left[\frac{\partial}{\partial z} - \frac{1}{B_0} (\mathbf{e}_z \times \nabla A_z) \cdot \nabla \right] \phi = 0. \quad (2.2)$$

Placing the reduced MHD equations in perspective relative to other reduced fluid models, Schekochihin *et al.* (2009) showed that they are valid for Alfvénic fluctuations (which decouple from density fluctuations) in a collisionless plasma at scales that are large compared to the ion scales, while their appropriateness for the study of magnetic reconnection at large scales was demonstrated in Eyink, Lazarian & Vishniac (2011).

In (2.1), (2.2) ϕ is the streamfunction (i.e. the electrostatic potential) and A_z is the z -component of the vector potential. The compressional component of the magnetic field perturbation, $\delta B_z \equiv B_z - B_0$, is governed by the pressure balance, $n_e T_e + n_i T_i + c^2 \epsilon_0 B^2/2 = \text{constant}$, where n_j and T_j are the density and the temperature of the particle species j ($j = e, i$). We study nonlinear processes whose characteristic perpendicular velocity of propagation is smaller than the ion thermal velocity, $d/dt \ll v_{Ti} \nabla_\perp$. For simplicity, they can be regarded as essentially isothermal and we set $T_e = T_i \equiv T = \text{constant}$. As the characteristic length is much bigger than the Debye length, $\nabla_\perp^{-1} \gg v_{Te}/\omega_{pe}$, we can neglect the charge separation and set $n_e = n_i \equiv n$. Then, for small perturbation of the plasma density and of the magnetic field intensity, $\delta n \ll n_0$ and $\delta B_z \ll B_0$, the pressure balance reduces to

$$2\beta \delta n/n_0 + \delta B_z/B_0 = 0. \quad (2.3)$$

Basic equations (2.1)–(2.3) are valid if $(v_{Ti}^2/\Omega_i^2) \nabla_\perp^2 = (\beta/2) (c^2/\omega_{pi}^2) \nabla_\perp^2 \ll 1$, that is satisfied if either plasma β is sufficiently small or the spatial scale sufficiently exceeds the ion inertial length c/ω_{pi} . In the solar wind, typical β -values lie in the range $1 \gtrsim \beta \gtrsim 0.1$ and we can set $(v_{Ti}^2/\Omega_i^2) \nabla_\perp^2 \rightarrow 0$ on a scale just 3–5 times bigger than the ion inertial length. Under such conditions we can neglect all thermal terms in ion

fluid equations [for details see e.g. Jovanovic *et al.* (2017)] and the ion continuity reduces to

$$\left[\frac{\partial}{\partial t} + \frac{1}{B_0} (\mathbf{e}_z \times \nabla \phi) \cdot \nabla \right] \left(\frac{\delta n}{n_0} - \frac{\delta B_z}{B_0} - \frac{\nabla_{\perp}^2 \phi}{\Omega_i B_0} \right) = 0, \quad (2.4)$$

where the second and the last terms come from the divergence of the $\mathbf{E} \times \mathbf{B}$ and the ion polarization drift, respectively. Equation (2.4) is solved as $\delta n/n_0 = \delta B_z/B_0 + \nabla_{\perp}^2 \phi / \Omega_i B_0$, which together with (2.3) yields

$$\frac{\delta B_z}{B_0} = -\frac{2\beta}{2\beta + 1} \frac{\nabla_{\perp}^2 \phi}{\Omega_i B_0}. \quad (2.5)$$

In low- β plasmas with $\beta \ll m_e/m_i$ and $\beta \ll 1$, the two-fluid version of (2.1), (2.2) were derived in Jovanovic *et al.* (1987), Schep, Pegoraro & Kuvshinov (1994) that included the shear-Alfvén phenomena on the electron inertial scale $c/\omega_{p,e}$. Their numerical solutions in configurations with magnetic shear and strong guide magnetic field (Cafaro *et al.* 1998; del Sarto *et al.* 2003; Porcelli *et al.* 2004), revealed the unfolding of collisionless (or fast) magnetic reconnection. The resulting growth of magnetic islands is followed by a very fast development of thin current sheets parallel to the guide field, whose thickness is comparable to or smaller than the electron plasma length $c/\omega_{p,e}$. A possible stationary state, associated with the fast reconnection, was studied analytically in Jovanovic & Vranjes (1994), Jovanović & Pegoraro (1998). It featured the usual chain of magnetic islands imbedded in the sheared magnetic field, coupled with a double chain of hydrodynamic vortices that fed the plasma into the regions around x -points, where the plasma had been depleted as the result of reconnection. Two-fluid equations for shear-Alfvén phenomena in low- β plasmas (Jovanovic *et al.* 1987; Jovanovic & Vranjes 1994) are very similar to the reduced MHD equations (2.1) and (2.2). The difference comes from a dispersive term due to the electron inertia in the parallel electron momentum equation that is neglected in (2.2) in the MHD scaling. The absence of dispersion prevents the appearance of a simple vortex chain (Jovanović & Pegoraro 1998) on the MHD spatial scale.

The Poisson bracket form of its nonlinear terms permits the reduced MHD system (2.1), (2.2) to be expressed in Hamiltonian form to which the Hamiltonian mechanics methodology was applied (Morrison & Greene 1980) and a number of invariants were found. The existence of these Casimir invariants can be used for the derivation of stability criteria for nonlinear solutions, see e.g. Jovanović & Horton (1994) for the stability study of electron scale shear Alfvén structures. As proposed by Morrison & Hazeltine (1984), Morrison, Caldas & Tasso (1984), the Hamiltonian formalism is also helpful in the construction of exact nonlinear reduced MHD equilibria using Casimirs, via the variational principle.

In this paper, using the method of Jovanovic & Vranjes (1994), we construct exact stationary reduced MHD stationary solutions whose characteristic length in the transverse direction exceeds the ion spatial scale. The stability study of these structures requires extensive three-dimensional numerical calculations and is outside of the scope of the present work. We seek a solution that is stationary in the Galilean frame travelling with the velocity $\mathbf{e}_y u_y$, which is strictly two-dimensional and tilted relative to the unperturbed magnetic field. We assume that the potentials depend only on two variables, *viz.* $\phi = \phi[x, y + (u_y/u_z)z - u_y t]$ and the same expression

for A_z . In other words, we seek a solution that is moving in the direction of the y axis with the velocity u_y , which is strictly two-dimensional in a plane that contains the x -axis, and is tilted to the z axis by the small angle $\arctan(u_y/u_z) \ll 1$. Using $\partial/\partial t = -u_z \partial/\partial z = -u_y \partial/\partial y = -(\mathbf{e}_z \times \nabla u_y x) \cdot \nabla$ equations (2.1) and (2.2) take the form

$$[\mathbf{e}_z \times \nabla(\phi - B_0 u_y x)] \cdot \nabla \nabla_\perp^2 \phi - c_A^2 \{\mathbf{e}_z \times \nabla[A_z - B_0(u_y/u_z)x]\} \cdot \nabla \nabla_\perp^2 A_z = 0, \quad (2.6)$$

$$(\mathbf{e}_z \times \nabla B_0 u_y x) \cdot \nabla A_z + \{\mathbf{e}_z \times \nabla[A_z - B_0(u_y/u_z)x]\} \cdot \nabla \phi = 0. \quad (2.7)$$

Conveniently introducing the dimensionless quantities

$$\mathbf{r}' = \mathbf{r}/l, \quad u'_z = u_z/c_A, \quad \varphi = \phi/(B_0 u_y l) - x/l, \quad (2.8a-c)$$

$$\psi = c_A A_z/(B_0 u_y l) - x c_A/(u_z l) + \psi_0, \quad \delta B'_z = c_A \delta B_z/(B_0 u_y), \quad (2.9a,b)$$

where l is an arbitrary length and ψ_0 is an arbitrary constant, equations (2.5)–(2.7) are rewritten as

$$\delta B'_z = -\sigma \nabla_\perp^2 \varphi, \quad \text{where } \sigma = (l \omega_{p,i}/c)[2\beta/(2\beta + 1)], \quad (2.10)$$

$$(\mathbf{e}_z \times \nabla' \varphi) \cdot \nabla' \nabla_\perp'^2 \varphi - (\mathbf{e}_z \times \nabla' \psi) \cdot \nabla' \nabla_\perp'^2 \psi = 0, \quad (2.11)$$

$$(\mathbf{e}_z \times \nabla' \psi) \cdot \nabla' \varphi = 0. \quad (2.12)$$

In the rest of the text, we omit the primes for simplicity. Equation (2.12) is readily integrated as

$$\varphi = F(\psi), \quad (2.13)$$

which after substitution into (2.11) gives

$$F'(\psi) \nabla_\perp'^2 \varphi - \nabla_\perp'^2 \psi = G(\psi), \quad (2.14)$$

where F and G are arbitrary functions of their argument and F' is the first derivative. These functions will be determined below from the appropriate asymptotic conditions for the magnetic field and for the plasma flow. For simplicity, we will study only field-aligned structures and in the rest of the paper we take $u_y/u_z = 0$.

Finally, substituting (2.13) into (2.14), we arrive at the following equation for the vector potential

$$\nabla_\perp \cdot [|F'^2(\psi) - 1|^{1/2} \nabla_\perp \psi] = -\frac{G(\psi)}{|F'^2(\psi) - 1|^{1/2}}, \quad (2.15)$$

which can be interpreted as a stationary, Liouville-like equation of heat transport (Polyanin & Zaitsev 2012), with a nonlinear source

$$\nabla_\perp^2 p(x, y) = g[p(x, y)], \quad (2.16)$$

in which the functions p and g are defined as

$$p(x, y) = P[\psi(x, y)], \quad g(p) = -\frac{G[P^{-1}(p)]}{|F'^2[P^{-1}(p)] - 1|^{1/2}}, \quad (2.17a,b)$$

with the function $P(\psi)$ defined through

$$P(\psi) = \int d\psi |F'^2(\psi) - 1|^{1/2}, \quad (2.18)$$

and P^{-1} being the inverse function. We note that, since the right-hand side of (2.15) is an arbitrary function of ψ , $g(p)$ is also an arbitrary function of its argument.

3. Chain of magnetic islands

If the source term $g(p)$ in our Liouville-like equation is adopted as $g(p) = A\kappa^2(1 - \gamma^2) \times \exp(-2p/A)$, equation (2.16) has the analytic solution in the form

$$p(x, y) = A \log \left(\frac{\cosh \kappa x + \gamma \cos \kappa y}{1 + \gamma} \right), \quad (3.1)$$

where without a loss of generality we may adopt $A > 0$ and $\kappa > 0$. The Kelvin–Stuart cat’s eyes solution, equation (3.1), has the form of a chain of islands imbedded in the sheared field p , which asymptotically, for $x \rightarrow \pm\infty$, behaves as $p \rightarrow A\kappa|x|$. The isolines $p(x, y) = p_c = \text{constant}$ are open (i.e. they stretch to the infinity in the x, y plane) for $p_c > 0$ and close on themselves when $p_c < 0$. Here γ is a free parameter that determines the amplitude of the islands. The cat’s eyes (3.1) have the same characteristic lengths along the vortex chain and perpendicularly to it, but chains with different parallel and perpendicular characteristic lengths are also possible, see Jovanovic & Vranjes (1994) for an elongated shear-Alfvén chain with small amplitude islands.

For a well-behaved function $P(\psi)$, the isolines of the vector potential, $\psi(x, y) = P^{-1}[p(x, y)] = \text{constant}$, coincide with the isolines of $p(x, y)$. Thus, the magnetic field resulting from the cat’s eyes solution, equation (3.1), features an inhomogeneous magnetic field which imbeds a chain of magnetic islands (possibly resulting from the saturation of a magnetic reconnection). We adopt an appropriate function $F(\psi)$ (that from (2.18) determines also the function $P(\psi)$), such that the dimensionless magnetic field $\delta\mathbf{B}$ and the flow velocity $\mathbf{v}$ have the asymptotic behaviour that corresponds to a current sheet with a finite thickness and to the absence of large-scale plasma flows

$$\left. \begin{aligned} \delta\mathbf{B} &= -\mathbf{e}_z \times \nabla_\perp P^{-1}[p(x, y)] \rightarrow \mathbf{e}_x A \kappa \tanh \kappa x, \\ \mathbf{v} &= \mathbf{e}_z \times \nabla_\perp [x + F(\psi)] \rightarrow 0. \end{aligned} \right\} \quad (3.2)$$

Taking that the function $F(\psi)$ tends to be linear when $\psi \rightarrow \infty$, i.e. that $\mathbf{v} = 0$ at $x \rightarrow \infty$, we have the following asymptotic expressions

$$\varphi = F(\psi) \sim F'(\infty)\psi \sim F'(\infty)|F'^2(\infty) - 1|^{-1/2}p, \quad (3.3)$$

which, after substitutions $\varphi \rightarrow -x$ and $p \rightarrow A\kappa|x|$ gives

$$F'(\infty) \rightarrow -\frac{\text{sign}(x)}{\sqrt{1 + A^2\kappa^2}}. \quad (3.4)$$

Obviously, the function $F(\psi)$ is multivalued, with different asymptotic forms at $x \rightarrow \pm\infty$. Transition between its branches occurs at the magnetic separatrix, determined by $P[\psi(x, y)] = 0$. The model function $F(\psi)$, featuring the appropriate asymptotic behaviour, can be adopted as

$$F(\psi) = F_1 \Psi \log \left(\cosh \frac{\psi}{\Psi} \right), \quad F_1 = \begin{cases} F_1^{\text{out}}, & \psi > 0, \\ F_1^{\text{in}}, & \psi < 0, \end{cases} \quad (3.5a,b)$$

where $\Psi > 0$ is an arbitrary constant. In the above, we allow the coefficient F_1 to take different constant values outside and inside the magnetic islands (i.e. for $\psi > 0$ and $\psi < 0$), respectively, denoted by the superscripts ‘out’ and ‘in’. These are given by

$F_1^{\text{out}} = F'(\infty) = -\text{sign}(x)/\sqrt{1 \pm A^2 \kappa^2}$ and $F_1^{\text{in}} = 0$. One easily verifies that for $\Psi \neq 0$, the function $F(\psi)$ is smooth, since it satisfies $F'(0^+) = F'(0^-) = 0$. Using (2.18), we obtain also the corresponding function $P(\psi)$, viz.

$$P(\psi) = F_1 \Psi \arcsin \left(F_1 \tanh \frac{\psi}{\Psi} \right) + \frac{\Psi}{2} \sqrt{1 - F_1^2} \left[\frac{2\psi}{\Psi} + \log \left(\frac{1 - F_1^2 \tanh \frac{\psi}{\Psi} + \sqrt{1 - F_1^2} \sqrt{1 - F_1^2 \tanh^2 \frac{\psi}{\Psi}}}{1 + F_1^2 \tanh \frac{\psi}{\Psi} + \sqrt{1 - F_1^2} \sqrt{1 - F_1^2 \tanh^2 \frac{\psi}{\Psi}}} \right) \right]. \quad (3.6)$$

This function is rather complicated and difficult to invert in the general case. To proceed analytically, we restrict ourselves to the limit $0 < \Psi \ll 1$, when we have $\Psi \log[\cosh(\psi/\Psi)] \rightarrow |\psi|$ and $F(\psi)$ simplifies to a part-by-part linear function with the constant slopes F_1^{out} and F_1^{in} outside and inside the magnetic islands, respectively. Then, potentials ψ and φ can be written in a simple form

$$\psi(x, y) = p(x, y) |F_1^2 - 1|^{-1/2}, \quad \varphi(x, y) = F_1 \psi(x, y), \quad (3.7a, b)$$

where p is given by (3.1) and $F_1 = F_1^{\text{in}}$ or F_1^{out} . The above streamfunction φ corresponds to a localized plasma flow that is laminar, i.e. without closed isolines. Inside the magnetic separatrix we have $\varphi = 0$, which implies that the fluid velocity is equal to the velocity of the coherent structure, i.e. plasma is trapped inside the magnetic islands and travels together with them.

The solution (3.7) is an approximation that provides a qualitative outline of the spatial profiles of the potentials ψ and φ . However, our simple choice $\Psi \rightarrow 0$ in (3.6), corresponding to a function $F(\psi)$ that is not smooth, gives rise also to the potentials ψ and φ that have discontinuous gradients at the magnetic separatrix, and to the presence of unphysical surface charges and surface currents at this location. Such apparent singularity vanishes if instead of $F(\psi) = F_1 |\psi|$, we use a smooth model function (3.5) that at the magnetic separatrix produces a current sheath and/or a plasma jet with finite width, determined by the parameter Ψ . On the electron spatial scale such singularity is naturally resolved (Jovanovic *et al.* 1987; Jovanovic & Vranjes 1994), since the additional dispersion coming from the electron inertia permits the arbitrary function $F(\psi)$ to have the same analytic expression on both sides of the separatrix.

Our results can be used to explain the structures observed in and around the current sheets in space plasmas, e.g. in the slow and the fast solar winds, planetary magnetosheaths, etc. They are displayed in figure 1, in a form that mimics the measurements made by four observers moving along straight lines that intersect the magnetic chain at the angle θ . We used the following set of typical dimensionless parameters: $A = 1.6$, $\kappa = 2.5$, $\gamma = 0.2$, $\sigma = 0.4$. The distance between neighbouring satellite trajectories is taken as $\Delta d = 0.15$ and their angle to the chain's axis $\theta = \pi/6$. Furthermore, for an easier comparison with the Cluster observations described in § 4 below, we plot our results in the 'solar wind reference frame' i.e. we decompose the magnetic field obtained from (3.1) and (3.7) into components that are parallel and perpendicular to an arbitrary vector $V_\perp$ perpendicular to the background magnetic field $B_0 \mathbf{e}_z$, viz. $\delta B_{R_{y1}} = \mathbf{e}_{V_\perp} \cdot [\nabla \times \mathbf{e}_z(\psi - \psi_X)]$ and $\delta B_{R_{x1}} = \mathbf{e}_{V_\perp} \cdot \nabla(\psi - \psi_X)$. Here

FIGURE 1. (a) “Cat’s eyes”. The contour plot (colour legend on top) of the vector potential ψ , see (3.1) and (3.7). The parameters are chosen as $A = 1.6$, $\kappa = 2.5$, $\gamma = 0.2$ and $\sigma = 0.4$. Four black parallel lines represent the trajectories of four spacecraft, C1–C4 (from left to right). (b–e) Rapid magnetic field $\delta \mathbf{B}_R = \nabla \times \mathbf{e}_z(\psi - \psi_X) + \mathbf{e}_z \delta B_z$, as would be recorded by four observers (‘Cluster satellites’) travelling along the trajectories indicated by parallel black lines in the first panel, with the typical parameters: distance between trajectories $\Delta d = 0.15$; angle to the chain’s axis $\theta = \pi/6$. The x_1 , y_1 , and z components of the rapid magnetic field are displayed in black, red and blue, respectively. The normalizations are given in (2.9).

FIGURE 2. The variation of the magnetic field intensity $|B|$. Data recorded by the four Cluster satellites C1–C4 are displayed as black, red, green and blue lines, respectively.

$V_{\perp}$ denotes the solar wind velocity component perpendicular to the background magnetic field, $e_{V_{\perp}}$ is a unit vector $e_{V_{\perp}} = V_{\perp}/|V_{\perp}|$, and $\psi_X(x)$ is the background, or the envelope vector potential (denoted in § 4 as the ‘1 s average’), given by $\psi_X(x) = A \log[(\cosh \kappa x + \gamma)/(1 + \gamma)]$ (see (3.1)). Parallel magnetic field perturbation δB_z is given by (2.10).

4. Cluster observations

As an illustrative example, we present results of satellite observations that are conformal with our structure (3.6)–(3.7), albeit not being of an exceedingly large size, *viz.* $c^2 \nabla_{\perp}^2 / \omega_{pi}^2 \sim 0.1$ – 0.25 . The measurements were done in the slow solar wind in the time interval analysed also in Perrone *et al.* (2016), using the fluxgate magnetometer (FGM) and the Spatio Temporal Analysis of Field Fluctuations experiment (STAFF) on board the Cluster satellites, whose description is given in Balogh *et al.* (2001), Cornilleau-Wehrin *et al.* (2003). A detailed explication of the measurements will be given in a separate publication, and here we present only those data that are relevant for our present study. The observed magnetic structures, detected as in Alexandrova *et al.* (2006), Alexandrova & Saur (2008), are displayed in figures 2–4. They feature a gentle ramp in the magnetic field intensity $|B| - \langle |B| \rangle_{10s}$, where $\langle \cdot \cdot \rangle_{\tau}$ denotes the moving average over the period τ , *viz.* $\langle f(t) \rangle_{\tau} = \int_{t-\tau/2}^{t+\tau/2} dt' f(t')$. The average strength of the magnetic field was evaluated adopting $\tau = 10$ s. An approximately linear growth by 25 % (from ~ 6 to ~ 7.5 nT) in the course of ~ 4 s was observed. The velocity of the slow solar wind was $V \sim 330$ km s $^{-1}$ while the velocity of the structures was calculated from the time delay between the signals recorded on the four spacecraft. It was established that the structures were mostly carried by the solar wind, since using two different methods their velocity in the plasma frame was determined to be 18 ± 32 km s $^{-1}$ and -3 ± 32 km s $^{-1}$, respectively. Likewise, from the comparison between the signals, it was estimated that their characteristic length in the direction perpendicular to their propagation was considerably bigger than the distance between satellites. Taking that the average magnetic field $\langle B \rangle_{10s}$ is frozen into the solar wind and carried by it with velocity V , where $|V| \sim 330$ km s $^{-1}$, the length of the ramp is estimated to be ~ 1500 km, which corresponds to ~ 30 ion plasma lengths ($c/\omega_{p,i} \sim 50$ km). From the raw satellite data, the variations of the magnetic field were determined on the (relatively) short and long temporal scales of $\tau = 1$ s and $\tau = 10$ s, respectively. The rapid part of the magnetic field δB_R is defined as $\delta B_R = B - \langle B \rangle_{1s}$, and the slow part δB_S as $\delta B_S = B - \langle B \rangle_{10s}$.

These data are displayed in figures 3 and 4, for each of the Cluster spacecraft. The local, time-independent, Cartesian reference frame used in these figures (the ‘solar wind frame’) is defined as follows. Its z -axis is adopted to be parallel to the magnetic field, averaged over the entire period of observation that lasted 12 s, the y -axis is taken to be perpendicular both to $\langle B \rangle_{12s}$ and to the direction of the solar wind, while

FIGURE 3. Slow variation of the magnetic field, δB_S , i.e. the deviation of the magnetic field from its 10 s average, $\delta B_S = \mathbf{B} - \langle \mathbf{B} \rangle_{10 \text{ s}}$. The x , y and z components are displayed in black, red and blue, respectively. From top to bottom: data from the Cluster spacecraft C1–C4.

FIGURE 4. Rapid variation of the magnetic field, δB_R , i.e. the deviation of the magnetic field from its 1 s average, $\delta B_R = \mathbf{B} - \langle \mathbf{B} \rangle_{1 \text{ s}}$. The x , y and z components are displayed in black, red and blue, respectively. (a–d) Data from the Cluster spacecraft C1–C4.

the x -axis is oriented along the solar wind velocity component $V_\perp$ that is perpendicular to the average magnetic field $\langle \mathbf{B} \rangle_{12 \text{ s}}$, viz.

$$\mathbf{e}_z = \langle \mathbf{B} \rangle_{12 \text{ s}} / |\langle \mathbf{B} \rangle_{12 \text{ s}}|, \quad \mathbf{e}_y = \mathbf{e}_z \times \mathbf{V} / |\mathbf{V}|, \quad \mathbf{e}_x = \mathbf{e}_y \times \mathbf{e}_z. \quad (4.1a-c)$$

We note from figures 3 and 4 that during the entire period of observations $0 \leq t \leq 12$ s, the slow magnetic field $\delta \mathbf{B}_S$ featured a very small ($\lesssim 5\%$) linear growth of its parallel component, δB_{S_z} . Conversely, its perpendicular components δB_{S_x} and δB_{S_y} exhibited broad maxima and minima with mostly flat tops and bottoms and with a rather abrupt change of sign at $t \simeq 6$ s. The slow magnetic field $\delta \mathbf{B}_S$ is mostly perpendicular to the solar wind, since $\delta B_{S_y} > \delta B_{S_x}, \delta B_{S_z}$. All three components of the rapid magnetic field $\delta \mathbf{B}_R$ exhibit similar profiles, with a relatively narrow minimum and maximum with the widths $\Delta t \sim 1$ s (i.e. ~ 330 km), and a rapid change of sign at $t \simeq 6$ s (2000 km). The magnitudes of the particular minima and maxima at various Cluster spacecraft are, to some degree, quantitatively different. We note that the perpendicular components of the slow and rapid magnetic fields are not parallel to each other, since $\delta B_{S_x}/\delta B_{S_y} \neq \delta B_{R_x}/\delta B_{R_y}$, which suggests a two-dimensional geometry of the latter. Large-scale sheared flows were not observed, but the small resolution and small sampling rate of their instruments could not exclude the existence of undetected short-scale plasma flows.

As a reasonable model for these observations, in the preceding section we assumed that the background plasma flow was asymptotically equal to zero, while the shape of the perpendicular slow magnetic field $\delta B_{S_x}, \delta B_{S_y}$ was modelled by infinitely stretching its broad maxima and minima to a $\sim \tanh$ shape, see (3.2). A qualitative agreement of our theoretical predictions, displayed in figure 1, with the observations is apparent. Notably, the components $\delta B_{R_{x1}}$ and $\delta B_{R_{y1}}$ feature mostly asymmetric bipolar signals, very similar to the satellite observations in figure 4. For larger angles θ ($\theta \gtrsim \pi/4$), the multiple maxima/minima vanish and we observe only asymmetric bipolar signals, which in a limited number of events degenerate into monopolar. However, many relevant parameters of the satellites' encounters with the structure are not known, such as the angle between the structure and the solar wind, ratio of the chain's longitudinal and perpendicular scale lengths, the relative position of the satellites to the structure's x -point, etc. Furthermore, a statistical analysis cannot be performed since only a small number of encounters have been recorded. Thus, an exact one-to-one quantitative matching between our theoretical predictions and the observational data is not possible to present here.

5. Concluding remarks

We have constructed a coherent stationary nonlinear solution of the Kadomtsev–Pogutse–Strauss equations of reduced MHD that satisfy the asymptotic boundary conditions observed by the Cluster spacecraft in the high- β plasma of the slow solar wind close to the Earth. An analytic cat's eyes-type solution is found in the form of a chain of magnetic vortices that is imbedded in a sheared magnetic field and in a localized laminar plasma flow. It possesses a significant compressional component of the magnetic field and traps the plasma fluid inside its magnetic separatrix. Such structure could have been created either in the turbulence cascade together with the current sheet in which it is imbedded, or it constitutes the signature of a past reconnection event. The apparent singularity of our solution is naturally resolved on the electron spatial scale, but it is also suppressed when a smooth nonlinear model function (3.5) is used instead of the applied simple part-by-part linear function suitable for analytic calculations. At the magnetic separatrix, the latter gives rise to a current sheet and/or a plasma jet whose widths are finite and determined by the parameter Ψ , which can be seen in (3.6). However, it should be noted that the measurements of the plasma flow from Cluster spacecraft were performed with a

relatively low resolution. Based on such relatively poor observational data available at present time, we are able to assert only the absence of significant large-scale flows associated with the magnetic structures in figures 2–4. Since, due to the low resolution, it was not possible to resolve the short-scale plasma flows, it is quite likely that the anticipated current sheet and/or plasma jet located at the separatrix of our cat's eyes solution would remain undetected.

An exact one-to-one quantitative matching between our theoretical predictions and the observational data is not possible to present here, because many relevant parameters of the satellites' encounter with the structure are not known and only a small number of encounters are recorded which does not provide sufficient data for statistical analysis. However, a striking qualitative agreement between them is apparent. Notably, the components δB_{x1} and δB_{y1} feature mostly asymmetric bipolar signals, very similar to the satellite observations in figure 4. For larger angles θ ($\theta \gtrsim \pi/4$), the multiple maxima/minima vanish and we observe only asymmetric bipolar signals, which in a limited number of events degenerate into monopolar. Their small but visible discontinuities in figure 1, occurring at the magnetic separatrix, result from our choice of a part-by-part linear function $F(\psi)$ and are readily removed by the use of a smooth model function, equation (3.5). Notwithstanding its discontinuity (more pronounced than that of δB_{x1} and δB_{y1} because it involves higher derivatives of the potential) that definitely vanishes when a smooth function $F(\psi)$ is used, our solution for δB_z features the parity and the spatial scaling that are in good agreement with the observations.

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